NAME: ______________________ _____ (   )   CLASS: 4 (   )

ANGLICAN HIGH SCHOOL
SECONDARY FOUR
PRELIMINARY EXAMINATIONS 2018
ADDITIONAL MATHEMATICS PAPER 1 [4047/01]

S4

11 September 2018 Tuesday **2 hours**

Additional Materials: 6 Writing Papers

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen on both sides of the writing paper provided.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters and glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Writing Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together and attach the question paper on top of the scripts.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

For Examiners' Use

Question	Marks	Question	Marks	Question	Marks
1		7		13	
2		8		Table of Penalties	
3		9			
4		10		Units	
5		11		Presentation	
6		12		Accuracy	
Parent's Name & Signature: Date:			Total:	80	

This paper consists of **6** printed pages.

[Turn over

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2\sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$\text{Area of } \Delta = \frac{1}{2}ab\sin C$$

Answer ALL questions

1. The product of the two positive numbers, x and y, where $x > y$, is 24. The difference between their squares is 14. Form two equations, and hence, find the exact values of the two numbers. [5]

2. Show that $\left(2+\sqrt{7}\right)^2 - \dfrac{18}{3-\sqrt{7}} = c + d\sqrt{7}$ where c and d are integers. [4]

3. **(a)** **(i)** Sketch the two curves $y = 0.5\sqrt[3]{x}$ and $y = \dfrac{8}{x}$ on the same axes for $x > 0$. [3]

(ii) Find the coordinates of the intersection point. [2]

(b) Solve the equation $2 = \left|e^{-x} - 3\right|$. [3]

4 **(i)** Given that the line $y = 2$ intersects the graph of $y = \log_{\frac{1}{5}} x$ at the point P, find the coordinates of P. [2]

(ii) Sketch the graph of $y = \log_{\frac{1}{5}} x$. [2]

(iii) State the range of values of x for which $y < 0$. [1]

5 **(i)** Sketch the graph of $y^2 = 169x$. [2]

(ii) Express $4x^2 - 181x = -9$ in the form $\left(px+q\right)^2 = 169x$, where p and q are constants. [2]

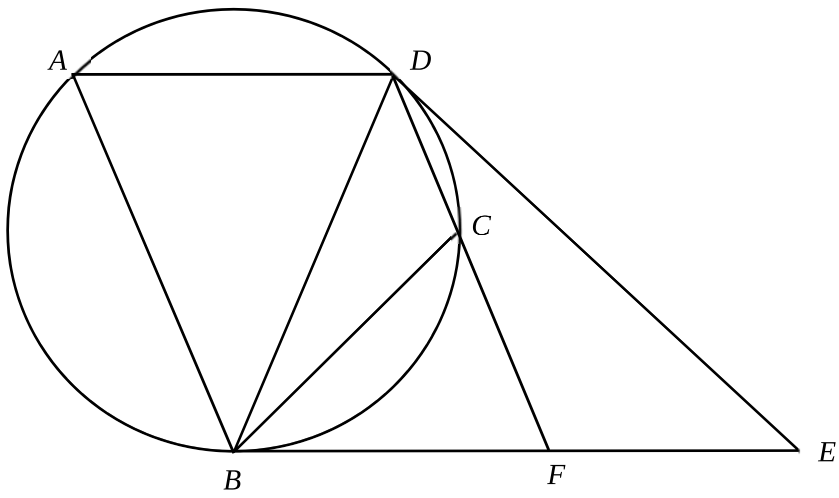
(iii) A suitable straight line can be drawn on the graph in **(i)** to solve the equation $4x^2 - 181x = -9$.

(a) State the equation of this straight line. [1]

(b) On the same axes, sketch the straight line and state the number of solutions of the equation $4x^2 - 181x = -9$. [2]

[Turn over

6.



The diagram shows a circle passing through points *A*, *B*, *C* and *D*. The tangents from *E* meets the circle at *B* and *D*. Given that $AD = BF$ and triangle *ABD* is isosceles, where $AB = BD$. Prove that

(i) *ABFD* is a parallelogram. [3]

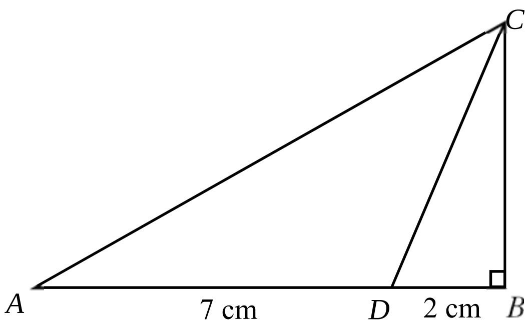
(ii) triangle *BCD* is similar to triangle *DFE*. [3]

(iii) $BD \times EF = CD \times DE$. [1]

7. (a) Sketch the graph of $y = 2\cos\left(\frac{x}{2}\right) - 1$, for the interval $0 \le x \le 2\pi$. [2]

(b) In the diagram, triangle *ABC* is a right-angle triangle, where $\angle ABC = 90°$.

D is a point of *AB* such that *AD* is 7 cm and *BD* is 2 cm.

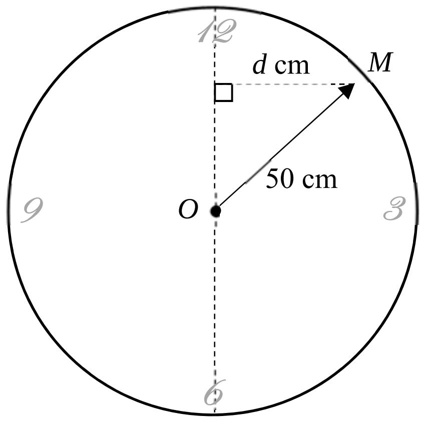


Given that $\cos\angle ADC = -\frac{1}{3}$,

(i) Find the exact length of *BC*. [1]

(ii) Find the value of $\tan\angle ACD$ in the form $a\sqrt{b}$, where *a* and *b* are integers. [2]

8.



The minute hand of a clock is 50 cm, measured from the centre of the clock, O, to the tip of the minute hand, M. The displacement, d cm, of M from the vertical line through O is given by $d = a \sin bt$, where t is the time in minutes past the hour.

(i) Find the exact value of a and of b. [3]

(ii) Find the duration, in each hour, where $|d| > 25$. [3]

9. **(i)** Prove that $\dfrac{\cos^4\theta - \sin^4\theta}{\sin^2\theta\cos^2\theta} = 4\cot 2\theta \operatorname{cosec} 2\theta$. [3]

(ii) Hence, or otherwise, solve

$$\frac{\cos^4\theta - \sin^4\theta}{\sin^2\theta\cos^2\theta} = 4\operatorname{cosec} 2\theta \text{ for } 0^\circ \le \theta \le 180^\circ.$$ [3]

10. A curve is such that $\dfrac{d^2y}{dx^2} = 6x - 2$ and $P(2, -8)$ is a point on the curve. The gradient of the normal at P is $-\dfrac{1}{2}$. Find the equation of the curve. [7]

11. Find and simplify $\dfrac{dy}{dx}$ for the following:

(i) $y = \ln \cos x$

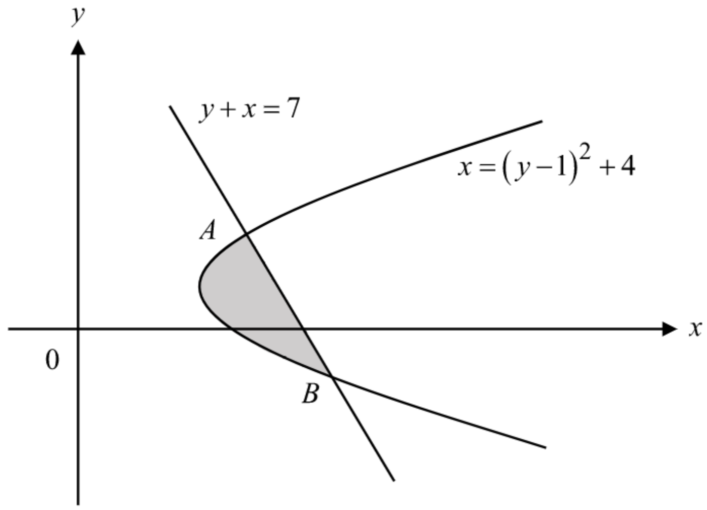
(ii) $y = e^{x^2} \times e^x$ [4]

[Turn over

12. In the diagram, the curve $x = (y-1)^2 + 4$ and the line $y + x = 7$ intersect at A and B.

(i) Find the coordinates of A and of B. [3]

(ii) Calculate the area of the shaded region. [4]



13. A particle moves in a straight line so that its acceleration, a ms^{-2}, is given by $a = 2t - 13$, where t is the time in seconds after passing a fixed point O. The particle first comes to instantaneous rest at $t = 5$ s . Find,

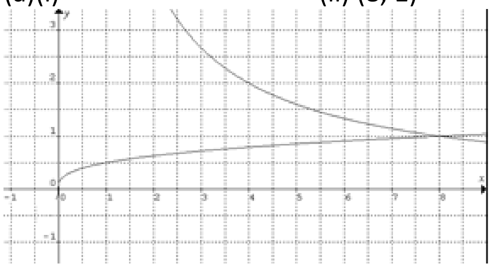
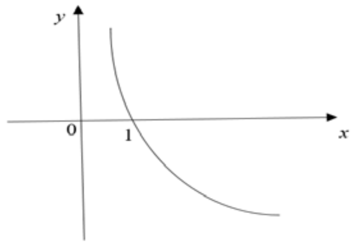
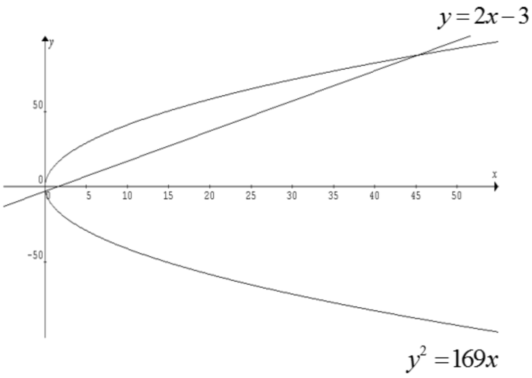
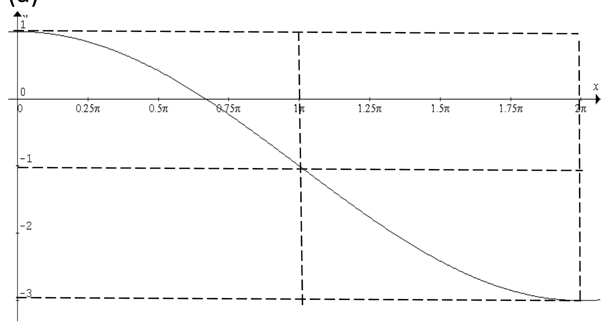
(i) the velocity when the particle passes through O. [2]

(ii) the total distance travelled by the particle when it next comes to rest. [5]

(iii) the minimum velocity of the particle. [2]

*** End of Paper ***

Answer key

1	$x = 4\sqrt{2}$ $y = 3\sqrt{2}$	2	$-16 - 5\sqrt{7}$
3	(a)(i) (ii) (8, 1)  (b) $x \approx -1.61$ or 0	4	(i) (0.04, 2) (iii) $x > 1$ (ii) 
5	(i)  (ii) $(2x-3)^2 = 169x$ (iii) (a) $y = 2x - 3$ (b) 2 solutions	7	(a)  (b) (i) $4\sqrt{2}$ cm (ii) $\frac{14\sqrt{2}}{25}$
8	(i) $a = 50$ $b = \frac{\pi}{30}$ (ii) 40 mins	9	(ii) $\theta = 22.5°$ or $112.5°$
10	$y = x^3 - 2x^2 - 6x$.	11	(i) $-\tan x$ (ii) $(2x+1)e^{x^2+x}$
12	(i) $A(5, 2)$, $B(8, -1)$ (ii) $4\frac{1}{2}$ units^2	13	(i) 40 ms^{-1} (ii) $83\frac{2}{3}m$ (iii) $-2\frac{1}{4}ms^{-1}$

2018 S4 AM Prel P1

1. The product of the two positive numbers, x and y, where $x > y$, is 24. The difference between their squares is 14. Form two equations, and hence, find the exact values of the two numbers. [5]

	Solutions	Marks
1	$xy = 24$	M1
	$y = \frac{24}{x}$ ... (1)	
	$x^2 - y^2 = 14$ ... (2)	M1
	Sub. (1) into (2):	
	$x^2 - \left(\frac{24}{x}\right)^2 = 14$	
	$x^2 - \frac{576}{x^2} = 14$	
	$x^4 - 14x^2 - 576 = 0$	M1
	$(x^2 - 32)(x^2 + 18) = 0$	
	$x^2 = 32$ or $x^2 = -18$ (rejected)	
	$x = \sqrt{32}$ $\left(-\sqrt{32}\text{ is rejected}\right)$	A1
	$= 4\sqrt{2}$	
	$\therefore\ y = \frac{24}{4\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$	A1
	$= 3\sqrt{2}$	

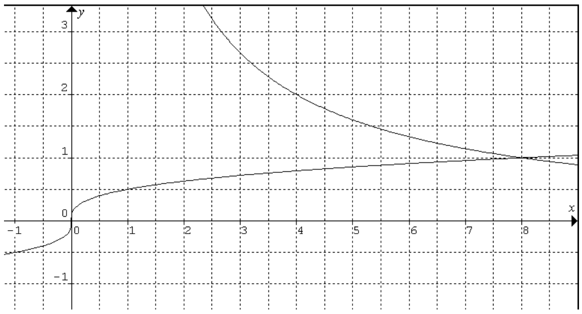
2. Show that $\left(2+\sqrt{7}\right)^2 - \frac{18}{3-\sqrt{7}} = c + d\sqrt{7}$ where c and d are integers. [4]

2	$\left(2+\sqrt{7}\right)^2 - \frac{18}{3-\sqrt{7}}$	
	$= 4 + 4\sqrt{7} + 7 - \frac{18(3+\sqrt{7})}{(3-\sqrt{7})(3+\sqrt{7})}$	M1, M1
	$= 11 + 4\sqrt{7} - \frac{54+18\sqrt{7}}{9-7}$	M1
	$= 11 + 4\sqrt{7} - \frac{1}{2}(54+18\sqrt{7})$	A1
	$= -16 - 5\sqrt{7}$	

3. (a) (i) Sketch the two curves $y = 0.5\sqrt[3]{x}$ and $y = \frac{8}{x}$ on the same axes for $x > 0$. [3]

(ii) Find the coordinates of the intersection point. [2]

(b) Solve the equation $2 = \left|e^{-x} - 3\right|$. [3]

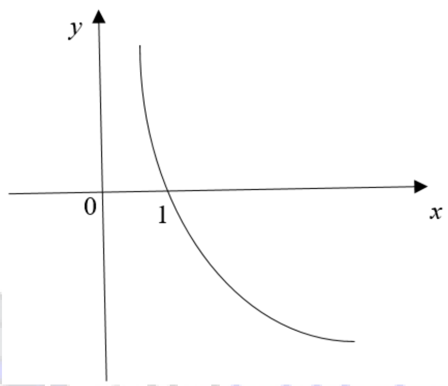
3(a) (i)		B1 B1 (for each curve) ONLY for $x > 0$ B1(label)
(ii)	$0.5\sqrt[3]{x} = \frac{8}{x}$ $x^{\frac{4}{3}} = 16$ $x = (2^4)^{\frac{3}{4}}$ $=8$ $y = \frac{8}{8} = 1$ (8, 1)	M1 A1
(b)	$2 = \left|e^{-x} - 3\right|$ $e^{-x} - 3 = 2$ or $e^{-x} - 3 = -2$ $e^{-x} = 5$ $\quad$ $e^{-x} = 1$ $e^{x} = 5^{-1}$ $\quad$ $e^{x} = 1$ $x = -\ln 5$ $\quad$ $x = \ln 1$ $= -1.6094$ $\quad$ $x = 0$ ≈ -1.61	M1 A1, A1

4 (i) Given that the line $y = 2$ intersects the graph of $y = \log_{\frac{1}{5}} x$ at the point *P*, find the coordinates of *P*. [2]

(ii) Sketch the graph of $y = \log_{\frac{1}{5}} x$. [2]

(iii) State the range of values of *x* for which $y < 0$. [1]

[Solution]

(i)	$\log_{\frac{1}{5}} x = 2$ $x = 0.2^2$ $= 0.04$ Coordinates of *P* are (0.04, 2)	**M1** **A1**
(ii)		* Shape – 1 mark * *x*-intercept – 1 mark
(iii)	$y < 0 \Rightarrow \log_{\frac{1}{5}} x < 0$ $\Rightarrow x > 1$	**B1**

5 (i) Sketch the graph of $y^2 = 169x$. [2]

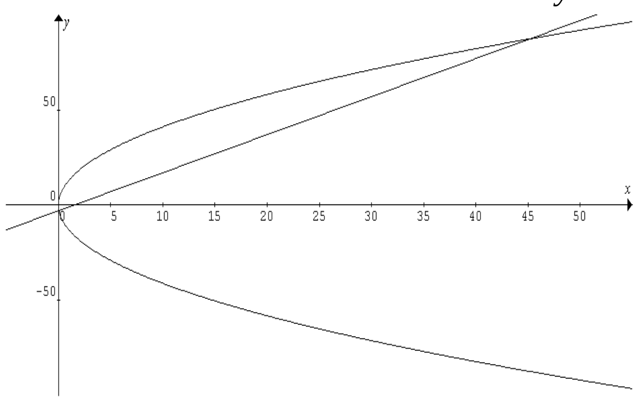
(ii) Express $4x^2 - 181x = -9$ in the form $(px + q)^2 = 169x$, where p and q are constants. [2]

(iii) A suitable straight line can be drawn on the graph in (i) to solve the equation $4x^2 - 181x = -9$.

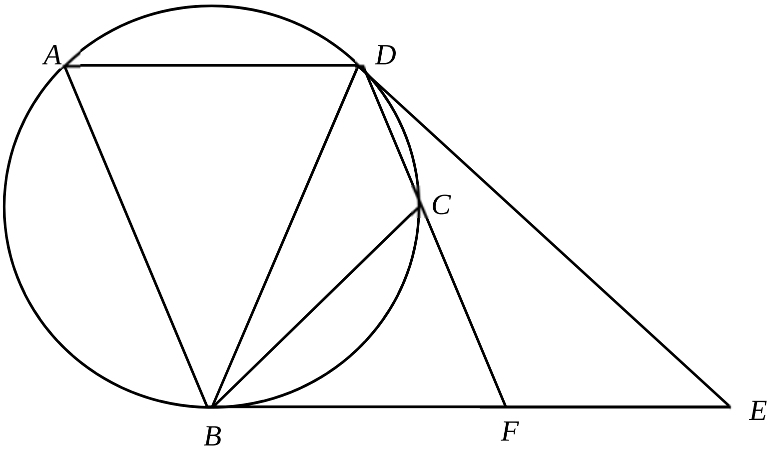
(a) State the equation of this straight line. [1]

(b) On the same axes, sketch the straight line and state the number of solutions of the equation $4x^2 - 181x = -9$. [2]

Solution

5(i)	$y = 2x - 3$  $y^2 = 169x$	* upper portion – 1 mark * lower portion – 1 mark * graph of y = 2x – 3
(ii)	$4x^2 - 181x = -9$ $4x^2 - 181x + 169x = -9 + 169x$ $4x^2 - 12x + 9 = 169x$ $(2x - 3)^2 = 169x$	**M1** **A1**
(iii)	(a) $y = 2x - 3$	**B1**
	2 solutions	**A1**

6



The diagram shows a circle passing through points *A*, *B*, *C* and *D*. The tangents from *E* meets the circle at *B* and *D*. Given that $AD = BF$ and triangle *ABD* is isosceles, where $AB = BD$. Prove that

i) *ABFD* is a parallelogram. [3]
ii) triangle *BCD* is similar to triangle *DFE*. [3]
iii) $BD \times EF = CD \times DE$. [1]

Solution

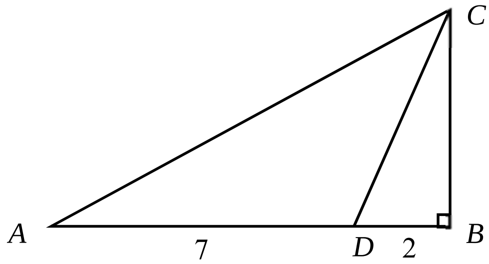
i)	$\angle DBF = \angle BAD$ (alt. seg. thm) $= \angle ADB$ (ΔABD is isosceles)	M1
	By alternate angles, $AD // BF$	M1
	Since $AD = BF$, $ABFD$ is a parallelogram.	M1
ii)	$\angle EDF = \angle DBC$ (alt. seg. thm) $\angle DFE = 180° - \angle BFD$ (adj $\angle$ on a str. line) $=180° - \angle BAD$ (opp. $\angle$ in parallelogram) $=180° - (180° - \angle DCB)$ ($\angle$ in opp. seg) $=\angle DCB$	M1 M1 M1
iii)	By AA, ΔBCD is similar to ΔDFE. $\frac{BD}{DE} = \frac{CD}{EF}$ $BD \times EF = CD \times DE$	M1

.

7. (a) Sketch the graph of $y = 2\cos\left(\frac{x}{2}\right) - 1$, for the interval $0 \le x \le 2\pi$. [2]

(b) In the diagram, triangle *ABC* is a right-angle triangle, where $\angle ABC = 90°$.

D is a point of *AB* such that *AD* is 7 cm and *BD* is 2 cm.

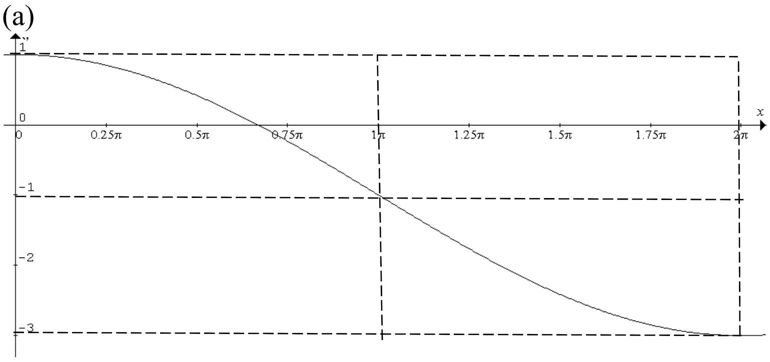


Given that $\cos \angle ADC = -\frac{1}{3}$,

(i) Find the exact length of *BC*. [1]

(ii) Find the value of $\tan \angle ACD$ in the form $a\sqrt{b}$, where *a* and *b* are integers. [2]

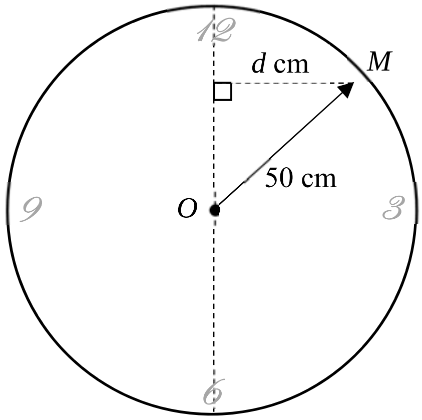
Solution

(a) 	G1: Correct shape. G1: Label all key points and axes clearly
(bi) $\cos \angle BDC = -\cos \angle ADC$ $\frac{BD}{CD} = -\left(-\frac{1}{3}\right)$ $\frac{2}{CD} = \frac{1}{3}$ $CD = 6$ cm $BC = \sqrt{6^2 - 2^2}$ $= 4\sqrt{2}$ cm	 A1
(bii)	

A1

$\tan\angle ACD = \tan(\angle ACB - \angle BCD)$ $= \dfrac{\tan\angle ACB - \tan\angle BCD}{1+(\tan\angle ACB)(\tan\angle BCD)}$ $= \dfrac{\frac{9}{4\sqrt{2}} - \frac{2}{4\sqrt{2}}}{1+\left(\frac{9}{4\sqrt{2}}\right)\left(\frac{2}{4\sqrt{2}}\right)}$ $= \dfrac{\frac{7}{4\sqrt{2}}}{\frac{25}{16}}$ $= \dfrac{14\sqrt{2}}{25}$	M1

8.



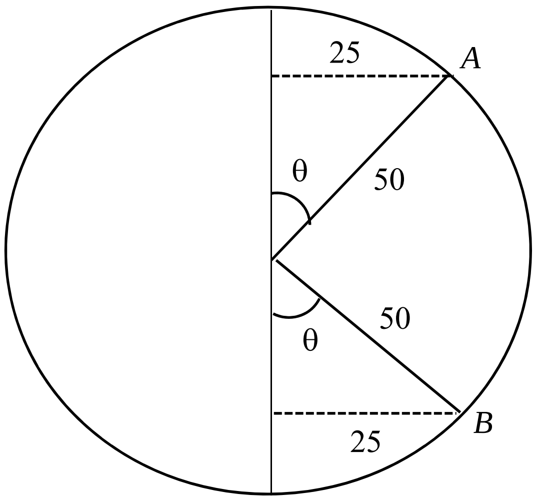
The minute hand of a clock is 50 cm, measured from the centre of the clock, *O*, to the tip of the minute hand, *M*. The displacement, *d* cm, of *M* from the vertical line through *O* is given by $d = a\sin bt$, where *t* is the time in minutes past the hour.

i) Find the exact value of *a* and of *b*. [3]

ii) Find the duration, in each hour, where $|d| > 25$. [3]

Solution

i)	$a = 50$ Period = 60 $\dfrac{2\pi}{b} = 60$ $b = \dfrac{\pi}{30}$	B1 -- for *a* M1 -- for period = 60 A1 – for *b*

ii)	$\lvert d \rvert > 25$ $\lvert\ \rvert$ when $d = 25$ $\left\lvert 50\sin\left(\frac{\pi}{30}t\right)\right\rvert = 25$ $50\sin\left(\frac{\pi}{30}t\right) = \pm 25$ $\sin\left(\frac{\pi}{30}t\right) = \pm\frac{1}{2}$ basic angle $= \sin^{-1}\left(\frac{1}{2}\right) = 30^\circ$ $\frac{\pi}{30}t = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$ $t = 5, 25, 35, 55$ For $\lvert d \rvert > 25$ Duration = (25 – 2) + (55 – 35) = 40 mins	M1 A1 A1
	Alternative Solution :  Observe that at the first instance when $d = 25$ at the point A, $\cos\theta = \frac{25}{50} = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6}$. This happened again at the point B. Between points A and B, $\lvert d \rvert > 25$. Time taken from A to B = $\frac{\pi - 2\theta}{\pi} \times 30 = \frac{\pi - \frac{\pi}{3}}{\pi} \times 30 = \frac{2}{3} \times 30 = 20$ minutes. By symmetry, the time for $\lvert d \rvert > 25$ in the other half of the clock face would be 20 minutes as well. Hence total time for $\lvert d \rvert > 25$ is 20 + 20 = 40 minutes.	

9. i) Prove that $\dfrac{\cos^4\theta-\sin^4\theta}{\sin^2\theta\cos^2\theta}=4\cot 2\theta \operatorname{cosec} 2\theta$. [3]

ii) Hence, or otherwise, solve

$$\frac{\cos^4\theta-\sin^4\theta}{\sin^2\theta\cos^2\theta}=4\operatorname{cosec}2\theta \text{ for } 0°\le\theta\le 180°.$$ [3]

Solution

i)	$LHS=\dfrac{\cos^4\theta-\sin^4\theta}{\sin^2\theta\cos^2\theta}$ $=\dfrac{(\cos^2\theta-\sin^2\theta)(\cos^2\theta+\sin^2\theta)}{(\sin\theta\cos\theta)^2}$ $=\dfrac{\cos 2\theta}{\left(\frac{1}{2}\sin 2\theta\right)^2}$ $=\dfrac{\cos 2\theta}{\frac{1}{4}\sin^2 2\theta}$ $=4\left(\dfrac{\cos 2\theta}{\sin 2\theta}\right)\left(\dfrac{1}{\sin 2\theta}\right)$ $=4\cot 2\theta \operatorname{cosec} 2\theta \quad (RHS)$	M1: factorise M1: double angle formulae M1: getting expression
ii)	$\dfrac{\cos^4\theta-\sin^4\theta}{\sin^2\theta\cos^2\theta}=4\operatorname{cosec}2\theta$ $4\cot 2\theta\operatorname{cosec}2\theta=4\operatorname{cosec}2\theta$ $4\operatorname{cosec}2\theta(\cot 2\theta-1)=0$ $\operatorname{cosec}\theta=0 \Rightarrow \dfrac{1}{\sin\theta}=0$ (no solution) OR $\cot 2\theta=1$ $\tan 2\theta=1$ basic angle $=\tan^{-1}1$ $=45°$ $0°\le\theta\le180° \Rightarrow 0°\le 2\theta\le 360°$ $2\theta=45°$ or $180°+45°$ $\theta=22.5°$ or $112.5°$	M1 M1 A1

10. A curve is such that $\dfrac{d^2y}{dx^2}=6x-2$ and $P(2,-8)$ is a point on the curve. The gradient of the normal at P is $-\dfrac{1}{2}$. Find the equation of the curve. [7]

Solution:

Given $\dfrac{d^2y}{dx^2}=6x-2$	

$\frac{dy}{dx} = \int (6x-2)dx$ $= 3x^2 - 2x + c$ Gradient of normal at (2, – 8) = $-\frac{1}{2}$ Gradient of tangent at $P = -\frac{1}{-\frac{1}{2}}$ $\frac{dy}{dx} = 2$ Sub $x = 2$, $3(2)^2 - 2(2) + c = 2$ $c = -6$ $\therefore \frac{dy}{dx} = 3x^2 - 2x - 6$ $y = \int (3x^2 - 2x - 6)dx$ $= x^3 - x^2 - 6x + c_1$ Sub (2, – 8), $-8 = (2)^3 - (2)^2 - 6(2) + c_1$ $c_1 = 0$ Hence, the equation of the curve is $y = x^3 - 2x^2 - 6x$.	M[1] – no mk if there is no 'c' B[1] –grad. of tangent at P M[1] –substitution A[1] – for 1st derivative A[1] – no mk if there is no 'c_1' M[1] –substitution A[1] - eqn

11. Find and simplify $\frac{dy}{dx}$ for the following:

(i) $y = \ln\cos x$

(ii) $y = e^{x^2} \times e^{x}$ [4]

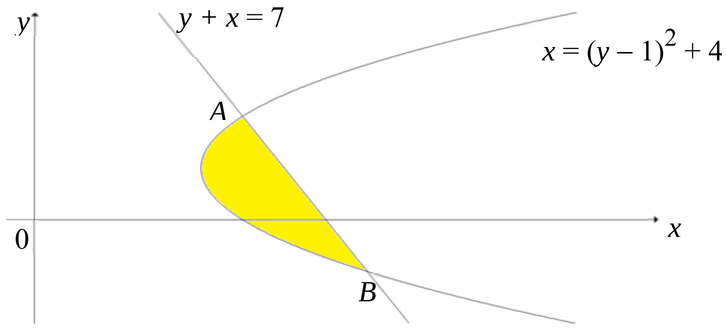
Solution:

(i)	$y = \ln\cos x$ $\frac{dy}{dx} = \frac{-\sin x}{\cos x}$ $= -\tan x$	M[1] A[1]
(ii)	$y = e^{x^2} \times e^{x}$ $y = e^{x^2+x}$ $\frac{dy}{dx} = (2x+1)e^{x^2+x}$	M[1] - Simplification A[1]
	OR $\frac{dy}{dx} = (2xe^{x^2})(e^{x}) + (e^{x^2})(e^{x})$ $= (2x+1) \times e^{x^2} e^{x}$ $= (2x+1)e^{x^2+x}$	OR M[1] A[1] - Simplification

12. In the diagram, the curve $x = (y - 1)^2 + 4$ and the line $y + x = 7$ intersect at A and B.

(i) Find the coordinates of A and of B. [3]

(ii) Calculate the area of the shaded region. [4]



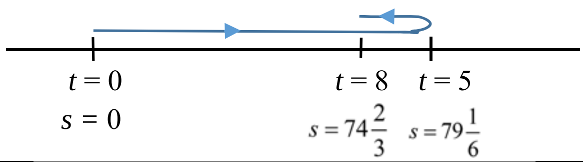
Solution:

(i) given $y + x = 7$ $y = -x + 7$ ....① sub ① into $x = (y-1)^2 + 4$ $x = (-x + 7 - 1)^2 + 4$ $= x^2 - 12x + 36 + 4$ $x^2 - 13x + 40 = 0$ $(x-5)(x-8) = 0$ $x = 5$ or $x = 8$ sub x into ①, $y = -5 + 7$ or $y = -8 + 7$ $= 2$ $= -1$ $\therefore A(5, 2)$, $B(8, -1)$	M[1] any QE $[x^2 - 13x + 40 = 0$ or $y^2 - y - 2 = 0]$ A[1] for 1st set of ans [both x or both y] A[1] ans in coordinates form
Area of shaded region $= \frac{1}{2}(2-(-1))(5+8) - \int_{-1}^{2}\left((y-1)^2+4\right)dy$ $= \frac{39}{2} - \left[\frac{(y-1)^3}{3} + 4y\right]_{-1}^{2}$ $= \frac{39}{2} - \left[\left(\frac{(2-1)^3}{3} + 4(2)\right) - \left(\frac{(-1-1)^3}{3} + 4(-1)\right)\right]$ $= 19\frac{1}{2} - 15$ $= 4\frac{1}{2}$ units^2	M[2]—1mk for each integration M[1] Substitution A[1]

13. A particle moves in a straight line so that its acceleration, a ms^{-2}, is given by $a = 2t - 13$, where t is the time in seconds after passing a fixed point O. The particle first comes to instantaneous rest at $t = 5$ s. Find,

i)	the velocity when the particle passes through O.	[2]
ii)	the total distance travelled by the particle when it next comes to rest.	[5]
iii)	the minimum velocity of the particle.	[2]

Solution

i)	$a = 2t - 13$ $v = \int 2t - 13\, dt$ $= t^2 - 13t + c$ When $t = 5$, $v = 0$. $0 = 5^2 - 13(5) + c$ $c = 40$ Velocity when passes through $O = 40$ ms^{-1}	M1 A1
ii)	$t^2 - 13t + 40 = 0$ $(t-5)(t-8) = 0$ $t = 5$ *or* $t = 8$ $v = t^2 - 13t + 40$ $s = \int (t^2 - 13t + 40)\, dt$ $= \frac{t^3}{3} - \frac{13t^2}{2} + 40t + c$ When $t = 0, c = 0$, $s = \frac{t^3}{3} - \frac{13t^2}{2} + 40t$ When $t = 5$, $s = \frac{5^3}{3} - \frac{13(5)^2}{2} + 40(5) = 79\frac{1}{6}$ When $t = 8$, $s = \frac{8^3}{3} - \frac{13(8)^2}{2} + 40(8) = 74\frac{2}{3}$ 	M1 M1 A1 M1 A1

	Total distance $= 79\frac{1}{6}+\left(79\frac{1}{6}-74\frac{2}{3}\right)$ $= 83\frac{2}{3}m$	
iii)	$a = 2t-13$ $2t-13=0$ $t=\frac{13}{2}$ $v=\left(\frac{13}{2}\right)^2-13\left(\frac{13}{2}\right)+40$ $=-2\frac{1}{4}ms^{-1}$	M1 A1

NAME: ________________________________ (   )   CLASS: 4 (   )

ANGLICAN HIGH SCHOOL
SECONDARY FOUR
PRELIMINARY EXAMINATIONS 2018
ADDITIONAL MATHEMATICS PAPER 2 [4047/02]

S4

14 September 2018   **2 hours 30 minutes**

Additional Materials: 8 Writing Papers and 1 Graph Paper

READ THESE INSTRUCTIONS FIRST
Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen on both sides of the writing paper provided.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters and glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate writing paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together and attach the question paper on top of the scripts.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

For Examiners' Use

Question	Marks	Question	Marks	Table of Penalties	
1		7			
2		8		Units	
3		9		Presentation	
4		10		Accuracy	
5		11		**Total:**	
6					
Parent's Name & Signature:					
Date:				100	

This paper consists of **6** printed pages.

[Turn over

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\cos ec^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\text{Area of } \Delta = \frac{1}{2}ab\sin C$$

Answer **all** questions.

1 **(a)** Given that the curve $y = x^2 + (3k-1)x + (2k+10)$ has a minimum value greater than 0, calculate the range of values of k. [4]

(b) Find the range of values of x for which $(x+4)(x-1)-6 \geq 0$. [2]

(c) The equation $2x^2 - x + 18 = 0$ has roots α and β. Find the quadratic equation whose roots are $\left(\frac{\alpha}{\beta}\right)^{\frac{1}{2}}$ and $\left(\frac{\beta}{\alpha}\right)^{\frac{1}{2}}$. [4]

2 **(a)** Simplify $\dfrac{25^p \times 10^{1+p}}{2^{p-1} \times 5^{2+3p}}$. [3]

(b) Given that n is a positive integer, show that $8^n + 8^{n+2} + 8^{n+4}$ is always divisible by 24. [2]

(c) Solve $2 - 2^a = 2^{a+3} - 4^{a+1}$. [4]

3 **(a)** Express $\dfrac{2x^3 - 3x - 1}{(x+3)(x-1)}$ as partial fractions. [5]

(b) The polynomial $P(x) = 2x^3 - hx^2 - 48x - 20$ leaves a remainder of 11 when divided by $x + 1$.

(i) Show that $h = 15$. [2]

(ii) Factorise $P(x)$ completely. [3]

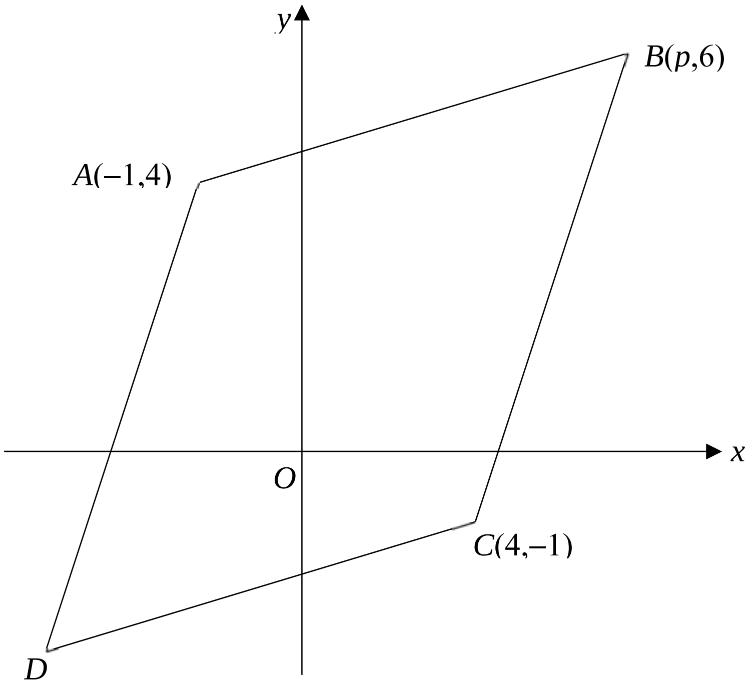
4 **(a)** **(i)** Write down, and simplify, the first 3 terms in the expansion of $(2-x)^8$ in ascending powers of x. [1]

(ii) Hence, determine the coefficient of y^2 in the expansion of $256(1-y)^8$. [3]

(b) **(i)** Write down the general term in the expansion of $\left(3x - \dfrac{1}{2x^2}\right)^{11}$. [1]

(ii) Hence, explain why the term in x^3 does not exist. [2]

5 **Solutions to this question by accurate drawing will not be accepted**.



The diagram shows a parallelogram with vertices $A(-1, 4)$, $B(p, 6)$, $C(4, -1)$ and D.

(i) Given that AC is perpendicular to BD, show that $p = 6$. [4]

(ii) Find the coordinates of D. [2]

(iii) Find the area of the parallelogram $ABCD$. [2]

6 A container in the shape of a pyramid has a volume of V cm^3, given by

$$V = \frac{1}{3}x(ax^2 + b),$$

where x is the height of the container in cm, and $(ax^2 + b)$ is the area of the rectangular base, of which a and b are unknown constants.
Corresponding values of x and V are shown in the table below.

x (cm)	5	10	15	20
V (cm^3)	150	600	1650	3600

(i) Using suitable variables, draw on graph paper, a straight line graph. [4]

(ii) Use your graph to estimate the value of a and of b. [4]

(iii) Explain how another straight line drawn on your graph can lead to an estimate of the value of x when the base area of the pyramid is three times the square of its height. Draw this line and find an estimate for the value of x. [3]

7 A circle has a diameter *AB*. The point *A* has coordinates (1, – 6) and the equation of the tangent to the circle at *B* is $3x+4y=k$.

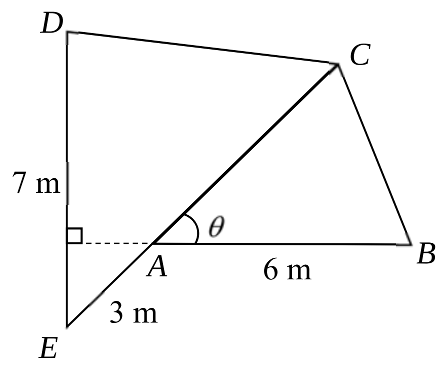
(i) Show that the equation of the normal to the circle at the point *A* is $4x-3y=22$. [3]

Given also that the line $x=-1$ touches the circle at the point (–1, –2).

(ii) Find the coordinates of the centre and the radius of the circle. [4]

(iii) Find the value of *k*. [3]

8 The diagram shows a lawn made up of two triangles, *ABC* and *CDE*. Triangle *ABC* is an isosceles triangle where *AB* = *AC* = 6 m. *DE* = 7 m, *AE* = 3 m, and *BA* produced is perpendicular to *DE*. Angle *BAC* is θ and the area of the lawn is *S* m^2.



(i) Show that $S=18\sin\theta+31.5\cos\theta$. [3]

(ii) Hence, express *S* as a single trigonometric term. [4]

(iii) Given that θ can vary, find the maximum area of the lawn and the corresponding value of θ. [2]

9 A curve has the equation $y=(1-x)\sqrt{1+2x}$.

(i) Find $\frac{dy}{dx}$ in its simplest form. [3]

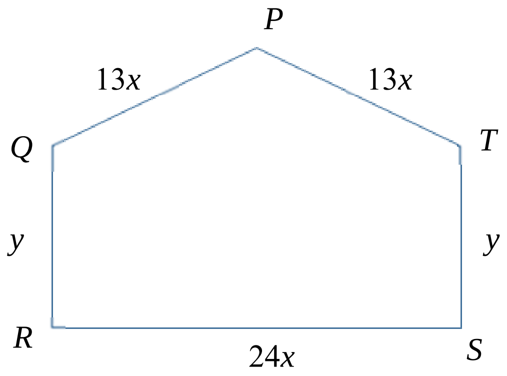
Hence,

(ii) determine the interval where *y* is increasing, [3]

(iii) find the rate of change of *x* when *x* = 4, given that *y* is decreasing at a constant rate of 2 units per second, [2]

(iv) evaluate $\int_1^4 \frac{x}{\sqrt{1+2x}}\,dx$. [2]

10 A piece of wire of length 180 cm is bent into the shape *PQRST* shown in the diagram.



Show that the area, A cm^2, enclosed by the wire is given by

$$A = 2160 - 540x^2.$$

Find the value of x and of y for which A is a maximum. [8]

11 **(a)** Find the following indefinite integrals.

(i) $\int \frac{e^{2x}}{2} dx$

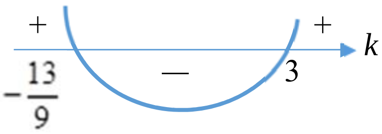
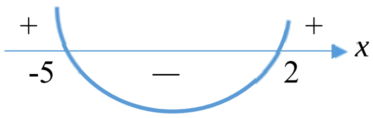
(ii) $\int \left(\frac{4}{x} + \frac{1}{x^2} \right) dx$ [3]

(b) Evaluate $\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \frac{1}{2\operatorname{cosec}^2 x} dx$, leaving your answer in terms of π. [5]

END OF PAPER.

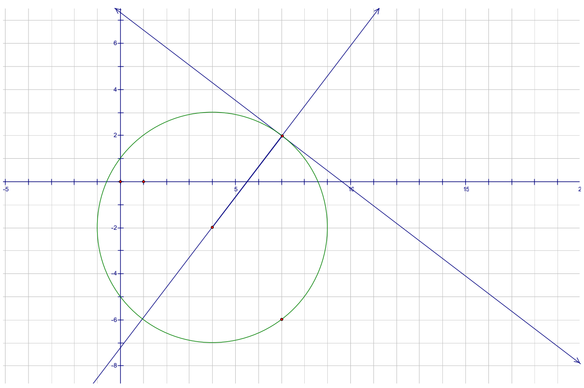
ANSWER KEY

1	(a) $-\frac{13}{9} < k < 3$ (b) $x \le -5$ or $x \ge 2$ (c) $x^2 - \frac{1}{6}x + 1 = 0$ or $6x^2 - x + 6 = 0$	2	(a) $\frac{4}{5}$ (b) $24 \times 1387 \times 8^{n-1}$ Since $n \ge 1$, $8^{n-1} \ge 1$, hence $8^n + 8^{n+2} + 8^{n+4}$ is divisible by 24. (c) $a = -2$ or $a = 1$
3	(a) $2x - 4 + \frac{23}{2(x+3)} - \frac{1}{2(x-1)}$ (b)(ii) $(x+2)(2x+1)(x-10)$		
4	(a)(i) $256 - 1024x + 1792x^2 + \ldots$ (ii) coefficient of $y^2 = 7168$ (b)(i) $T_{r+1} = \binom{11}{r}(3x)^{11-r}\left(-\frac{1}{2x^2}\right)^r$ (ii) $r = \frac{8}{3}$. As r is a not a whole number, the term in x^3 does not exist.	5	(ii) $(-3, -3)$ (iii) 45 units^2
		7	(ii) centre is $(4, -2)$ radius = 5 units (iii) $k = 29$
		8	(ii) $36.3\sin(\theta + 60.3°)$ (iii) Max $S \approx 36.3$ m^2 $\theta \approx 29.7°$
9	(i) $\frac{dy}{dx} = -\frac{3x}{\sqrt{1+2x}}$ (ii) y in increasing when $-0.5 < x < 0$. (iii) $\frac{dx}{dt} = \frac{1}{2}$ units/sec (iv) 3	10	$x = 2$ cm and $y = 40$ cm when A is a maximum.
11	(a) (i) $\int \frac{e^{2x}}{2} dx = \frac{e^{2x}}{4} + c$ (ii) $\int \left(\frac{4}{x} + \frac{1}{x^2}\right) dx = 4\ln x - \frac{1}{x} + c$	11	(b) $\frac{\pi}{12} - \frac{\sqrt{3}}{8}$ or $\frac{2\pi - 3\sqrt{3}}{24}$

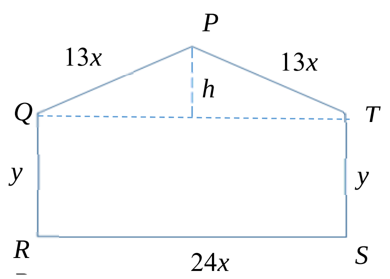
	Solutions
1(a)	$x^2+(3k-1)x+(2k+10)>0$ $b^2-4ac<0$ $(3k-1)^2-4(1)(2k+10)<0\ 9k^2-6k+1-8k-40<0$ $9k^2-14k-39<0$ $(9k+13)(k-3)<0$  $-\frac{13}{9}<k<3$
(b)	$(x+4)(x-1)-6\geq 0$ $x^2+3x-4-6\geq 0$ $x^2+3x-10\geq 0$ $(x+5)(x-2)\geq 0$  $x\leq -5$ or $x\geq 2$
(c)	$2x^2-x+18=0$ $\alpha+\beta=\frac{1}{2}$ $\alpha\beta=9$ $\left(\frac{\alpha}{\beta}\right)^{\frac{1}{2}}+\left(\frac{\beta}{\alpha}\right)^{\frac{1}{2}}=\frac{\alpha+\beta}{\alpha^{\frac{1}{2}}\beta^{\frac{1}{2}}}$ $=\frac{(\alpha+\beta)}{(\alpha\beta)^{\frac{1}{2}}}$ $=\frac{\frac{1}{2}}{9^{\frac{1}{2}}}$ $=\frac{1}{6}$ $\left(\frac{\alpha}{\beta}\right)^{\frac{1}{2}}\times\left(\frac{\beta}{\alpha}\right)^{\frac{1}{2}}=\left(\frac{\alpha\beta}{\beta\alpha}\right)^{\frac{1}{2}}$ $=1$ Required equation is $x^2-\frac{1}{6}x+1=0$ or $6x^2-x+6=0$

2(a)	$\frac{25^p \times 10^{1+p}}{2^{p-1} \times 5^{2+3p}} = \frac{5^{2p} \times (2\times 5)^{1+p}}{2^{p-1} \times 5^{2+3p}}$ $= \frac{5^{2p} \times 2^{1+p} \times 5^{1+p}}{2^{p-1} \times 5^{2+3p}}$ $= 2^{1+p-(p-1)} \times 5^{2p+1+p-(2+3p)}$ $= 2^2 \times 5^{-1}$ $= \frac{4}{5}$
(b)	$8^n + 8^{n+2} + 8^{n+4} = 8^n + 8^n \times 8^2 + 8^n \times 8^4$ $= 8^n(1+64+4096)$ $= 8^n(4161)$ $= 8^1 \times 8^{n-1} \times 3 \times 1387$ $= 24 \times 1387 \times 8^{n-1}$ Since $n \geq 1, 8^{n-1} \geq 1$ and $24 \times 1387 \times 8^{n-1}$ is divisible by 24. o.e.
(c)	$2 - 2^a = 2^{a+3} - 4^{a+1}$ $2 - 2^a = 2^3(2^a) - 2^{2(a+1)}$ $2 - 2^a = 8(2^a) - 2^2(2^{2a})$ $2 - 2^a = 8(2^a) - 4(2^a)^2$ Let u be 2^a. $2 - u = 8u - 4u^2$ $4u^2 - 9u + 2 = 0$ $(4u-1)(u-2) = 0$ $u = \frac{1}{4}$ or $u = 2$ $2^a = 2^{-2}$ or $2^a = 2$ $a = -2$ or $a = 1$
3(a)	$\frac{2x^3 - 3x - 1}{(x+3)(x-1)} = \frac{2x^3 - 3x - 1}{x^2 + 2x - 3}$ Long division of $2x^3 - 0x^2 - 3x - 1$ by $x^2 + 2x - 3$, quotient $2x - 4$: $-(2x^3 + 4x^2 - 6x)$ $-4x^2 + 3x - 1$ $-(-4x^2 - 8x + 12)$ $11x - 13$ $\frac{2x^3 - 3x - 1}{(x+3)(x-1)} = 2x - 4 + \frac{11x - 13}{(x+3)(x-1)}$

	$\frac{11x-13}{(x+3)(x-1)}=\frac{P}{x+3}+\frac{Q}{x-1}$ $=\frac{P(x-1)+Q(x+3)}{(x+3)(x-1)}$ $\Rightarrow\quad 11x-13=Px-P+Qx+3Q$ $=(P+Q)x+(-P+3Q)$ $P+Q=11 \quad \ldots (1)$ $-P+3Q=-13 \quad \ldots (2)$ (1)+(2): $4Q=-2 \quad \Rightarrow Q=-\frac{1}{2}$ $\therefore\quad P-\frac{1}{2}=11 \quad \Rightarrow\quad P=\frac{23}{2}$ $\frac{2x^3-3x-1}{(x+3)(x-1)}=2x-4+\frac{23}{2(x+3)}-\frac{1}{2(x-1)}$
(b) (i)	$P(x)=2x^3-hx^2-48x-20$ $P(-1)=11$ $2(-1)^3-h(-1)^2-48(-1)-20=11$ $-2-h+48-20=11$ $h=15$ (shown)
(ii)	$P(x)=2x^3-15x^2-48x-20$ By trial and error, $x+2$ is a factor. $2x^3-15x^2-48x-20$ $=(x+2)(ax^2+bx+c)$ $=ax^3+bx^2+cx+2ax^2+2bx+2c$ $=ax^3+(b+2a)x^2+(c+2b)x+2c$ By comparing coefficients of x^3: $a=2$ x^2: $b+2(2)=-15$ $b=-19$ constant: $2c=-20$ $c=-10$ $\therefore P(x)=(x+2)(2x^2-19x-10)$ $=(x+2)(2x+1)(x-10)$
4(a) (i)	$(2-x)^8=2^8+\binom{8}{1}2^7(-x)+\binom{8}{2}2^6(-x)^2+\ldots$ $=256-1024x+1792x^2+\ldots$
(ii)	$256(1-y)^8=2^8(1-y)^8$ $=[2(1-\text{y})]^8$ $=(2-2\text{y})^8$ Taking $x=2y$, $(2-2y)^8=256-1024(2y)+1792(2y)^2+\ldots$ Hence, coefficient of $y^2=1792\times 2^2=7168$.

(b) (i)	$T_{r+1} = \binom{11}{r}(3x)^{11-r}\left(-\frac{1}{2x^2}\right)^r$
(ii)	For term in x^3, $11-r-2r=3$ $3r=8 \Rightarrow r=\frac{8}{3}$ As r is a not a whole number, the term in x^3 does not exist. o.e.
5 (i)	Grad $AC = \frac{4-(-1)}{-1-4} = -1$ Mid-point of $AC = \left(\frac{-1+4}{2}, \frac{4-1}{2}\right) = \left(\frac{3}{2}, \frac{3}{2}\right)$ As BD and AC share the same mid-point (property of parallelogram), gradient of $BD = \frac{6-\frac{3}{2}}{p-\frac{3}{2}} = \frac{9}{2p-3}$ $\left(\frac{9}{2p-3}\right)(-1) = -1$ $2p-3=9 \Rightarrow 2p=12 \Rightarrow p=6$ (shown)
(ii)	Let D be (a, b). $\left(\frac{a+6}{2}, \frac{b+6}{2}\right) = \left(\frac{3}{2}, \frac{3}{2}\right)$ Comparing coordinates, $\frac{a+6}{2} = \frac{3}{2}$ $a+6=3 \Rightarrow a=-3$. Similarly, $b=-3$. Therefore, coordinates of D are $(-3, -3)$.
(iii)	Area of the parallelogram $ABCD$ $=\frac{1}{2}\begin{vmatrix} -3 & 4 & 6 & -1 & -3 \\ -3 & -1 & 6 & 4 & -3 \end{vmatrix}$ $=\frac{1}{2}\{[(-3)(-1)+4(6)+6(4)+(-1)(-3)]$ $-[4(-3)+6(-1)+(-1)(6)+(-3)(4)]\}$ $= 45$ units^2
7 (i)	 The normal to the circle at point A will pass through the centre of the circle, and point B also, and is perpendicular to the tangent to the circle at B. $3x+4y=k \Rightarrow y=-\frac{3}{4}x+\frac{k}{4}$ Grad of tangent at $B = -\frac{3}{4}$ Grad of normal at $A = \frac{4}{3}$ Equation of normal at A: $y-(-6) = \frac{4}{3}(x-1)$ $y = \frac{4}{3}x - \frac{22}{3}$ $\Rightarrow 4x-3y=22$. (shown)
(ii)	Since the line $x=-1$ touches the circle at the point $(-1, -2)$, so the equation of the normal at $(-1, -2)$ is $y=-2$. Solving the equations $4x-3y=22$ and

	$y = -2$, $4x - 3(-2) = 22 \Rightarrow 4x = 16 \Rightarrow x = 4$. Thus the centre is (4, –2). Radius $= \sqrt{(4-1)^2 + [(-2)-(-6)]^2}$ $= \sqrt{9+16}$ = 5 units
(iii)	Let the coordinates of B be (p, q). $\left(\frac{p+1}{2}, \frac{q-6}{2}\right) = (4, -2)$ $p = 2(4) - 1 = 7$ and $q = 2(-2) + 6 = 2$ Therefore, B is (7, 2). Sub. (7, 2) into $3x + 4y = k$, $k = 3(7) + 4(2) = 29$
8 (i)	Area $\Delta ABC = \frac{1}{2}(6)^2 \sin\theta$ $= 18\sin\theta$ Area $\Delta CDE = \frac{1}{2}(7\times 9)\sin(90° - \theta)$ $= 31.5\cos\theta$ $S = 18\sin\theta + 31.5\cos\theta$ (shown)
(ii)	$S = 18\sin\theta + 31.5\cos\theta$ $R = \sqrt{18^2 + 31.5^2}$ $= 36.28016$ $\tan\alpha = \frac{31.5}{18}$ $\alpha = \tan^{-1}\left(\frac{31.5}{18}\right)$ $= 60.2551187°$ $S = 36.28016\sin(\theta + 60.2551187°)$ $\approx 36.3\sin(\theta + 60.3°)$
(iii)	$S = 36.28016\sin(\theta + 60.2551187°)$ Max $S \approx 36.3$ m^2 $\sin(\theta + 60.2551187°) = 1$ $0° < \theta < 90°$ $60.2551187° < \theta + 60.2551187° < 150.2551187°$ $\theta + 60.2551187° = 90°$ $\theta = 29.7448813°$ $\approx 29.7°$

9 (i)	$y=(1-x)\sqrt{1+2x}$ $\frac{dy}{dx}=(1-x)\left(\frac{1}{2}\right)(1+2x)^{-\frac{1}{2}}(2)+(1+2x)^{\frac{1}{2}}(-1)$ $=(1+2x)^{-\frac{1}{2}}(1-x-1-2x)$ $=-\frac{3x}{\sqrt{1+2x}}$
(ii)	For $\frac{dy}{dx}>0$, $-\frac{3x}{\sqrt{1+2x}}>0$ $\Rightarrow$ $1+2x>0$ and $-3x>0$ $x>-0.5$ $x<0$ $\therefore$ y in increasing when $-0.5<x<0$.
(iii)	$\frac{dy}{dt}=\frac{dy}{dx}\times\frac{dx}{dt}$ When $x=4$, $\frac{dy}{dt}=-2$, $-2=-\frac{3(4)}{\sqrt{1+2(4)}}\times\frac{dx}{dt}$ $\frac{dx}{dt}=\frac{1}{2}$ units/sec
(iv)	$\int_1^4 \frac{x}{\sqrt{1+2x}}dx$ $=\left[-\frac{1}{3}(1-x)\sqrt{1+2x}\right]_1^4$ $=\left(-\frac{1}{3}(1-(4))\sqrt{1+2(4)}\right)$ $-\left(-\frac{1}{3}(1-1)\sqrt{1+2(1)}\right)$ $=3$
10	$13x+13x+y+24x+y=180$ $50x+2y=180$ $y=90-25x$ 

	Let h cm be the perpendicular distance from P to QT. $h^2 = (13x)^2 - (\frac{24x}{2})^2$ $= 25x^2$ $h = 5x$ Area $= y(24x) + \frac{1}{2}(24x)(5x)$ $A = (90 - 25x)(24x) + 60x^2$ $= 2160x - 600x^2 + 60x^2$ $= 2160x - 540x^2$ (shown) $\frac{dA}{dx} = 2160 - 1080x$ When $\frac{dA}{dx} = 0$, $2160 - 1080x = 0$ $x = 2160 \div 1080$ $= 2$ Sub $x = 2$, into $y = 90 - 25x$ $y = 90 - 25(2)$ $= 40$ $\frac{d^2A}{dx^2} = -1080$, $\therefore$ A is a maximum. $x = 2$ cm and $y = 40$ cm when A is a maximum.
11(a)(i)	$\int \frac{e^{2x}}{2} dx = \frac{e^{2x}}{4} + c$
(ii)	$\int \left(\frac{4}{x} + \frac{1}{x^2} \right) dx = 4 \ln x - \frac{1}{x} + c$

(b)	$\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \frac{1}{2\operatorname{cosec}^2 x} dx$ $= \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \frac{\sin^2 x}{2} dx$ $= \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \frac{1}{2} \times \frac{1}{2}(1-\cos 2x)\ dx$ $= \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \frac{1}{4}(1-\cos 2x)\ dx$ $= \left[\frac{1}{4}x - \frac{1}{8}\sin 2x\right]_{-\frac{\pi}{6}}^{\frac{\pi}{6}}$ $= \left(\frac{1}{4}\left(\frac{\pi}{6}\right) - \frac{1}{8}\sin 2\left(\frac{\pi}{6}\right)\right) -$ $\left(\frac{1}{4}\left(-\frac{\pi}{6}\right) - \frac{1}{8}\sin 2\left(-\frac{\pi}{6}\right)\right)$ $= \frac{\pi}{24} - \frac{\sqrt{3}}{16} + \frac{\pi}{24} - \frac{\sqrt{3}}{16}$ $= \frac{\pi}{12} - \frac{\sqrt{3}}{8}$ or $\frac{2\pi - 3\sqrt{3}}{24}$

BUKIT PANJANG GOVERNMENT HIGH SCHOOL

Preliminary Examination 2018

SECONDARY 4 EXPRESS/ 5 NORMAL

ADDITIONAL MATHEMATICS — 4047/1

Paper 1

Date: 3 August, 2018
Duration: 2 h
Time**: 1030 – 1230**

Additional Materials: *Answer Paper*

READ THESE INSTRUCTIONS FIRST

Write your class, register number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 80.

Setter: Mr Choo Kong Lum **[Turn over]**

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \cdots + \binom{n}{r}a^{n-r}b^r + \cdots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = cosAcosB \mp \sin AsinB$$

$$\tan(A \pm B) = \frac{tanA \pm B}{1 \mp tanAtanB}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \tfrac{1}{2} ab \sin C$$

Answer* ALL *the questions.

1(a) Given that $(\sqrt{3}+1)x = \sqrt{3}-1$, find the value of $x + \frac{1}{x}$ without using a calculator. [4]

1(b) Given that $2\sqrt{2} - 3 = \frac{\sqrt{h-k\sqrt{2}}}{1+\sqrt{2}}$, find the values of *h* and *k*. [3]

2(a) Show that for all real values of *p* and of *q*, $y = -(1+p^2)x^2 + 2pqx - (2q^2+1)$ is always negative for all real values of *x*. [4]

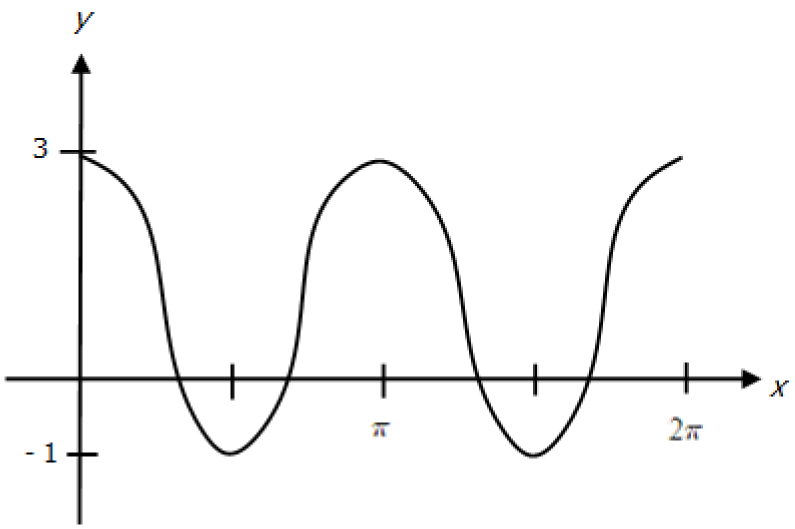
2(b) Find the range of values of *m* for which $\frac{-4}{m^2+3m+2} < 0$ [2]

3(a) (i) For the function $y = \sin x$, where $-1 \le y \le 1$, state the principal values of *x*, in radians. [1]

(ii) For the function $y = \cos x$, where $-1 \le y \le 1$, state the principal values of *x*, in radians. [1]

(iii) For the function $y = \tan x$, state the principal values of *x*, in radians. [1]

3(b) The diagram shows part of the graph for the function $y = a\cos bx + c$.



(i) Find the values of *a*, *b* and *c*. [3]

(ii) Copy the diagram and draw the line $y = \frac{x}{\pi} - 1$ on the same diagram. Hence state the number of solutions when $a\cos bx + c = \frac{x}{\pi} - 1$. [2]

4. (i) Sketch the graph of $y = x^{\frac{2}{3}}$ for $x \geq 0$. [1]

(ii) Find the equation of the line that must be inserted in the graph above in order to solve the equation $3x^{\frac{2}{3}} + 9x = 6$. [2]

5. Express $\frac{4x^5+2x^4+3x^3-x^2-x+1}{x^3+x}$ in partial fractions. [6]

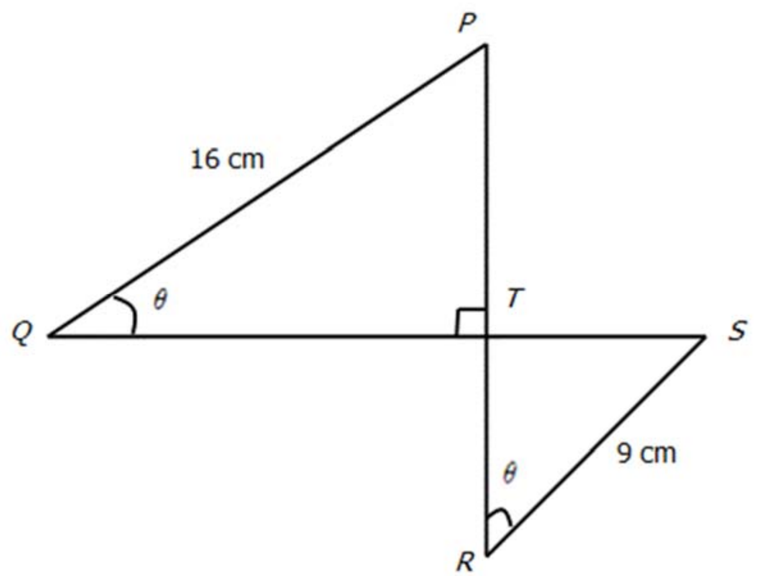
6. (i) Sketch the graphs of $y = |x-2| + 1$ and $y = x^2 + 3$ on the same diagram. For each graph, indicate the coordinates of the minimum point on the diagram. [4]

(ii) Find the coordinates of the point of intersection. [4]

7(a) Given that $y = \ln\sqrt{\frac{3x+1}{3x-1}}$, find an expression for $\frac{dy}{dx}$ and simplify your answer as a single fraction. [3]

7(b) Given that $y = 2e^{x^2+3}$, find the coordinates of the stationary point, leaving your answer in exact form. Determine the nature of the stationary point. [5]

8. The diagram shows two lines *PR* and *QS* which are perpendicular to each other. *RS* = 9 cm, *PQ* = 16 cm and $\angle PQT = \angle SRT = \theta$ radians.



(i) Show that $QS = 16cos\theta + 9sin\theta$. [1]

(ii) Express *QS* in the form of $Rsin(\theta + \alpha)$. [3]

(iii) Find the value of θ for which *QS* = 12 cm. [3]

(iv) Show that the area of the quadrilateral *PQRS* is $\frac{288+337sin2\theta}{4}$ cm^2 [4]

9. (i) Differentiate $(x-5)\sqrt{2x-1}$ with respect to *x* and simplify your answer as a single fraction. [2]

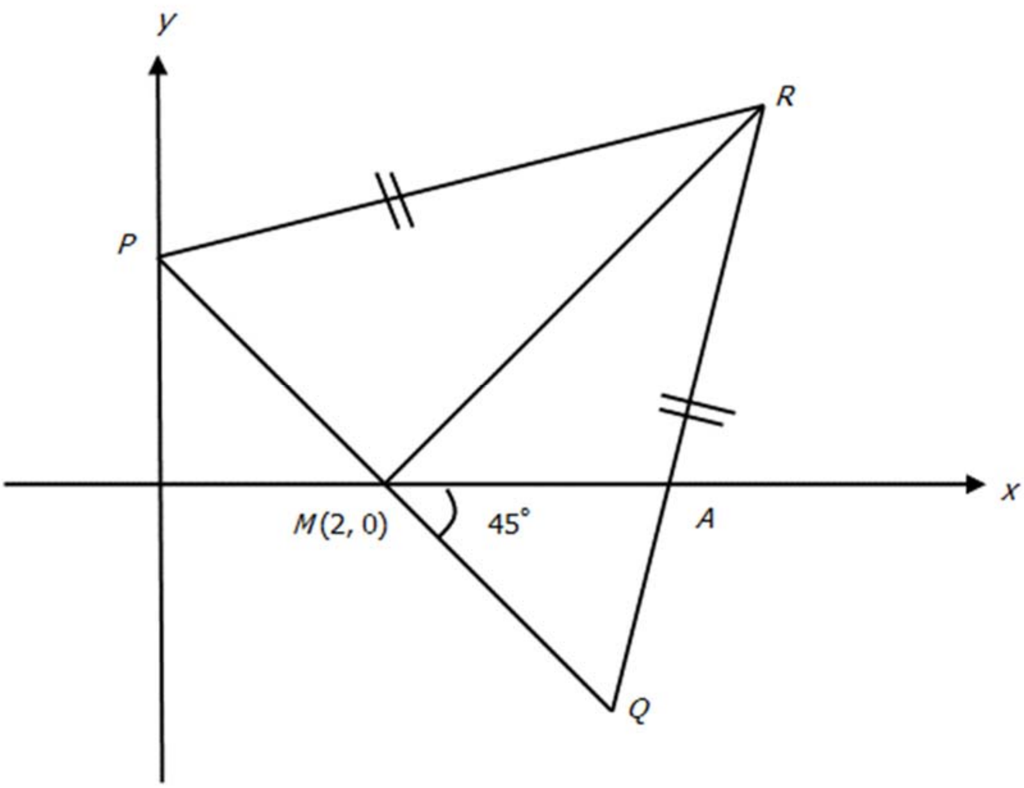
(ii) Hence evaluate $\int_1^2 \frac{3x-9}{\sqrt{2x-1}}\,dx$, leaving your answer in exact form. [4]

10. (i) Given that $\frac{dy}{dx}=\frac{5}{1+\cos 2x}$. Find the equation of the curve if the curve passes through the *y* – axis at $y=1$. [4]

(ii) Find the equation of the normal to the curve at $x=\frac{\pi}{4}$. [3]

11. **Solutions to this question by accurate drawing will not be accepted.**

The following diagram shows an isosceles triangle *PQR*, where *PR* = *QR*. It is given that *M* (2, 0) is the midpoint of *PQ*. The line *QR* intersects the *x* - axis at point *A* such that $\angle AMQ = 45°$.



(i) Show that the gradient of the line *MR* is 1. [1]

(ii) Find the equation of the line *PQ*. [2]

(iii) Find the coordinates of *Q*. [2]

(iv) Given that the area of ΔPQR is 20 units², find the coordinates of *R*. [5]

END OF PAPER

ANSWERS (SEC 4 EXP / 5 NA AM PAPER 1 – PRELIM 2018)

1(a)	4
1(b)	$h = 3,\ k = 2$
2(b)	$m < -2$ or $m > -1$
3(a)(i)	$-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$
3(a)(ii)	$0 \leq x \leq \pi$
3(a)(iii)	$-\frac{\pi}{2} < x < \frac{\pi}{2}$
3(b)(i)	$a = 2,\ \ b = 2,\ \ c = 1$
3(b)(ii)	4
4(ii)	$y = -3x + 2$
5.	$4x^2 + 2x - 1 + \frac{1}{x} - \frac{4x}{x^2+1}$
6(ii)	(2, 1) and (0, 3)
6(iii)	(0, 3) and (-1, 4)
7(a)	$\frac{-3}{(3x+1)(3x-1)}$ or $\frac{3}{(1+3x)(1-3x)}$
7(b)	$(0,\ 2e^3)$ minimum point
8(ii)	$\sqrt{337}\sin(\theta + 1.06)$ or $18.4\sin(\theta + 1.06)$
8(iii)	1.37 radians
9(i)	$\frac{3x-6}{\sqrt{2x-1}}$
9(ii)	$7 - 6\sqrt{3}$
10(i)	$y = \frac{5}{2}\tan x + 1$
10(ii)	$y = -\frac{1}{5}x + \frac{\pi}{20} + \frac{7}{2}$
11(ii)	$y = -x + 2$
11(iii)	(4, - 2)
11(iv)	(7, 5)

Name : ____( ) Class : __________

BUKIT PANJANG GOVERNMENT HIGH SCHOOL

Preliminary Examinations 2018

SECONDARY FOUR EXPRESS/FIVE NORMAL

ADDITIONAL MATHEMATICS

4047/02

Paper 2

Date: 13 August, 2018
Duration: 2 hours 30 min
Time: 07 45 – 10 15

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 100.

Setter: Mrs Chiu H W **[Turn over]**

This paper consists of **5** printed pages.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \cdots + \binom{n}{r} a^{n-r} b^r + \cdots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

1 Expand $(1 + a\,x)^4(1 - 4x)^3$ in ascending powers of x up to and including the term containing x^2. [4]

Given that the first two terms in the above expansion are $p + qx^2$, where p and q are constants, find the value of p and of q. [3]

2 (i) Given that $u = 4^x$, express $4^x - 3(4^{1-x}) = 11$ as an equation in u. [2]

(ii) Hence find the value(s) of x for which $4^x - 3(4^{1-x}) = 11$. [4]

(iii) Given that $p > 0$, determine the number of real roots in the equation $4^x - 3(4^{1-x}) = p$. **Show your working clearly.** [3]

3 (i) Show that $\frac{1}{\operatorname{cosec} x - 1} - \frac{1}{\operatorname{cosec} x + 1} = 2\tan^2 x$. [3]

(ii) Hence solve $\frac{1}{\operatorname{cosec} x - 1} - \frac{1}{\operatorname{cosec} x + 1} = 4 + \sec x$ for $0° < x < 360°$. [4]

4 A curve has the equation $y = \frac{2x - 7}{x - 1} - 20x$.

(i) Obtain an expression for $\frac{dy}{dx}$. [3]

(ii) Determine the values of x for which y is a decreasing function. [3]

The variables are such that, when $x = 2$, y is decreasing at the rate of 1.5 units per second.

(iii) Find the rate of change of x when $x = 2$. [2]

It is given further that the variable z is such that $z = \frac{2}{y}$.

(iv) Find the rate of change of z when $x = 2$. [3]

5 It is given that $f(x) = (kx + 1)(x^2 - 3x + k)$.

(a) (i) Find the value(s) of k if $3 - x$ is a factor of $f(x)$. [2]

(ii) For the values(s) of k found in (i), write down an expression for $f(x)$ with $(3 - x)$ as a factor. [2]

(b) Find the smallest integer value of k such that there is only one real solution for $(kx + 1)(x^2 - 3x + k) = 0$. [3]

6 The table below shows values of the variables x and y which are related by the equation $ay = x(1 - bx)$ where a and b are constants. One of the values of y is believed to be inaccurate.

x	2	3.5	4.5	6	7
y	5.0	9.1	14.0	21.0	26.3

(i) Plot $\frac{y}{x}$ against x and draw a straight line graph. [3]

(ii) Determine which value of y is inaccurate and estimate its correct value. [2]

(iii) Estimate the value of a and b. [4]

An alternative method for obtaining straight line graph for the equation $ay = x(1 - bx)$ is to plot x on the vertical axis and $\frac{y}{x}$ on the horizontal axis.

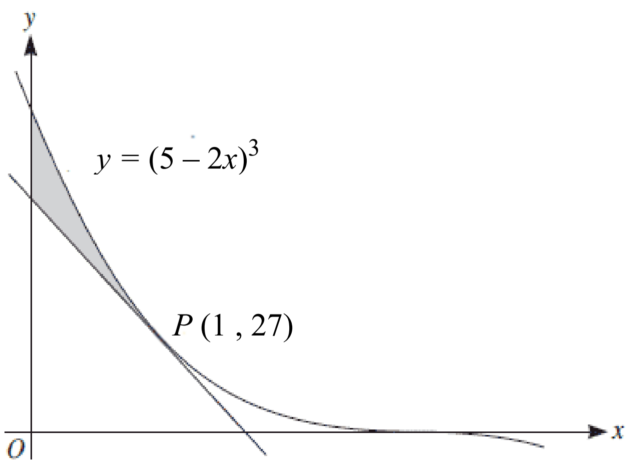
(iv) Without drawing a second graph, use your values of a and b to estimate the gradient and intercept on the vertical axis of the graph of x plotted against $\frac{y}{x}$. [3]

7 The roots of the quadratic equation $x^2 - 4x + 2 = 0$ are α and β.

(i) Find the exact value of $\alpha - \beta$ if $\alpha < \beta$. [4]

(ii) Form a quadratic equation with roots $\frac{\alpha - 1}{\beta}$ and $\frac{\beta - 1}{\alpha}$. [5]

8



The diagram shows the curve $y = (5 - 2x)^3$ and the tangent to the curve at the point $P(1, 27)$.

(i) Find the equation of the tangent to the curve at P. [4]

(ii) Find the area of the shaded region. [5]

9 A particle moves in a straight line so that t seconds after leaving a fixed point O, its velocity, v m s^{-1}, is given by $v = 2\left(3 - e^{-t/2}\right)$.

(i) Find the initial velocity of the particle. [1]

(ii) Find the acceleration of the particle when $v = 5$. [3]

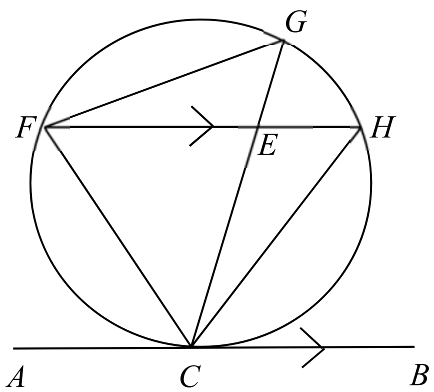
(iii) Calculate the displacement of the particle from O when $t = 10$. [3]

(iv) Does the particle reverses its direction of motion? Justify your answer with working clearly shown. [2]

10 The diagram shows a point C on a circle and line ACB is a tangent to the circle. Points F, G and H lie on the circle such that FH is parallel to AB. The lines GC and FH intersect at E.

(i) Prove that triangles ECF and FCG are similar.
Hence show that $(EC)(CG) = (CF)^2$. [4]

(ii) By using similar triangles, show that $(FE)(EH) = CF^2 - EC^2$. [5]



11 The equation of a circle, C_1, with centre A, is given by $x^2 + y^2 + 4x + 6y - 12 = 0$.

(i) Find the coordinates of A and the radius of C_1. [2]

Given that the circle passes through a point $P(-5, -7)$ and a point Q such that PQ is the diameter of the circle

(ii) write down the coordinates of Q. [2]

The tangent to the circle at point Q intersects the x-axis at point R.
A second circle, C_2, centre B, is drawn passing through A, Q and R.

(iii) Find the coordinates of R. [3]

(iv) Determine the coordinates of the centre, B and the radius of C_2. [4]

– END –

BPGH Preliminary Examination 2019 (Sec 4E/5N)
Additional Mathematics Paper 2 (Answers)

1 $(1 + ax)^4(1 - 4x)^3 = 1 + (4a - 12)x + (48 - 48a + 6a^2)\,x^2$
$p = 1 \quad a = 3 \quad q = -42$

2 (i) $u - \frac{12}{u} = 11$ (ii) $x = 1.79$, $4^x = -1$ (no real solution)

(iii) $u - \frac{12}{u} = p$
$u^2 - pu - 12 = 0$
$u = \frac{p + \sqrt{p^2 + 48}}{2}$ or $\frac{p - \sqrt{p^2 + 48}}{2}$
$4^x = \frac{p + \sqrt{p^2 + 48}}{2}$ or $4^x = \frac{p - \sqrt{p^2 + 48}}{2}$
Since $\frac{p+\sqrt{p^2+48}}{2} > 0$, $4^x > 0$ and there is real solution for x.
Since $\frac{p-\sqrt{p^2+48}}{2} < 0$, $4^x < 0$ and there is NO real solution for x.

Number of real solutions = 1

3 (i) L.H.S $= \frac{1}{cosec\,x-1} - \frac{1}{cosec x+1}$ (ii) $x = 30°, 131.8°, 228.2°, 300°$

$= \frac{cosec\,x + 1 - (cosec\,x - 1)}{cosec^2 x - 1}$

$= \frac{2}{cosec^2 x - 1}$

$= \frac{2}{cot^2 x - 1}$
$= 2\,tan^2 x$

4 (i) $\frac{dy}{dx} = \frac{5}{(x-1)^2} - 20$ (ii) $x < \frac{1}{2}$ or $x > \frac{3}{2}$

(iii) $\frac{dx}{dt} = 0.1\ units/s$

(iv) $\frac{dz}{dy} = -\frac{2}{y^2}$
When $x = 2$, $y = -43$
$\frac{dz}{dt} = \frac{dz}{dy} \times \frac{dy}{dt} = 1.62 \times 10^{-3}\ units/s$

5 (a) (i) $k = 0$, $k = -\frac{1}{3}$
(ii) When $k = 0$, $f(x) = -x(3 - x)$

When $k = -\frac{1}{3}$, $f(x) = \frac{1}{3}(3 - x)\left(x^2 - 3x - \frac{1}{3}\right)$

(b) $x^2 - 3x + k = 0$
No real solution when $b^2 - 4ac < 0$, $k > 2\frac{1}{4}$. Smallest integer value of k is 3.

6 (ii) Inaccurate value of $y = 9.1$
Correct value of $\frac{y}{x} = 2.9$. When $x = 3.5$, correct value of $y = 2.9 \times 3.5 = 10.15$

(iii) Equation is $\frac{y}{x} = \frac{1}{a} - \frac{b}{a}x$
From graph, $\frac{1}{a} = 2$, $a = \frac{1}{2}$
$-\frac{b}{a} = 0.25$, $b = -0.125$

(iv) Equation is $x = \frac{1}{b} - \frac{a}{b}\left(\frac{y}{x}\right)$
Gradient $= -\frac{a}{b} = 4$
Intercept on vertical axis $= \frac{1}{b} = -8$

7 (i) $\alpha - \beta = -\sqrt{8}$ (given $\alpha < \beta$)

(ii) $\frac{\alpha-1}{\beta} + \frac{\beta-1}{\alpha} = 4$, $\left(\frac{\alpha-1}{\beta}\right)\left(\frac{\beta-1}{\alpha}\right) = -\frac{1}{2}$
Equation is $x^2 - 4x - \frac{1}{2} = 0$ or $2x^2 - 8x - 1 = 0$

8 (i) $\frac{dy}{dx} = -6(5-2x)^2$, equation of tangent is $y = -54x + 81$
(ii) Shaded area $= \int_0^1 (5-2x)^3 dx - \int_0^1 (-54x+81)\, dx = 68 - 54 = 14$ units^2

9 (i) $v = 4\ m\ s^{-1}$ (ii) $a = \frac{1}{2}\ m\ s^{-2}$ (iii) $s = 6t + 4e^{-t/2} - 4 = 56.0$ m (when $t = 10$ s)
(iv) When $v = 0$, $t = -2.20\ s$. Since time cannot have a negative value, the particle did not reverse its direction of motion.

10 (i) ∠ACF = ∠FGC (alternate segment theorem/tangent-chord theorem)
∠ACF = ∠EFC (alternate angles)
∴ ∠FGC = ∠EFC

∠EFC = ∠FCG (common angle)
ΔECF and ΔFCG are similar triangles (AA similarity test)
$\frac{EC}{FC} = \frac{CF}{CG}$
$(EC)(CG) = (CF)^2$

(ii) ∠GEF = ∠HEC (vertically opposite angles)
∠FGE = ∠CHE (angles in same segment)
ΔFGE and ΔCHE are similar triangles (AA similarity test)
$\frac{FE}{EC} = \frac{EG}{EH}$
$(FE)(EH) = (EG)(EC)$
$= (CG - EC)(EC)$
$= (CG)(EC) - (EC)^2$
$= CF^2 - EC^2$ [$(EC)(CG) = (CF)^2$ in (i)]

11 (i) Centre, A = (−2 , −3) , radius = 5 units (ii) Q(1 , 1)
(iii) R$\left(\frac{7}{3}, 0\right)$ (iv) B$\left(\frac{1}{6}, -\frac{3}{2}\right)$, radius = 2.64 units

Name:	Class:	Class Register Number:

中正中学

CHUNG CHENG HIGH SCHOOL (MAIN)

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PRELIMINARY EXAMINATION 2018

SECONDARY 4

ADDITIONAL MATHEMATICS 4047/01

Paper 1 **17 September 2018**

2 hours

Additional Materials: Answer Paper

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number clearly on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

/ 80

This document consists of **6** printed pages.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

1 A curve is such that $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = ax - 2$, where a is a constant. The curve has a minimum gradient at $x = \frac{1}{3}$.

(i) Show that $a = 6$. [1]

The tangent to the curve at $(1,\ 4)$ is $y = 2x + 2$.

(ii) Find the equation of the curve. [6]

2 The roots of the quadratic equation $3x^2 + 2x + 4 = 0$ are α and β.

(i) Show that $\alpha^2 + \beta^2 = -\frac{20}{9}$. [3]

(ii) Find a quadratic equation with roots $\frac{\alpha^2}{\beta}$ and $\frac{\beta^2}{\alpha}$. [4]

3 It is given that $\mathrm{f}(x) = (x+h)^2(x-1) + k$, where h and k are constants and $h < k$. When $\mathrm{f}(x)$ is divided by $x + h$, the remainder is 6. It is given that $\mathrm{f}(x)$ is exactly divisible by $x + 5$.

(i) State the value of k and show that $h = 4$. [4]

(ii) Find the range of values of the constant b for which the graph of $y = \mathrm{f}(x) + bx$ is an increasing function for all values of x. [4]

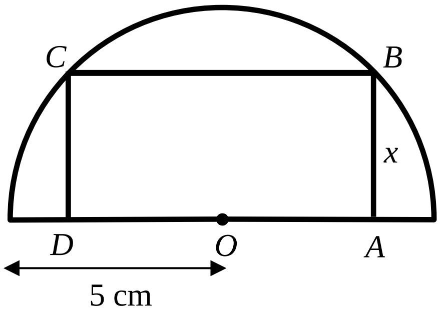
4 Given that $\tan(x+y) = -\frac{120}{119}$ and $\cos x = \frac{5}{13}$, where x and y are acute angles, show that $x = y$ **without** finding the values of x and y. [4]

5 The variables x and y are such that when $\frac{x}{y}$ are plotted against x, a straight line l_1 of gradient 2 is obtained. It is given that $y = \frac{1}{5}$ when $x = 3$.

(i) Express y in terms of x. [3]

(ii) When the graph of $x = 2y$ is plotted on the same axes as the line l_1, the two lines intersect at one point. Find the coordinates of the point of intersection. [2]

6 The figure shows a semicircle of radius 5 cm and centre, *O*. A rectangle *ABCD* is inscribed in the semicircle such that the four vertices *A*, *B*, *C* and *D* touch the edge of the semicircle. The length of *AB* = *x* cm.



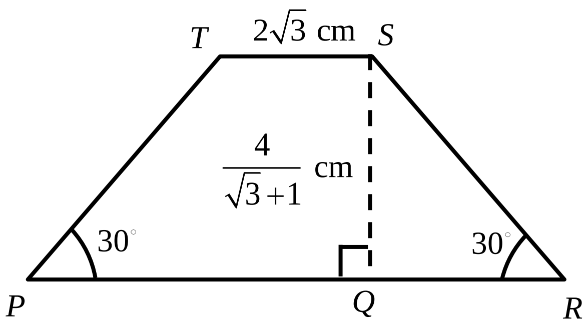
(i) Show that the perimeter, *P* cm, of rectangle *ABCD* is given by

$$P = 2x + 4\sqrt{25 - x^2}$$ [2]

(ii) Given that *x* can vary, find the value of *x* when the perimeter is stationary. [4]

7 In the diagram below, *PQRST* is a trapezium where angle *QRS* = angle *TPR* = 30°. *SQ* is the height of the trapezium and the length of *SQ* is $\frac{4}{\sqrt{3}+1}$ cm. The length of *TS* is $2\sqrt{3}$ cm.

By rationalising $\frac{4}{\sqrt{3}+1}$, find the area of trapezium *PQRST* in the form $\left(a\sqrt{3}-12\right)$ cm^2, where *a* is an integer. [5]



8 A particle moving in a straight line passes a fixed point *A* with a velocity of –8 cms^{-1}. The acceleration, a cms^{-2} of the particle, *t* seconds after passing *A* is given by $a = 10 - kt$, where *k* is a constant. The particle first comes to instantaneous rest at $t = 1$ and reaches maximum speed at *T* seconds (The particle does not come instantaneously to rest at $1 < t < T$).

(i) Find the value of *k*. [3]

(ii) Find the total distance travelled by the particle when $t = T$. [5]

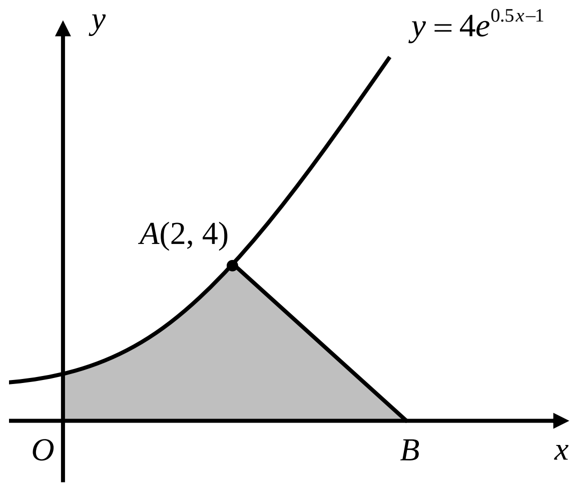
9 It is given that $y = 1 - 3\sin 2x$ for $-\frac{\pi}{2} \le x \le \pi$.

(i) State the period of *y*. [1]

(ii) Sketch the graph of $y = 1 - 3\sin 2x$. [3]

(iii) By drawing a straight line on the same diagram as in part **(ii)**, find the number of solutions to the equation $3\sin 2x + 1\frac{1}{2} = \frac{3x}{\pi}$ for $-\frac{\pi}{2} \le x \le \pi$. [3]

10



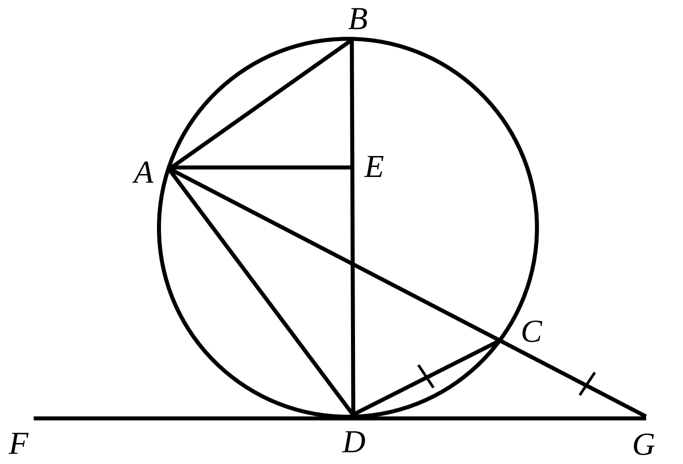
The diagram shows part of the curve $y = 4e^{0.5x-1}$. The normal to the curve at point $A(2, 4)$ cuts the *x*-axis at point *B*.

Find

(i) the coordinates of *B*, [4]

(ii) the area of the shaded region. [3]

11

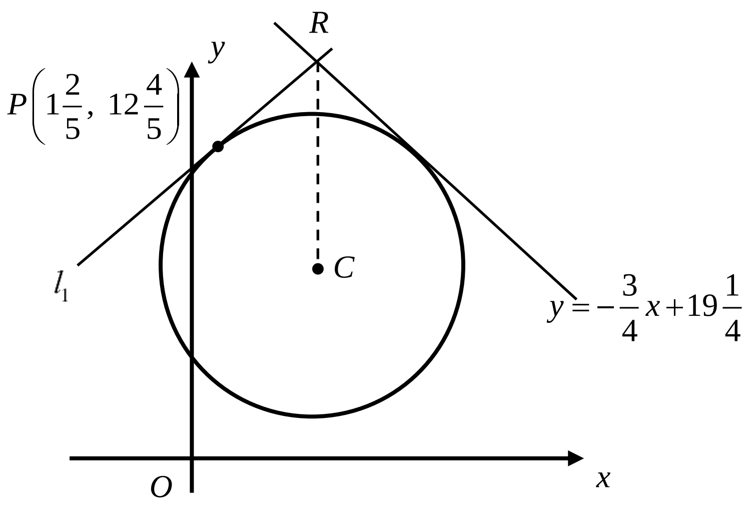


In the diagram, *BD* and *AC* are chords of the circle. *FD* is a tangent to the circle at *D*. *AC* and *FD* are produced to meet at *G* such that *CG* = *CD*. *E* is a point along *BD*. Triangle *BAE* is similar to triangle *ADE*.

(i) By showing that triangle *BAD* and triangle *AED* are similar, prove that *AB* is perpendicular to *AD*. [4]

(ii) Show that angle *ADB* = 90° – 2 × (angle *CGD*) . [4]

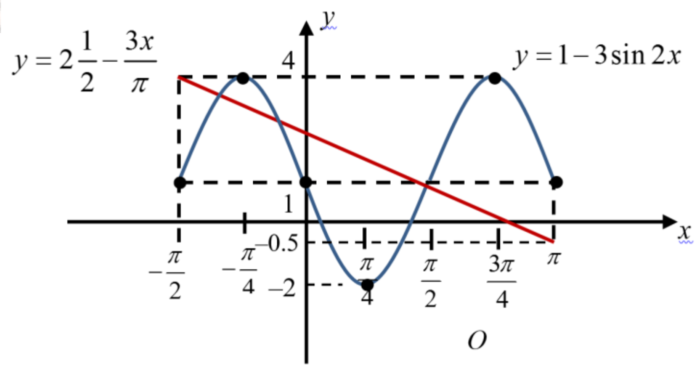
12 The line $y=-\frac{3}{4}x+19\frac{1}{4}$ is a tangent to the circle, centre *C*. Another line, l_1 is tangent to the circle at point $P\left(1\frac{2}{5},\ 12\frac{4}{5}\right)$. The two tangents intersect at point *R*, which is directly above the centre of the circle.



(i) Show that the coordinates of *R* are $\left(5,\ 15\frac{1}{2}\right)$. [4]

(ii) Find the equation of the circle. [4]

Answer Key

1	**(i)**	Show question
	(ii)	$y = x^3 - x^2 + x + 3$
2	**(i)**	Show question
	(ii)	$x^2 - \frac{16}{9}x + \frac{4}{3} = 0$ or any other equivalent equation
3	**(i)**	$k = 6$; $h = 4$ (show question)
	(ii)	$b > 8\frac{1}{3}$
4		Show question
5	**(i)**	$y = \frac{x}{2x+9}$
	(ii)	$\left(-3\frac{1}{2},\ 2\right)$
6	**(i)**	Show question
	(ii)	$x = \sqrt{5}$ or 2.24 (3 s.f.)
7		$\left(12\sqrt{3} - 12\right)$ cm^2
8	**(i)**	$k = 4$
	(ii)	$8\frac{1}{6}$ m
9	**(i)**	π
	(ii)	
	(iii)	3 solutions
10	**(i)**	$B(10, 0)$
	(ii)	$\left(24 - \frac{8}{e}\right)$ units^2 or 21.1 units^2 (3 s.f)
11	**(i), (ii)**	Show question
12	**(ii)**	$(x-5)^2 + (y-8)^2 = 36$

Name:	Class:	Class Register Number:

中正中学

CHUNG CHENG HIGH SCHOOL (MAIN)

PRELIMINARY EXAMINATION 2018

SECONDARY 4

ADDITIONAL MATHEMATICS 4047/01

Paper 1

17 September 2018

2 hours

Additional Materials: Answer Paper

MARK SCHEME

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[Turn Over

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1. A curve is such that $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = ax - 2$, where a is a constant. The curve has a minimum gradient at $x = \frac{1}{3}$.

(i) Show that $a = 6$. [1]

The tangent to the curve at $(1,\ 4)$ is $y = 2x + 2$.

(ii) Find the equation of the curve. [6]

Marking Scheme

(i) At minimum gradient, $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 0$

$a\left(\frac{1}{3}\right) - 2 = 0$

$\frac{a}{3} = 2$

$a = 6$

(ii) $\frac{\mathrm{d}y}{\mathrm{d}x} = \int (6x - 2)\ \mathrm{d}x$

$= 3x^2 - 2x + c$ where c is an arbitrary constant

$y = 2x + 2$

Gradient of tangent $= 2$

$3(1)^2 - 2(1) + c = 2$

$c = 1$

$y = \int (3x^2 - 2x + 1)\ \mathrm{d}x$

$= x^3 - x^2 + x + c_1$ where c_1 is an arbitrary constant

Sub. (1, 4)

$4 = 1^3 - 1^2 + 1 + c_1$

$c_1 = 3$

Equation of curve is $y = x^3 - x^2 + x + 3$

2. The roots of the quadratic equation $3x^2+2x+4=0$ are α and β.

(i) Show that $\alpha^2+\beta^2=-\dfrac{20}{9}$. [3]

(ii) Find a quadratic equation with roots $\dfrac{\alpha^2}{\beta}$ and $\dfrac{\beta^2}{\alpha}$. [4]

Marking Scheme

(i) $\alpha+\beta=-\dfrac{2}{3}$

$\alpha\beta=\dfrac{4}{3}$

$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$

$\alpha^2\beta^2=\left(-\dfrac{2}{3}\right)^2-2\left(\dfrac{4}{3}\right)$

$=\dfrac{4}{9}-\dfrac{8}{3}$

$=-\dfrac{20}{9}$ (shown)

(ii) Sum of roots $=\dfrac{\alpha^2}{\beta}+\dfrac{\beta^2}{\alpha}$

$=\dfrac{\alpha^3+\beta^3}{\alpha\beta}$

$=\dfrac{(\alpha+\beta)(\alpha^2-\alpha\beta+\beta^2)}{\alpha\beta}$

$=\dfrac{\left(-\dfrac{2}{3}\right)\left(-\dfrac{20}{9}-\dfrac{4}{3}\right)}{-\dfrac{4}{3}}$

$=\dfrac{16}{9}$

Product of roots $=\left(\dfrac{\alpha^2}{\beta}\right)\left(\dfrac{\beta^2}{\alpha}\right)$

$=\alpha\beta$

$=\dfrac{4}{3}$

The quadratic equation is $x^2-\dfrac{16}{9}x+\dfrac{4}{3}=0$

OR $9x^2-16x+12=0$

3. It is given that $\mathrm{f}(x)=(x+h)^2(x-1)+k$, where h and k are constants and $h<k$. When $\mathrm{f}(x)$ is divided by $x+h$, the remainder is 6. It is given that $\mathrm{f}(x)$ is exactly divisible by $x+5$.

(i) State the value of k and show that $h=4$. [4]

(ii) Find the range of values of the constant b for which the graph of $y=\mathrm{f}(x)+bx$ is an increasing function for all values of x. [4]

Marking Scheme

(i) $k=6$ B1

$\mathrm{f}(-5)=0$

$(-5+h)^2(-5-1)+6=0$

$(h-5)^2(-6)=-6$

$(h-5)^2=1$

$h-5=-1$ or 1

$h=4$ or 6 (rejected as $h<k$)

(ii) $y=(x+4)^2(x-1)+6+bx$

$\dfrac{dy}{dx}=2(x+4)(x-1)+(x+4)^2+b$

$=(x+4)[2(x-1)+(x+4)]+b$

$=(x+4)(3x+2)+b$

For increasing function, $\dfrac{dy}{dx}>0$

$(x+4)(3x+2)+b>0$

$3x^2+14x+8+b>0$

Discriminant <0

$(14)^2-4(3)(8+b)<0$

$196-96-12b<0$

$12b>100$

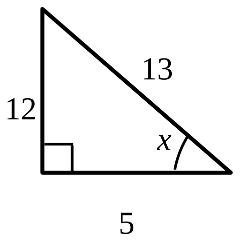
$b>8\frac{1}{3}$

4. Given that $\tan(x+y) = -\frac{120}{119}$ and $\cos x = \frac{5}{13}$, where x and y are acute angles, show that $x = y$ **without** finding the values of x and y. [4]

Marking Scheme

$$\tan(x+y) = -\frac{120}{119}$$

$$\frac{\tan x + \tan y}{1 - \tan x \tan y} = -\frac{120}{119}$$



$$\tan x = \frac{12}{5}$$

$$\frac{\frac{12}{5} + \tan y}{1 - \frac{12}{5}\tan y} = -\frac{120}{119}$$

$$\frac{12}{5} + \tan y = -\frac{120}{119} + \frac{288}{119}\tan y$$

$$\frac{2028}{595} = \frac{169}{119}\tan y$$

$$\tan y = \frac{12}{5}$$

Since $\tan x = \tan y$ and x and y are both acute, $x = y$.

5. The variables x and y are such that when $\frac{x}{y}$ are plotted against x, a straight line l_1 of gradient 2 is obtained. It is given that $y=\frac{1}{5}$ when $x=3$.

(i) Express y in terms of x. [3]

(ii) When the graph of $x=2y$ is plotted on the same axes as the line l_1, the two lines intersect at one point. Find the coordinates of the point of intersection. [2]

<u>Marking Scheme</u>

(ii) $\frac{x}{y}=2x+c$

$\frac{3}{\frac{1}{5}}=2(3)+c$

$c=9$

$\frac{x}{y}=2x+9$

$\frac{y}{x}=\frac{1}{2x+9}$

$y=\frac{x}{2x+9}$

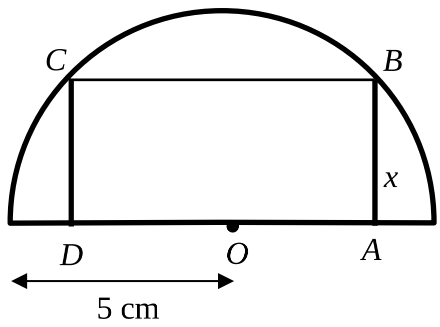
(iii) $x=2y \Rightarrow \frac{x}{y}=2$

$2x+9=2$

$x=-3\frac{1}{2}$

The point of intersection is $\left(-3\frac{1}{2},\ 2\right)$.

6. The figure shows a semicircle of radius 5 cm and centre, *O*. A rectangle *ABCD* is inscribed in the semicircle such that the four vertices *A*, *B*, *C* and *D* touch the edge of the semicircle. The length of *AB* = *x* cm.



(i) Show that the perimeter, P cm, of rectangle *ABCD* is given by

$$P = 2x + 4\sqrt{25 - x^2}$$ [2]

(ii) Given that *x* can vary, find the value of *x* when the perimeter is stationary. [4]

Marking Scheme

(i) $OB = 5$ cm (radius of circle)

$OB^2 = OA^2 + AB^2$

$25 = OA^2 + x^2$

$OA = \sqrt{25 - x^2}$

$P = AB + CD + AD + BC$

$= 2AB + 4OA$

$= 2x + 4\sqrt{25 - x^2}$ (shown)

(ii) $P = 2x + 4\sqrt{25 - x^2}$

$\frac{\mathrm{d}P}{\mathrm{d}x} = 2 + 4\left(\frac{1}{2}\right)\left(25 - x^2\right)^{-\frac{1}{2}}(-2x)$

$= 2 - \frac{4x}{\sqrt{25 - x^2}}$

At stationary P, $\frac{\mathrm{d}P}{\mathrm{d}x} = 0$

$2 - \frac{4x}{\sqrt{25 - x^2}} = 0$

$\frac{4x}{\sqrt{25 - x^2}} = 2$

$\frac{16x^2}{25 - x^2} = 4$

$4x^2 = 25 - x^2$

$5x^2 = 25$

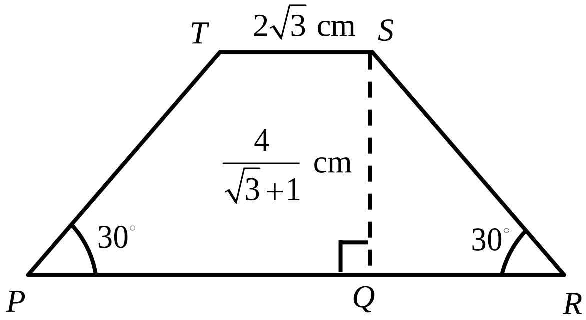
$x^2 = 5$

$x = \sqrt{5}$ or $-\sqrt{5}$ (rejected)

or 2.24 (3 s.f)

7. In the diagram below, *PQRST* is a trapezium where angle *QRS* = angle *TPR* = 30°. *SQ* is the height of the trapezium and the length of *SQ* is $\dfrac{4}{\sqrt{3}+1}$ cm. The length of *TS* is $2\sqrt{3}$ cm.

By rationalising $\dfrac{4}{\sqrt{3}+1}$, find the area of trapezium *PQRST* in the form $\left(a\sqrt{3}-12\right)$ cm^2, where *a* is an integer. [5]



Marking Scheme

$$\frac{4}{\sqrt{3}+1}=\frac{4}{\sqrt{3}+1}\times\frac{\sqrt{3}-1}{\sqrt{3}-1}$$
$$=\frac{4\sqrt{3}-4}{3-1}$$
$$=2\sqrt{3}-2$$

$$\tan 30^\circ=\frac{2\sqrt{3}-2}{QR}$$
$$\frac{1}{\sqrt{3}}=\frac{2\sqrt{3}-2}{QR}$$
$$QR=2(3)-2\sqrt{3}$$
$$=\left(6-2\sqrt{3}\right)\text{ cm}$$

$$\text{Area of trapezium}=\frac{1}{2}\left[2\left(6-2\sqrt{3}\right)+2\left(2\sqrt{3}\right)\right]\left(2\sqrt{3}-2\right)$$
$$=\frac{1}{2}\left(12-4\sqrt{3}+4\sqrt{3}\right)\left(2\sqrt{3}-2\right)$$
$$=\frac{1}{2}(12)\left(2\sqrt{3}-2\right)$$
$$=6\left(2\sqrt{3}-2\right)$$
$$=\left(12\sqrt{3}-12\right)\text{ cm}^2$$

8. A particle moving in a straight line passes a fixed point A with a velocity of $-8\ \text{cms}^{-1}$. The acceleration, $a\ \text{cms}^{-2}$ of the particle, t seconds after passing A is given by $a = 10 - kt$, where k is a constant. The particle first comes to instantaneous rest at $t = 1$ and reaches maximum speed at T seconds (The particle does not comes instantaneous to rest at $1 < t < T$).

(i) Find the value of k. [3]

(ii) Find the total distance travelled by the particle when $t = T$. [5]

<u>Marking Scheme</u>

(i) $a = 10 - kt$

$v = \int (10 - kt)\ dt$

$= 10t - \frac{kt^2}{2} + c$ where c is an arbitary constant

When $t = 0$, $v = -8$

$-8 = c$

$\therefore v = 10t - \frac{kt^2}{2} - 8$

When $t = 1$, $v = 0$

$0 = 10 - \frac{k}{2} - 8$

$k = 4$

(ii) $a = 10 - 4t$

At maximum speed, $a = 0$

$10 - 4t = 0$

$t = 2\frac{1}{2}$

$s = \int (10t - 2t^2 - 8)\ dt$

$= 5t^2 - \frac{2t^3}{3} - 8t + c_1$ where c_1 is an arbitary constant

When $t = 0$, $s = 0$, $c_1 = 0$

$\therefore s = 5t^2 - \frac{2t^3}{3} - 8t$

When $t = 0$, s = 0

When $t = 1$, $s = -\frac{11}{3}$

When $t = 2\frac{1}{2}$, $s = \frac{5}{6}$

Total distance travelled $= \left(\frac{11}{3}\right) \times 2 + \frac{5}{6}$

$= 8\frac{1}{6}$ m

9. It is given that $y = 1 - 3\sin 2x$ for $-\frac{\pi}{2} \le x \le \pi$.

(i) State the period of y. [1]

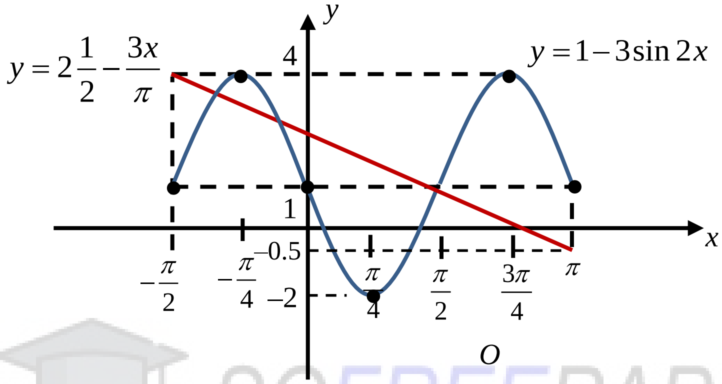
(ii) Sketch the graph of $y = 1 - 3\sin 2x$. [3]

(iii) By drawing a straight line on the same diagram as in part **(ii)**, find the number of solutions to the equation $3\sin 2x + 1\frac{1}{2} = \frac{3x}{\pi}$ for $-\frac{\pi}{2} \le x \le \pi$. [3]

Marking Scheme

(i) 180° or π

(ii)



$3\sin 2x + 1\frac{1}{2} = \frac{3x}{\pi}$

$3\sin 2x = \frac{3x}{\pi} - 1\frac{1}{2}$

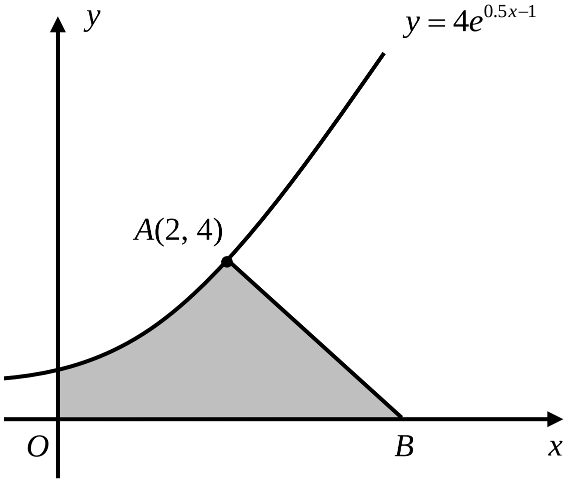
$3\sin 2x - 1 = \frac{3x}{\pi} - 2\frac{1}{2}$

$1 - 3\sin 2x = 2\frac{1}{2} - \frac{3x}{\pi}$

Draw the line of $y = 2\frac{1}{2} - \frac{3x}{\pi}$.

From the graph, there are 3 points of intersections, thus there are 3 solutions

10.



The diagram shows part of the curve $y = 4e^{0.5x-1}$. The normal to the curve at point $A(2, 4)$ cuts the x-axis at point B.

Find

(i) the coordinates of B, [4]

(ii) the area of the shaded region. [3]

<u>Marking Scheme</u>

(i) $y = 4e^{0.5x-1}$

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 4(0.5)e^{0.5x-1}$$

$$= 2e^{0.5x-1}$$

When $x = 2$, $\frac{\mathrm{d}y}{\mathrm{d}x} = 2$

Gradient of normal $= -\frac{1}{2}$

Let $B(x, 0)$.

$$\frac{4-0}{2-x} = -\frac{1}{2}$$

$8 = -2 + x$

$x = 10$

$\therefore B(10, 0)$

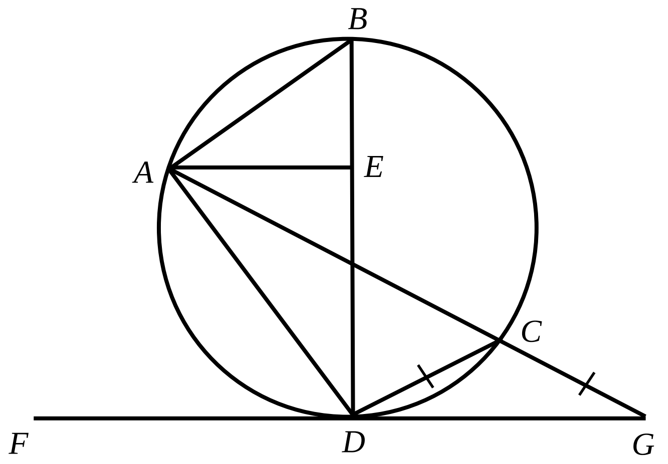
(ii) Area of shaded region $= \int_0^2 4e^{0.5x-1}\ \mathrm{d}x + \frac{1}{2}(10-2)(4)$

$$= \left[\frac{4e^{0.5x-1}}{0.5}\right]_0^2 + 16$$

$$= 8e^0 - 8e^{-1} + 16$$

$$= \left(24 - \frac{8}{e}\right) \text{ units}^2 \quad \text{or} \quad 21.1 \text{ units}^2 \quad (3 \text{ s.f})$$

11.



In the diagram, *BD* and *AC* are chords of the circle. *FD* is a tangent to the circle at *D*. *AC* and *FD* are produced to meet at *G* such that *CG* = *CD*. *E* is a point along *BD*. Triangle *BAE* is similar to triangle *ADE*.

(i) By showing that triangle *BAD* and triangle *AED* are similar, prove that *AB* is perpendicular to *AD*. [4]

(ii) Show that angle *ADB* = 90° – 2 × (angle *CGD*) [4]

Marking Scheme

(i) $\angle ABE = \angle DAE$ (corresponding angles of similar triangles *BAE* and *ADE*)

$\angle ADE = \angle BDA$ (common angle)

By AA similarity rule, triangles *BAD* and *AED* are similar.

$\angle BEA = \angle AED$ (corresponding angles of similar triangles *BAE* and *ADE*)

$= 90^\circ$ (adjacent $\angle$s on straight line)

$\therefore \angle BAD = \angle AED$ (corresponding angles of similar triangles *BAD* and *AED*)

$= 90^\circ$

$AB \perp AD$ (shown)

(ii) Let $\angle CGD = a$.

$\angle CDG = \angle CGD$ (base $\angle$s of isosceles Δ)

$= a$

BD is a diameter (right-angle in a semicircle)

$\therefore \angle EDG = 90^\circ$ (tangent $\perp$ radius)

$\angle DAC = \angle CDG$ ($\angle$s in alternate segment)

$= a$

Consider ΔADG,

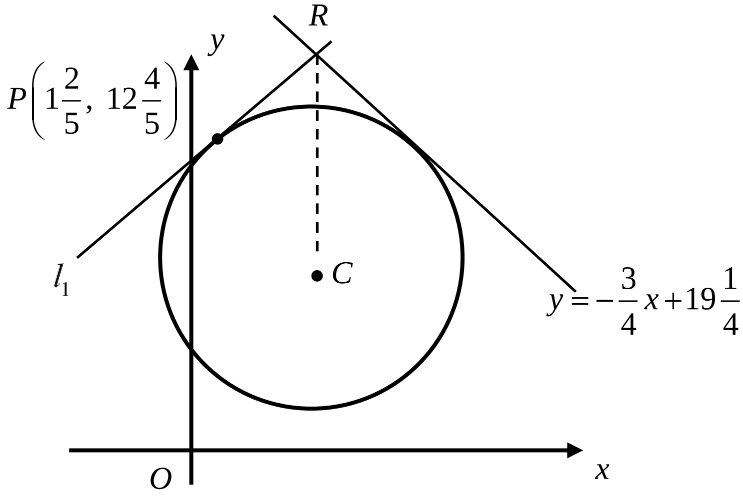
$\angle ADB = 180^\circ - \angle DAC - \angle CGD - \angle EDG$ (sum of $\angle$s in Δ)

$= 180^\circ - a - a - 90^\circ$

$= 90^\circ - 2a$

$= 90^\circ - 2 \times \angle CGD$ (shown)

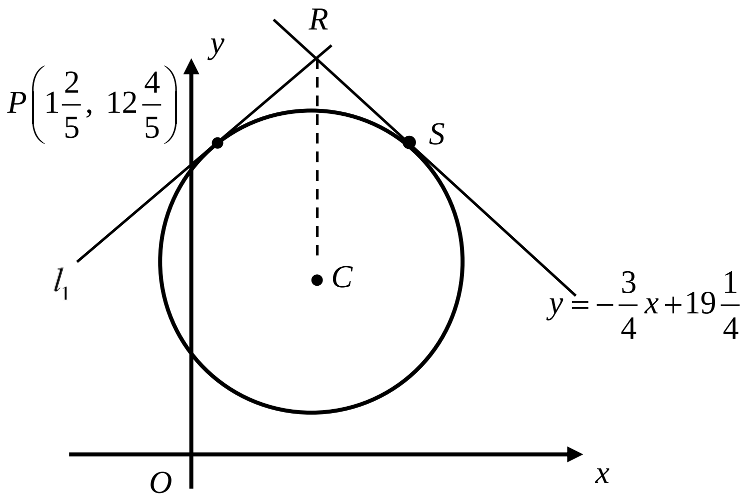
12. The line $y = -\frac{3}{4}x + 19\frac{1}{4}$ is a tangent to the circle, centre C. Another line, l_1 is tangent to the circle at point $P\left(1\frac{2}{5},\ 12\frac{4}{5}\right)$. The two tangents intersect at point R, which is directly above the centre of the circle.



(i) Show that the coordinates of R are $\left(5,\ 15\frac{1}{2}\right)$. [4]

(ii) Find the equation of the circle. [4]

Marking Scheme



y-coordinate of $S = 12\frac{4}{5}$

$12\frac{4}{5} = -\frac{3}{4}x + 19\frac{1}{4}$

$x = 8\frac{3}{5}$

$\therefore S\left(8\frac{3}{5},\ 12\frac{4}{5}\right)$

$$x_C = \frac{8\frac{3}{5}+1\frac{2}{5}}{2}$$
$$= 5$$

$$y = -\frac{3}{4}(5) + 19\frac{1}{4}$$
$$= 15\frac{1}{2}$$

$\therefore R\left(5, 15\frac{1}{2}\right)$ (shown)

<u>Alternative Method</u>

Gradient of $l_1 = \frac{3}{4}$

Equation of l_1 is $y - 12\frac{4}{5} = \frac{3}{4}\left(x - 1\frac{2}{5}\right)$ --------- (1)

$y = -\frac{3}{4}x + 19\frac{1}{4}$ ----------- (2)

Sub. (2) into (1),

$$-\frac{3}{4}x + 19\frac{1}{4} - 12\frac{4}{5} = \frac{3}{4}\left(x - 1\frac{2}{5}\right)$$

$$-\frac{3}{4}x + \frac{129}{20} = \frac{3}{4}x - \frac{21}{20}$$

$$-\frac{3}{2}x = -\frac{15}{2}$$

$x = 5$ sub. into (2)

$$y = 15\frac{1}{2}$$

$\therefore R\left(5, 15\frac{1}{2}\right)$ (shown)

(ii) Gradient of normal at $S = \frac{4}{3}$

Equation of normal is $y - 12\frac{4}{5} = \frac{4}{3}\left(x - 8\frac{3}{5}\right)$

When $x = 5$,

$y - 12\frac{4}{5} = \frac{4}{3}\left(5 - 8\frac{3}{5}\right)$

$y = 8$

$\therefore C(5,8)$

Radius $= \sqrt{\left(5 - 8\frac{3}{5}\right)^2 + \left(8 - 12\frac{4}{5}\right)^2}$

$= 6$ units

Equation of circle is $(x-5)^2 + (y-8)^2 = 36$.

Alternative Method

Gradient of normal at $P = -\frac{4}{3}$

Gradient of normal at P is $y - 12\frac{4}{5} = -\frac{4}{3}\left(x - 1\frac{2}{5}\right)$

Sub. $x = 5$,

$y = 8$

Centre of circle is $(5,8)$

Radius of circle $= \sqrt{\left(5 - 1\frac{2}{5}\right)^2 + \left(8 - 12\frac{4}{5}\right)^2}$

= 6 units

Equation of circle is $(x-5)^2 + (y-8)^2 = 36$

Name:	Class:	Class Register Number:

中正中学

CHUNG CHENG HIGH SCHOOL (MAIN)

____
Parent's Signature

PRELIMINARY EXAMINATION 2018
SECONDARY 4

ADDITIONAL MATHEMATICS 4047/02

Paper 2 **18 September 2018**

2 hours 30 minutes

Additional Materials: Answer Paper

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number clearly on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

/ 100

This document consists of **6** printed pages.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

1 An empty, inverted cone has a height of 600 cm. The radius of the top of the cone is 200 cm. Water is poured into the cone at a constant rate.

(i) When the depth of the water in the cone is h cm, find the volume of the water in the cone in terms of π and h. [4]

The water level is rising at a rate of 3 cm per minute when the depth of the water is 120 cm.

(ii) Find the rate at which water is being poured into the cone, leaving your answer in terms of π. [3]

2 It is given that $y = x - \ln(\sec x + \tan x)$, $0 < x < \frac{\pi}{2}$.

(i) Show that $\frac{\mathrm{d}}{\mathrm{d}x}(\sec x) = \sec x \tan x$. [3]

(ii) Hence, express $\frac{\mathrm{d}y}{\mathrm{d}x}$ in the form $a + b\sec x$, where a and b are integers. [3]

(iii) Deduce that y is a decreasing function. [2]

3 **(a)** Prove that $\frac{1+\sin 2x+\cos 2x}{\cos x+\sin x} = 2\cos x$. [3]

(b) Given that $\frac{\sec^2 x}{2\tan^2 x+1} = \frac{3}{4}$, where $180° < x < 270°$, find the exact value of $\sin x$. [5]

4 **(a)** Solve, for x and y, the simultaneous equations

$$2^x = 8\left(2^y\right),$$
$$\lg(2x+y) = \lg 63 - \lg 3.$$

[4]

(b) Express $\log_{\sqrt{2}} y = 3 - \log_2(y-6)$ as a cubic equation. [4]

5 **(i)** Express $\frac{2x^2-7}{(x+1)(x^2-x-6)}$ in partial fractions. [4]

(ii) Hence, find $\int_4^5 \frac{8x^2-28}{(x+1)(x^2-x-6)}\,\mathrm{d}x$. [4]

6 **(a)** **(i)** Sketch the graph of $y = |(x-1)(x-5)|$. [3]

(ii) Determine the set of values of a for which the line $y = a$ intersects the graph of $y = |(x-1)(x-5)|$ at four points. [2]

(b) Find the range of values of k for which the line $y = kx - 3$ does not intersect the curve $y = 2x^2 - 6x + 5$. [4]

7 **(i)** Show that $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\ln 3x}{2x^2}\right) = \frac{1}{2x^3} - \frac{\ln 3x}{x^3}$. [3]

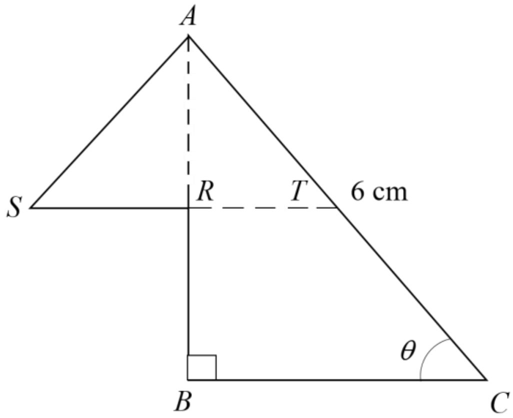
(ii) Hence, integrate $\frac{\ln 3x}{x^3}$ with respect to x. [3]

(iii) Given that the curve $y = \mathrm{f}(x)$ passes through the point $\left(\frac{1}{3}, \frac{3}{4}\right)$ and is such that $\mathrm{f}'(x) = \frac{\ln 3x}{x^3}$, find $\mathrm{f}(x)$. [2]

8 **(i)** Find the coefficient of x^4 in the expansion of $(6 - x^2)^5 \left(2x^2 + \frac{1}{3}\right)$. [4]

(ii) In the expansion of $(2 + x)^n$, the ratio of the coefficients of x and x^2 is $2:3$. Find the value of n. [5]

9 In the diagram, triangle ABC is a right angle triangle where angle $ACB = \theta$ and $AC = 6$ cm. R is a point on AB and T is the mid-point of AC. RT is parallel to BC and AR is a line of symmetry of triangle AST.

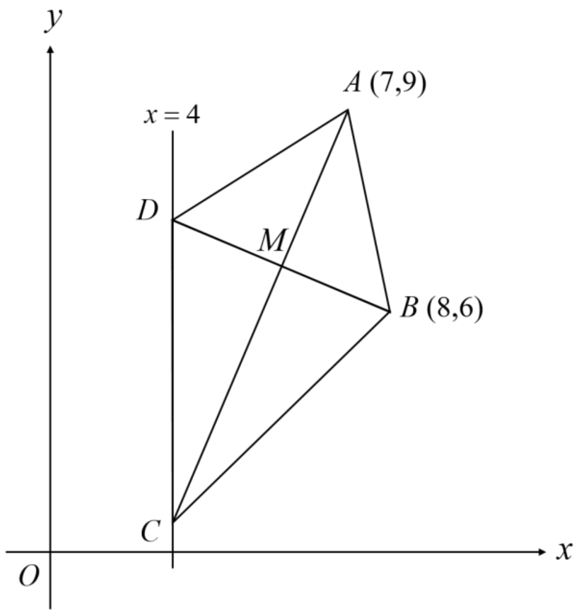


(a) Show that the perimeter, P cm, of the above diagram is $P = 9\cos\theta + 3\sin\theta + 9$. [2]

(b) **(i)** By expressing P in the form $m + n\cos(\theta - \alpha)$, find the value of θ for which $P = 15$. [6]

(ii) Hence, state the maximum value of P and find the corresponding value of θ. [3]

10 The diagram shows a quadrilateral $ABCD$ where the coordinates of vertices A and B are $(7,9)$ and $(8,6)$ respectively. Both vertices C and D lie on the line $x = 4$. AC passes through M, the midpoint of BD.



(i) Given that $AB = AD$, find the coordinates of C and D. [7]

(ii) Hence or otherwise, prove that quadrilateral $ABCD$ is a kite. [2]

(iii) Find the area of the kite $ABCD$. [2]

11 **(a)** The amount of caffeine, C mg, left in the body t hours after drinking a certain cup of coffee is represented by $C = 100\text{e}^{-kt}$.

(i) Given that the amount of caffeine left in the body is 20 mg after 2.5 hours, find the value of k . [2]

(ii) Find the number of hours, correct to 3 significant figures, for half the initial amount of caffeine to be left in the body. [3]

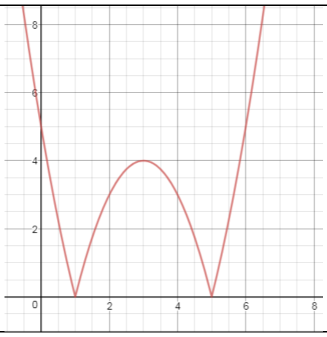
(b) The curve $y = ax^4 + bx^3 + 7$, where a and b are constants, has a minimum point at $(1, 6)$.

Find

(i) the value of a and of b , [4]

(ii) the coordinates of the other stationary point on the curve and determine the nature of this stationary point. [4]

Answer Key

1	**(i)**	$v=\dfrac{\pi h^3}{27}$
	(ii)	$4800\pi\ \text{cm}^3/\text{min}$
2	**(ii)**	$1-\sec x$
3	**(b)**	$\sin x=-\dfrac{\sqrt{3}}{3}$
4	**(a)**	$x = 8, y = 5$
	(b)	$y^3-6y^2-8=0$
5	**(i)**	$\dfrac{2x^2-7}{(x+1)(x^2-x-6)}=\dfrac{5}{4(x+1)}+\dfrac{11}{20(x-3)}+\dfrac{1}{5(x+2)}$
	(ii)	2.56
6	**(ai)**	
	(aii)	$0<a<4$
	(b)	$-14<k<2$
7	**(ii)**	$-\dfrac{1}{4x^2}-\dfrac{2\ln 3x}{x^2}+c$
	(iii)	$f(x)=\dfrac{\ln 3x}{-2x^2}-\dfrac{1}{4x^2}+3$
8	**(i)**	-12440
	(ii)	$n = 7$
9	**(bi)**	$P=9+\sqrt{90}\cos(\theta-18.43495°)$; $\theta=69.2°$
	(bii)	maximum value of $P=9+\sqrt{90}$, corresponding value of $x=18.4°$
10	**(i)**	$D(4,8)$, $C(4,3)$
	(ii)	Since $M_{AC} \bullet M_{BD}=-1$, diagonals AC and BD are perpendicular to each other. AC bisects BD. $\therefore$ quadrilateral $ABCD$ is a kite.
	(iii)	$15\ \text{units}^2$
11	**(ai)**	$k=0.644$
	(aii)	$t=1.08$ hours
	(bi)	$a=3$ and $b=-4$
	(bii)	$(0,7)$, point of inflexion

	Working	Common Issues
1 (i)	$\frac{h}{600}=\frac{r}{200}$ (ratio of corresponding sides are equal) $r=\frac{h}{3}$ $V=\frac{1}{3}\pi\left(\frac{h}{3}\right)^2 h$ $v=\frac{\pi h^3}{27}$	
(ii)	$\frac{\mathrm{d}h}{\mathrm{d}t}=3$ cm/s $\frac{\mathrm{d}V}{\mathrm{d}h}=\frac{\pi}{27}\left(3h^2\right)$ $=\frac{\pi h^2}{9}$ $\frac{\mathrm{d}V}{\mathrm{d}t}=\frac{\mathrm{d}V}{\mathrm{d}h}\times\frac{\mathrm{d}h}{\mathrm{d}t}$ $=\frac{\pi h^2}{9}\times 3$ $=4800\pi$ cm^3/min	

	Working	**Common Issues**
2 (i)	$y = x - \ln(\sec x + \tan x)$ $\frac{d}{dx}(\sec x) = \frac{d}{dx}\left(\frac{1}{\cos x}\right)$ $= \frac{(\cos x)(0) - (1)(-\sin x)}{\cos^2 x}$ $= \frac{\sin x}{\cos^2 x}$ $= \sec x \tan x$	
(ii)	$\frac{dy}{dx} = 1 - \frac{1}{\sec x + \tan x}(\sec x \tan x + \sec^2 x)$ $= 1 - \frac{\sec x \tan x + \sec^2 x}{\sec x + \tan x}$ $= 1 - \frac{\sec x(\tan x + \sec x)}{\sec x + \tan x}$ $= 1 - \sec x$	
(iii)	$\frac{dy}{dx} = 1 - \sec x$ $= 1 - \frac{1}{\cos x}$ $= \frac{\cos x - 1}{\cos x}$ Numerator: $0 < \cos x < 1$ $\therefore \cos x - 1$ will always be negative. Denominator: $0 < \cos x < 1$ $\therefore \cos x$ will always be positive. $\therefore \frac{dy}{dx} < 0,$ y is a decreasing function.	

	Working	Common Issues
3 (a)	$\text{LHS} = \dfrac{1+\sin 2x+\cos 2x}{\cos x+\sin x}$ $= \dfrac{1+(2\sin x\cos x)+(2\cos^2 x-1)}{\cos x+\sin x}$ $= \dfrac{2\cos^2 x+2\sin x\cos x}{\cos x+\sin x}$ $= \dfrac{2\cos x(\cos x+\sin x)}{\cos x+\sin x}$ $= 2\cos x$ $= \text{RHS (proven)}$	
(b)	$\dfrac{\sec^2 x}{2\tan^2 x+1} = \dfrac{3}{4}$ $\dfrac{4}{\cos^2 x} = 6\tan^2 x+3$ $\dfrac{4}{\cos^2 x} = \dfrac{6\sin^2 x}{\cos^2 x}+\dfrac{3\cos^2 x}{\cos^2 x}$ $4 = 6\sin^2 x+3\cos^2 x$ $4 = (3\sin^2 x+3\cos^2 x)+3\sin^2 x$ $4 = 3+3\sin^2 x$ $\sin^2 x = \dfrac{1}{3}$ $\sin x = \sqrt{\dfrac{1}{3}}$ (reject as $180° < x < 270°$) or $\sin x = -\dfrac{\sqrt{3}}{3}$ **<u>Alternative:</u>** $\dfrac{1+\tan^2 x}{2\tan^2 x+1} = \dfrac{3}{4}$ $4+4\tan^2 x = 6\tan^2 x+3$ $\tan^2 x = \dfrac{1}{2}$ $\dfrac{\sin^2 x}{\cos^2 x} = \dfrac{1}{2}$ $\dfrac{\sin^2 x}{(1-\sin^2 x)} = \dfrac{1}{2}$ $2\sin^2 x = 1-\sin^2 x$ $\sin^2 x = \dfrac{1}{3}$ $\sin x = \sqrt{\dfrac{1}{3}}$ (reject as $180° < x < 270°$) or $\sin x = -\dfrac{\sqrt{3}}{3}$	

	Working	Common Issues
4 (a)	$2^x = 8(2^y)$ ----- (1) $\lg(2x+y) = \lg 63 - \lg 3$ ----- (2) From (1), $2^x = 2^3 \times 2^y$ $x = 3 + y$ ----- (3) From (2), $\lg(2x+y) = \lg\left(\frac{63}{3}\right)$ $2x + y = 21$ ----- (4) Sub (3) into (4), $2(3+y) + y = 21$ $y = 5$ $x = 8$	
4 (b)	$\log_{\sqrt{2}} y = 3 - \log_2(y-6)$ $\log_{2^{\frac{1}{2}}} y = 3 - \log_2(y-6)$ $\frac{\lg y}{\lg 2^{\frac{1}{2}}} = 3 - \frac{\lg(y-6)}{\lg 2}$ $\frac{\lg y}{\frac{1}{2}\lg 2} + \frac{\lg(y-6)}{\lg 2} = 3$ $2\lg y + \lg(y-6) = 3\lg 2$ $\lg y^2 + \lg(y-6) = \lg 2^3$ $\lg\left[y^2(y-6)\right] = \lg 8$ $y^3 - 6y^2 - 8 = 0$	

	Working	**Common Issues**

5 (i)

$$\frac{2x^2-7}{(x+1)(x^2-x-6)}=\frac{2x^2-7}{(x+1)(x-3)(x+2)}$$

$$=\frac{A}{x+1}+\frac{B}{x-3}+\frac{C}{x+2}$$

$$A(x-3)(x+2)+B(x+1)(x+2)+C(x+1)(x-3)=2x^2-7$$

When $x=1$,

$$A(-4)(1)=2(-1)^2-7$$

$$A=\frac{5}{4}$$

When $x=-2$,

$$C(-1)(-5)=2(-2)^2-7$$

$$C=\frac{1}{5}$$

When $x=3$,

$$B(4)(5)=2(3)^2-7$$

$$B=\frac{11}{20}$$

$$\frac{2x^2-7}{(x+1)(x^2-x-6)}=\frac{5}{4(x+1)}+\frac{11}{20(x-3)}+\frac{1}{5(x+2)}$$

(ii)

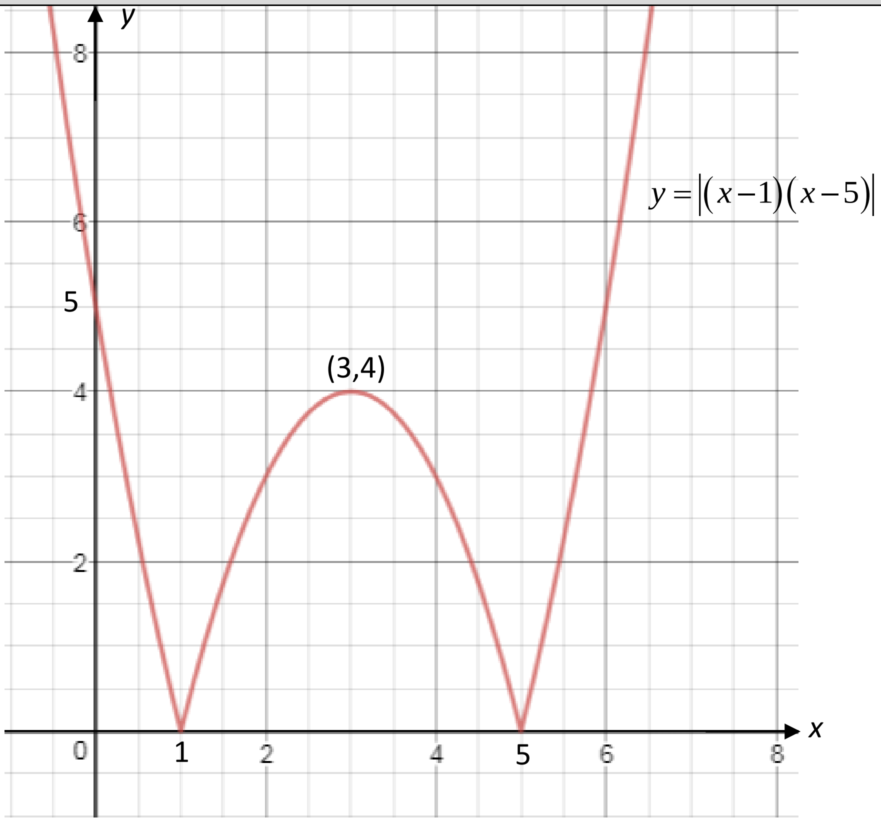
$$\int_4^5\frac{8x^2-28}{(x+1)(x^2-x-6)}\mathrm{d}x$$

$$=4\int_4^5\left[\frac{5}{4(x+1)}+\frac{11}{20(x-3)}+\frac{1}{5(x+2)}\right]\mathrm{d}x$$

$$=4\left[\frac{5}{4}\ln(x+1)+\frac{11}{20}\ln(x-3)+\frac{1}{45}\ln(x+2)\right]_4^5$$

$$=\left[\left(5\ln 6+\frac{11}{5}\ln 2+\frac{4}{5}\ln 7\right)-\left(5\ln 5+\frac{11}{5}\ln 1+\frac{4}{5}\ln 6\right)\right]$$

$$=2.56\text{ (3sf)}$$

	Working	Common Issues
6 (a i)		
(aii)	$0<a<4$	
(b)	$2x^2-6x+5=kx-3$ $2x^2-(6+k)x+8=0$ $b^2-4ac<0$ $\because$ no intersection $\left[-(6+k)\right]^2-4(2)(8)<0$ $36+12k+k^2-64<0$ $k^2+12k-28<0$ $(k+14)(k-2)<0$ $-14<k<2$	

	Working	Common Issues
7 **(i)**	$\frac{d}{dx}\left(\frac{\ln 3x}{2x^2}\right)=\frac{1}{2}\frac{d}{dx}\left(\frac{\ln 3x}{x^2}\right)$ $=\frac{1}{2}\left[\frac{x^2\left(\frac{3}{3x}\right)-(2x)(\ln 3x)}{x^4}\right]$ $=\frac{1}{2}\left[\frac{x-2x(\ln 3x)}{x^4}\right]$ $=\frac{x-2x(\ln 3x)}{2x^4}$ $=\frac{1}{2x^3}-\frac{\ln 3x}{x^3}$ (shown)	
(ii)	$\int \frac{1}{2x^3}-\frac{\ln 3x}{x^3}\,dx=\frac{\ln 3x}{2x^2}+c$ $\int \frac{\ln 3x}{x^3}\,dx=\frac{1}{2}\int x^{-3}dx-\frac{\ln 3x}{2x^2}+c$ $=\frac{1}{2}\left(\frac{x^{-2}}{-2}\right)-\frac{\ln 3x}{2x^2}+c$ $=-\frac{1}{4x^2}-\frac{2\ln 3x}{x^2}+c$	
(iii)	$f'(x)=\frac{\ln 3x}{x^3}$ $f(x)=\frac{\ln 3x}{-2x^2}-\frac{1}{4x^2}+c$ Given $f\left(\frac{1}{3}\right)=\frac{3}{4}$, $\frac{3}{4}=0-\frac{1}{4\left(\frac{1}{3}\right)^2}+c$ $c=3$ $\therefore f(x)=\frac{\ln 3x}{-2x^2}-\frac{1}{4x^2}+3$	

	Working	Common Issues
8 (i)	$\left(6-x^2\right)^5=\binom{5}{0}(6)^5\left(x^2\right)^0-\binom{5}{1}(6)^4\left(x^2\right)^1+\binom{5}{2}(6)^3\left(x^2\right)^2+\ldots.$ $=7776-6480x^2+2160x^4+\ldots.$ Coefficient of $x^4=(-6480)(2)+(2160)\left(\frac{1}{3}\right)$ $=-12960+720$ $=-12240$	
(ii)	For x term, $r=1$ $T_2=\binom{n}{1}\left(2^{n-1}\right)x$ $=\frac{2^n(n)}{2}x$ For x^2 term, $r=2$ $T_3=\binom{n}{2}\left(2^{n-2}\right)x^2$ $=\frac{2^n(n)(n-1)}{8}x^2$ $\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}=\frac{\frac{2^n(n)}{2}}{\frac{2^n(n)(n-1)}{8}}=\frac{2}{3}$ $\frac{2^n(3n)}{2}=\frac{2^n(n)(n-1)}{4}$ $2^n(6n)=2^n(n)(n-1)$ $2^n(n)(n-1)-2^n(6n)=0$ $2^n(n)\left[(n-1)-6\right]=0$ $2^n=0$ (reject as $2^n>0$) $n=0$ (reject as $n\neq0$) $n=7$	

	Working	Common Issues
9 **(a)**	$AB = 6\sin x$ $BC = 6\cos x$ $RB = \frac{6\sin x}{2} = 3\sin x$ (ratio of corresponding sides are equal) $SR = RT = \frac{6\cos x}{2} = 3\cos x$ (ratio of corresponding sides are equal) $P = 6 + 6\cos x + 3\sin x + 3\cos x + 3$ $= 9 + 9\cos x + 3\sin x$ (shown)	
(bi)	$P = 9 + 9\cos\theta + 3\sin\theta$ $= 9 + n\cos(\theta - \alpha)$ $= 9 + n(\cos\theta\cos\alpha + \sin\theta\sin\alpha)$ Comparing coefficients, $n\cos\alpha = 9$, $n\sin\alpha = 3$ $\tan\alpha = \frac{1}{3}$ $\alpha = \tan^{-1}\frac{1}{3}$ $= 18.43495°$ $n^2 = 9^2 + 3^2$ $n = \sqrt{90}$ $\therefore P = 9 + \sqrt{90}\cos(\theta - 18.43495°)$ $9 + \sqrt{90}\cos(\theta - 18.43495°) = 15$ $\cos(\theta - 18.43495°) = \frac{15-9}{\sqrt{90}}$ Basic angle $= \cos^{-1}\left(\frac{6}{\sqrt{90}}\right) = 50.7685°$ $\theta - 18.43495° = 50.7685°$ or $\theta - 18.43495° = 360° - 50.7685°$ (reject) $\theta = 69.2°$	
(bii)	Maximum value of P is when $\cos(x - 18.43495°) = 1$ $\therefore$ maximum value of $P = 9 + \sqrt{90}$ corresponding value of $x = 18.4°$	

	Working	Common Issues
10 (i)	Length of $AD=\sqrt{(y-9)^2+(4-7)^2}$ Length of $AB=\sqrt{(6-9)^2+(8-7)^2}$ $(y-9)^2+9=9+1$ $(y-9)^2=1$ $y-9=1$ or $y-9=-1$ $y=10$ (reject) or $y=8$ $\therefore$ coordinates of $D(4,8)$. Coordinates of $M=\left(\frac{8+4}{2},\frac{6+8}{2}\right)$ $=(6,7)$ Gradient of AM $=\frac{9-7}{7-6}$ $=2$ Equation of AC: $9=2(7)+c$ $c=-5$ $y=2x-5$ When $x=4,\ y=3$ $\therefore$ coordinates of $C(4,3)$.	
(ii)	$\text{M}_{AC}=\text{M}_{AM}=2$ $\text{M}_{BD}=\frac{6-8}{8-4}=-\frac{1}{2}$ Since $\text{M}_{AC}\bullet\text{M}_{BD}=-1$, diagonals AC and BD are perpendicular to each other. $\therefore$ quadrilateral $ABCD$ is a kite.	
(iii)	Area of $ABCD=\frac{1}{2}\begin{vmatrix}7 & 4 & 4 & 8 & 7\\ 9 & 8 & 3 & 6 & 9\end{vmatrix}$ $=\frac{1}{2}\{[(7\times8)+(4\times3)+(4\times6)+(8\times9)]-[(9\times4)+(8\times4)+(3\times8)+(6\times7)]\}$ $=\frac{1}{2}\times30$ $=15\text{ units}^2$	

	Working	Common Issues
11 (ai)	$20 = 100e^{-k(2.5)}$ $\ln\frac{1}{5} = -2.5k$ $k = 0.644$	
(aii)	$100e^{-0.643775t} = \frac{1}{2}100e^{0}$ $-0.643775t = \ln\frac{1}{2}$ $t = 1.08$ hours	
(bi)	$y = ax^4 + bx^3 + 7$ $\frac{dy}{dx} = 4ax^3 + 3bx^2$ Sub $x = 1$ into $\frac{dy}{dx}$, $4a + 3b = 0$ ----- (1) Sub $(1,6)$ into curve y, $6 = a + b + 7$ $a = -b - 1$ ----- (2) Sub (2) into (1), $4(-b-1) + 3b = 0$ $b = -4,\quad a = 3$	
(bii)	$\frac{dy}{dx} = 4(3)x^3 + 3(-4)x^2$ $= 12x^3 - 12x^2$ When $\frac{dy}{dx} = 0$, $12x^3 - 12x^2 = 0$ $12x^2(x-1) = 0$ $x = 0$ or $x = 1$ When $x = 0$, $y = 7$ $\therefore$ the other stationary point is $(0,7)$.	

	Working	**Common Issues**
	$\frac{d^2y}{dx^2} = 36x^2 - 24x$ When $x = 0$, $\frac{d^2y}{dx^2} = 0$ (not conclusive) | | ***x* = -0.1** | ***x* = 0** | ***x* = 0.1** | | $\frac{dy}{dx}$ | negative | 0 | negative | Using the first derivative test, the gradient changes from negative to negative, thus $(0,7)$ is a point of inflexion.	

End of Paper

CEDAR GIRLS' SECONDARY SCHOOL
Preliminary Examination 2018
Secondary Four

ADDITIONAL MATHEMATICS 4047/01

Paper 1

17 August 2018

2 hours

Additional Materials: Answer Paper

READ THESE INSTRUCTIONS FIRST

Write your Centre number, index number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

This document consists of **6** printed pages and **1** cover page.

[Turn over

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ \ldots\ (n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$cosec^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

Answer **all** the questions.

1 A cone has curved surface area $\pi\left(17-\sqrt{3}\right)$ cm^2 and slant height $\left(7-3\sqrt{3}\right)$ cm.
Without using a calculator, find the diameter of the base of the cone, in cm, in the form of $a+b\sqrt{3}$, where a and b are integers. [4]

2 The roots of the quadratic equation $5x^2-3x+1=0$ are $\dfrac{1}{\alpha}$ and $\dfrac{1}{\beta}$.

Find a quadratic equation with roots α^3 and β^3. [6]

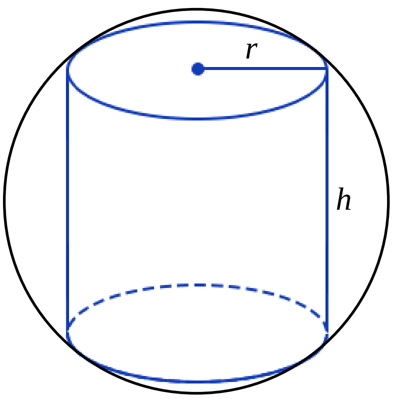
3 **(i)** Show that $2x^2+1$ is a factor of $2x^3-4x^2+x-2$. [2]

(ii) Express $\dfrac{11x-5x^2-11}{2x^3-4x^2+x-2}$ in partial fractions. [5]

4 **(i)** Sketch the graph of $y=\dfrac{4}{\sqrt{x}}$ for $x>0$. [2]

(ii) Find the coordinates of the point(s) of intersection of $y=\dfrac{4}{\sqrt{x}}$ and $y^2=81x$. [4]

5 The diagram shows a cylinder of height *h* cm and base radius *r* cm inscribed in a sphere of radius 35 cm.



(i) Show that the height of the cylinder, *h* cm, is given by $h=2\sqrt{1225-r^2}$. [2]

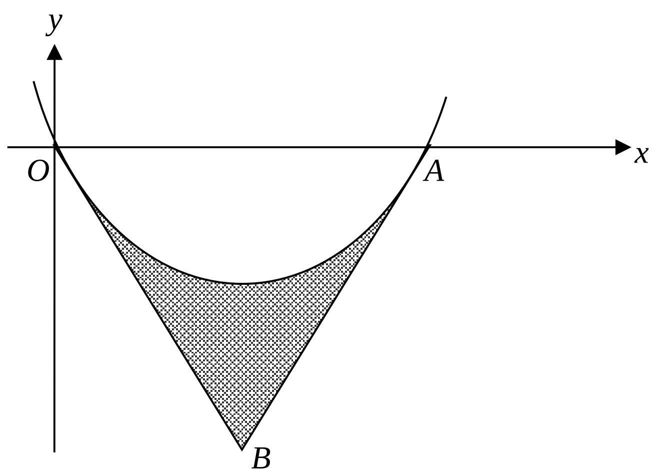
(ii) Given that *r* can vary, find the maximum volume of the cylinder. [4]

6 **(i)** Show that $\dfrac{2-\sec^2 x}{2\tan x+\sec^2 x}=\dfrac{\cos x-\sin x}{\cos x+\sin x}$. [3]

(ii) Hence find, for $0 \le x \le 2\pi$, the values of x for which $\dfrac{6-3\sec^2 x}{2\tan x+\sec^2 x}=\dfrac{3}{2}$. [3]

7 A curve is such that $\dfrac{\mathrm{d}^2 y}{\mathrm{d}x^2}=\dfrac{2}{e^{2x-3}}$ and the point $P(1.5, 2)$ lies on the curve.
The gradient of the normal to the curve at P is 10. Find the equation of the curve. [6]

8 The diagram shows the graph of $y = x^{\frac{3}{2}} - 4x$ which passes through the origin O and cuts the x-axis at the point $A(16, 0)$. Tangents to the curve at O and A meet at the point B.



(i) Show that B is the point $\left(5\dfrac{1}{3}, -21\dfrac{1}{3}\right)$. [3]

(ii) Find the area of the shaded region bounded by the curve and the lines OB and AB. [4]

9 A tram, moving along a straight road, passes station O with a velocity of 975 m/min. Its acceleration, a m/min^2, t mins after passing through station O, is given by $a = 2t - 80$.
The tram comes to instantaneous rest, first at station A and later at station B.
Find

(i) the acceleration of the tram at station A and at station B, [3]

(ii) the greatest speed of the tram as it travels from station A to station B, [2]

(iii) the distance between station A to station B. [2]

10 **(i)** By considering the general term in the binomial expansion of $\left(x^4 - \dfrac{1}{kx^2}\right)^6$, where k is a positive constant, explain why there are only even powers of x in this expansion. [2]

(ii) Given that the term independent of x in this binomial expansion is $\dfrac{5}{27}$, find the value of k. [2]

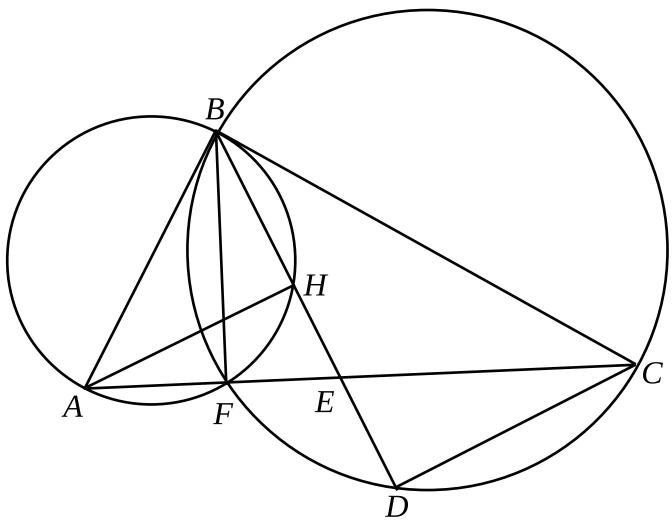
(iii) Using the value of k found in part **(ii)**, hence obtain the coefficient of x^{18} in $\left(2 - 3x^6\right)\left(x^4 - \dfrac{1}{kx^2}\right)^6$. [4]

11 M and N are two points on the circumference of a circle, where M is the point (6, 8) and N is the point (10, 16). The centre of the circle lies on the line $y = 2x + 1$.

(i) Find the equation of the circle in the form $x^2 + y^2 + ax + by + c = 0$, where a, b and c are constants. [6]

(ii) Explain whether the point (9, 10) lie inside the circle. Justify your answer with mathematical calculations. [2]

12



In the diagram, two circles intersect at *B* and *F*. *BC* is the diameter of the larger circle and is the tangent to the smaller circle at *B*.
Point *A* lies on the smaller circle such that *AFEC* is a straight line.
Point *D* lies on the larger circle such that *BHED* is a straight line.
Prove that

(i) *CD* is parallel to *AH*, [3]

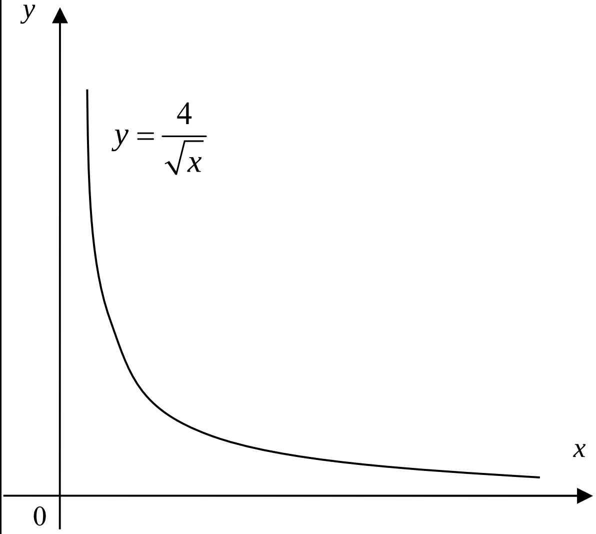
(ii) *AB* is a diameter of the smaller circle, [2]

(iii) triangles *ABC* and *BFC* are similar, [2]

(iv) $AC^2 - AB^2 = CF \times AC$. [2]

End of Paper

CEDAR GIRLS' SECONDARY SCHOOL
SECONDARY 4 ADDITIONAL MATHEMATICS
Answer Key for 2018 Preliminary Examination

PAPER 4047/1			
1	$\left(10+4\sqrt{3}\right)$ cm	**10ii**	$k=3$
2	$x^2+18x+125=0$	**10iii**	Coefficient of $x^{18}=2\left(-2\right)+(-3)\left(\frac{5}{3}\right)=-9$
3i	$2x^3-4x^2+x-2=(2x^2+1)(x-2)$ It is divisible by $2x^2+1$ with no remainder.	**11i**	$x^2+y^2-12x-26y+180=0$
3ii	$\frac{-5x^2+11x-11}{2x^3-4x^2+x-2}=-\frac{1}{x-2}+\frac{5-3x}{2x^2+1}$	**11ii**	Length of point to centre of circle = 4.24 < 5. Yes, the point lies inside the circle as its length from the centre of the circle is less than the radius.
4i		**12i**	$\angle AHD=\angle HDC$ (alternate angles)
		12ii	AB is a diameter of the smaller circle ($\angle$ in semicircle).
		12iii	Triangle ABC is similar to triangle BFC as all corresponding angles are equal.
		12iv	$\frac{BC}{FC}=\frac{AC}{CB}$ (ratio of similar triangles) $BC^2=CF\times AC$ $BC^2=AC^2-AB^2$ (Pythagoras' Theorem) $\therefore AC^2-AB^2=CF\times AC$ (shown)
4ii	$\left(\frac{4}{9},6\right)$		
5i	Using Pythagoras' Theorem: $\left(\frac{h}{2}\right)^2+r^2=35^2$		
5ii	104 000 cm^3 (3 s.f.)		
6ii	$x=0.322$ or $x=3.46$ (3 s.f.)		
7	$y=\frac{1}{2}e^{3-2x}+\frac{9}{10}x+\frac{3}{20}$		
8ii	68.3 units^2 (3 s.f.)		
9i	Acceleration at A = – 50 m/min^2 Acceleration at B = 50 m/min^2		
9ii	Greatest speed = 625 m/min		
9iii	20.8 km (3 s.f.)		
10i	General term $=\binom{6}{r}\left(x\right)^{24-6r}\left(-\frac{1}{k}\right)^r$ Since $6r$ is an even number, $24-6r$ will be even.		

2018 Preliminary Examination 2
Additional Mathematics 4047 Paper 1
Solutions

Qn	Working
1	$\pi rl = \pi\left(17-\sqrt{3}\right)$ $r = \dfrac{\left(17-\sqrt{3}\right)}{7-3\sqrt{3}}$ $r = \dfrac{\left(17-\sqrt{3}\right)}{7-3\sqrt{3}} \times \dfrac{7+3\sqrt{3}}{7+3\sqrt{3}}$ $r = \dfrac{110+44\sqrt{3}}{22}$ $r = 5+2\sqrt{3}$ Diameter = $=10+4\sqrt{3}$ cm
2	$\dfrac{1}{\alpha}+\dfrac{1}{\beta} = -\dfrac{-3}{5}$ $=\dfrac{3}{5}$ $\dfrac{1}{\alpha\beta} = \dfrac{1}{5}$ $\alpha\beta = 5$ $\dfrac{1}{\alpha}+\dfrac{1}{\beta} = \dfrac{\alpha+\beta}{\alpha\beta}$ $\dfrac{\alpha+\beta}{5} = \dfrac{3}{5}$ $\alpha+\beta = 3$ $\alpha^3+\beta^3 = (\alpha+\beta)(\alpha^2-\alpha\beta+\beta^2)$ $=3[(\alpha+\beta)^2-3\alpha\beta]$ $=3[(3)^2-3(5)]$ $=-18$ $\alpha^3\beta^3 = (\alpha\beta)^3$ $=125$ Equation: $x^2+18x+125=0$

Qn	Working

3i

$2x^3 - 4x^2 + x - 2 = (2x^2+1)(x-2)$

It is divisible by $2x^2+1$ with no remainder.

3ii

$$\frac{-5x^2+11x-11}{2x^3-4x^2+x-2} = \frac{A}{x-2} + \frac{Bx+C}{2x^2+1}$$

$-5x^2+11x-11 = A(2x^2+1) + (Bx+C)(x-2)$

When $x = 2$,

$A = -1$

Comparing x^2: $-5 = 2A + B$

$-5 = -2 + B$

$B = -3$

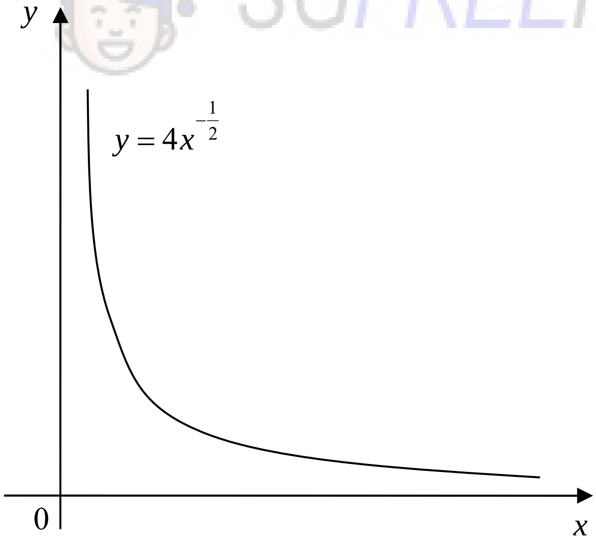
Comparing constant: $-11 = A - 2C$

$-11 = -1 - 2C$

$C = 5$

$$\frac{-5x^2+11x-11}{2x^3-4x^2+x-2} = -\frac{1}{x-2} + \frac{5-3x}{2x^2+1}$$

4



4ii

$$\left(\frac{4}{\sqrt{x}}\right)^2 = 81x$$

$$\frac{16}{x} = 81x$$

$$81x^2 = 16$$

$$x = \pm\frac{4}{9}$$

$$x = \frac{4}{9}$$

$$y = 6$$

Point of intersection = $\left(\frac{4}{9}, 6\right)$

5i

$$\left(\frac{h}{2}\right)^2 + r^2 = 35^2 \text{ (Pythagoras' Theorem)}$$

$$\frac{h^2}{4} = 1225 - r^2$$

$$h^2 = 4(1225 - r^2)$$

$$h = 2\sqrt{1225 - r^2}$$

(shown)

5ii

$$V = \pi r^2 (2\sqrt{1225 - r^2})$$

$$V = 2\pi r^2 (1225 - r^2)^{\frac{1}{2}}$$

$$\frac{dV}{dr} = 2\pi r^2 (\frac{1}{2}(-2r)(1225 - r^2)^{-\frac{1}{2}}) + (1225 - r^2)^{\frac{1}{2}}(4\pi r)$$

$$= -2\pi r^3 (1225 - r^2)^{-\frac{1}{2}} + 4\pi r(1225 - r^2)^{\frac{1}{2}}$$

$$-2\pi r^3 (1225 - r^2)^{-\frac{1}{2}} + 4\pi r(1225 - r^2)^{\frac{1}{2}} = 0$$

$$r^3 = 2r(1225 - r^2)$$

$$3r^3 = 2450r$$

$r = 28.577$ (reject $r = 0$ and -ve r)

Using First Derivative Test,

x	28.577 (-)	28.577	28.577 (+)
Sign of $\frac{dV}{dr}$	+ve	0	-ve
slope	/	—	\

V is maximum at $r = 28.577$

	Maximum volume: $V=\pi(28.577)^2(2\sqrt{1225-(28.577)^2})$ $=103\,688$ $=104\,000$ $=104\,000\text{ cm}^3$ (3 s.f.)

Qn	**Working**
6i	LHS: $\dfrac{2-\sec^2 x}{2\tan x+\sec^2 x}=\dfrac{2-(\tan^2 x+1)}{2\tan x+(\tan^2 x+1)}$ $=\dfrac{1-\tan^2 x}{2\tan x+\tan^2 x+1}$ $=\dfrac{(1-\tan x)(1+\tan x)}{(\tan x+1)^2}$ $=\dfrac{1-\tan x}{1+\tan x}$ $=\dfrac{1-\dfrac{\sin x}{\cos x}}{1+\dfrac{\sin x}{\cos x}}$ $=\dfrac{\cos x-\sin x}{\cos x}\times\dfrac{\cos x}{\cos x+\sin x}$ $=\dfrac{\cos x-\sin x}{\cos x+\sin x}$ (shown)
6ii	$3\times\dfrac{\cos x-\sin x}{\cos x+\sin x}=\dfrac{3}{2}$ $\dfrac{\cos x-\sin x}{\cos x+\sin x}=\dfrac{1}{2}$ $2\cos x-2\sin x=\cos x+\sin x$ $\cos x=3\sin x$ $\tan x=\dfrac{1}{3}$ $x=0.322$ or $x=3.46$ (3 s.f.)

Qn	Working
7	$\frac{d^2y}{dx^2}=2e^{3-2x}$ $\frac{dy}{dx}=2[-\frac{1}{2}e^{3-2x}]+c$ $\frac{dy}{dx}=-e^{3-2x}+c$ Gradient at tangent at $P=-\frac{1}{10}$ $-e^{3-2x}+c=-\frac{1}{10}$ when $x=1.5$ $c=\frac{9}{10}$ $\frac{dy}{dx}=-e^{3-2x}+\frac{9}{10}$ $y=\frac{1}{2}e^{3-2x}+\frac{9}{10}x+c$ $2=\frac{1}{2}e^{3-2(1.5)}+\frac{9}{10}(1.5)+c$ $c=\frac{3}{20}$ Eqn: $y=\frac{1}{2}e^{3-2x}+\frac{9}{10}x+\frac{3}{20}$

Qn	Working
8i	$\frac{dy}{dx}=\frac{3}{2}x^{\frac{1}{2}}-4$ At O, $x=0$, $\frac{dy}{dx}=-4$ Equation OB: $y=-4x$ …(1) At A, $x=16$, $\frac{dy}{dx}=2$ $y=2x+c$ $0=2(16)+c$ $c=-32$ Equation AB: $y=2x-32$ $2x-32=-4x$ $x=5\frac{1}{3}$ Sub into (1), $y=-21\frac{1}{3}$ $B=\left(5\frac{1}{3},-21\frac{1}{3}\right)$ (shown)
ii	Area of curve = $\left\lvert\int_0^{16} x^{\frac{3}{2}}-4x\,dx\right\rvert=\left\lvert\left[\frac{2}{5}x^{\frac{5}{2}}-2x^2\right]_0^{16}\right\rvert$ $=102.4$ units^2 Area of triangle OAB = $\frac{1}{2}\times16\times21\frac{1}{3}$ $=170\frac{2}{3}$ units^2 Area of shaded region $=170\frac{2}{3}-102.4$ $=68.3$ units^2 (3 s.f.)

Qn	Working
9i	$a = 2t - 80$ $v = t^2 - 80t + c$ $t = 0$, $v = 975$ $975 = (0)^2 - 80(0) + c$ $c = 975$ $v = t^2 - 80t + 975$ When $v = 0$, $t^2 - 80t + 975 = 0$ $(t-15)(t-65) = 0$ $t = 15$, $t = 65$ Acceleration at a = 2(15) – 80 = – 50 m/min^2 Acceleration at a = 2(65) – 80 = 50 m/min^2
9ii	When $a = 0$, $t = \frac{15+65}{2}$ $t = 40$ $v = (40)^2 - 80(40) + 975$ $v = -625$ m/min Greatest speed = 625 m/min
9iii	Distance AB = $\left\| \int_{15}^{65} t^2 - 80t + 975\, dt \right\|$ $= \left\| \left[\frac{t^3}{3} - 40t^2 + 975t \right]_{15}^{65} \right\|$ $= 20833\frac{1}{3}$ m = 20 800 m (3 s.f.) = 20.8 km

Qn	Working
10(i)	General Term $=\binom{6}{r}\left(x^4\right)^{6-r}\left(-\frac{1}{k}x^{-2}\right)^r$ $=\binom{6}{r}(x)^{24-6r}\left(-\frac{1}{k}\right)^r$ Since $6r$ is an even number, $24-6r$ will be even.
(ii)	For independent term, $24-6r=0 \Rightarrow r=4$ $\binom{6}{4}\left(-\frac{1}{k}\right)^4=\frac{5}{27}$ $\frac{15}{k^4}=\frac{5}{27}$ $k=+\sqrt[4]{\frac{27\times 15}{5}}=3$ (as $k>0$)
(iii)	$\left(2-3x^6\right)(\ldots+\text{Term in } x^{18}+\text{Term in } x^{12}+\ldots)$ For term in x^{18}, $24-6r=18 \Rightarrow r=1$ Therefore, term in $x^{18}=\binom{6}{1}\left(-\frac{1}{3}\right)x^{18}=-2x^{18}$ For term in x^{12}, $24-6r=12 \Rightarrow r=2$ Therefore, term in $x^{12}=\binom{6}{2}\left(-\frac{1}{3}\right)^2 x^{12}=\frac{5}{3}x^{12}$ Coefficient of $x^{18}=2(-2)+(-3)\left(\frac{5}{3}\right)=-9$

Qn	Working
11i	Let *MN* be a chord of circle. Midpoint of *MN* $=\left(\frac{10+6}{2},\frac{16+8}{2}\right)$ = (8, 12) Gradient of *MN* $=\frac{16-8}{10-6}$ = 2 Gradient of perpendicular bisector $=-\frac{1}{2}$ Equation of perpendicular bisector of *MN*: $y-12=-\frac{1}{2}(x-8)$ $y=-\frac{1}{2}x+16$ $-\frac{1}{2}x+16=2x+1$ $x=6$ $y=13$ Centre of circle = (6, 13) Radius = 13 – 8 = 5 units Equation of circle: $(x-6)^2+(y-13)^2=5^2$ $x^2+y^2-12x-26y+180=0$
11ii	Length of point to centre of circle $=\sqrt{(9-6)^2+(10-13)^2}$ $=\sqrt{18}$ $=4.24$ units < 5 (radius) Yes, the point lies inside the circle as its length from the centre of the circle is less than the radius.

Qn	Working
12i	$\angle BDC = 90°$ ($\angle$ in semicircle) $\angle BFC = 90°$ ($\angle$ in same segment) or ($\angle$ in semicircle) $\angle BFA = 180° - 90°$ (adj $\angle s$ on straight line) $= 90°$ $\angle BHA = \angle BFA = 90°$ ($\angle$ in same segment) $\angle AHD = 180° - 90°$ (adj $\angle s$ on straight line) $= 90°$ $\angle AHD = \angle BDC = \angle HDC$ (alternate angles) $\therefore CD // AH$
12ii	$\angle BHA = \angle BFA = 90°$ ($\angle$ in same segment) AB is a diameter of the smaller circle ($\angle$ in semicircle).
12iii	Since AB and BC are tangents to the smaller and bigger circle respectively, $\angle ABC = 90°$ (tan $\perp$ rad) $\angle ABC = \angle BFC$ $\angle BCA = \angle FCB$ (common $\angle$) Triangle ABC is similar to triangle BFC as all corresponding angles are equal.
12iv	$\frac{BC}{FC} = \frac{AC}{CB}$ (ratio of similar triangles) $BC^2 = CF \times AC$ $BC^2 = AC^2 - AB^2$ (Pythagoras' Theorem) $\therefore AC^2 - AB^2 = CF \times AC$ (shown)

CEDAR GIRLS' SECONDARY SCHOOL
Preliminary Examination
Secondary Four

ADDITIONAL MATHEMATICS

4047/02
20 August 2018

2 hours 30 minutes

Additional Materials: Answer Paper
Graph paper (1 sheet)

READ THESE INSTRUCTIONS FIRST

Write your Centre number, index number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ \ldots\ (n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$cosec^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

$$\sin A + \sin B = 2\sin \tfrac{1}{2}(A+B)\cos \tfrac{1}{2}(A-B)$$

$$\sin A - \sin B = 2\cos \tfrac{1}{2}(A+B)\sin \tfrac{1}{2}(A-B)$$

$$\cos A + \cos B = 2\cos \tfrac{1}{2}(A+B)\cos \tfrac{1}{2}(A-B)$$

$$\cos A - \cos B = -2\sin \tfrac{1}{2}(A+B)\sin \tfrac{1}{2}(A-B)$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

Answer **all** the questions.

1 **(a)** Given that $3\lg\left(x\sqrt[3]{y}\right) = 2 + 2\lg x - \lg y$, where x and y are positive numbers, express, in its simplest form, y in terms of x. [3]

(b) Given that $p = \log_8 q$, express, in terms of p,

(i) $\log_8\left(\dfrac{1}{q}\right)$, [2]

(ii) $\log_2 4q$. [2]

2 **(i)** Show that $\dfrac{\mathrm{d}}{\mathrm{d}x}(\sin x \cos x) = 2\cos^2 x - 1$. [2]

(ii) Hence, without using a calculator, find the value of each of the constants a and b for which

$$\int_0^{\frac{\pi}{4}} \cos^2 x \, \mathrm{d}x = a + b\pi.$$ [4]

3 The variables x and y are such that when values of $\dfrac{1}{y} + \dfrac{1}{x}$ are plotted against $\dfrac{1}{x}$, a straight line with gradient m is obtained. It is given that $y = \dfrac{1}{6}$ when $x = 1$ and that $y = \dfrac{1}{2}$ when $x = \dfrac{1}{2}$.

(i) Find the value of m. [4]

(ii) Find the value of x when $\dfrac{3}{y} + \dfrac{3}{x} = 3$. [2]

(iii) Express y in terms of x. [2]

4 The equation of a curve is $y = x^3 + px^2$, where p is a positive constant.

(i) Show that the origin is a stationary point on the curve and find the x-coordinate of the other stationary point in terms of p. [3]

(ii) Find the nature of each of the stationary points. [3]

Another curve has equation $y = x^3 + px^2 + px$.

(iii) Find the set of values of p for which this curve has no stationary points. [3]

5 A quadratic function f(x) is given by $\mathrm{f}(x) = k(x-2)^2 - (x-3)(x+2)$, where k is a constant and $k \neq 1$.

(i) Find the value of k such that the graph of $y = \mathrm{f}(x)$ touches the x-axis at one point. [3]

(ii) Find the range of values of k for which the function possesses a maximum point. [1]

(iii) Find the range of values of k for which the value of the function never exceeds 18. [3]

6 **(a)** A substance is decaying in such a way that its mass, m kg, at a time t years from now is given by the formula

$$m = 240e^{-0.04t}.$$

(i) Find the time taken for the substance to halve its mass. [2]

(ii) Find the value of t for which the mass is decreasing at a rate of 2.1 kg per year. [3]

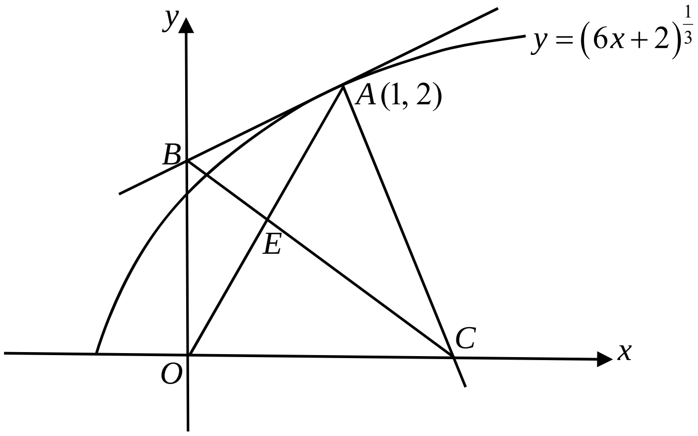
(b) The noise rating, N and its intensity, I are connected by the formula

$$N = 10\left(\lg\frac{I}{k}\right), \text{ where } k \text{ is a constant.}$$

A hot water pump has a noise rating of 50 decibels.
A dishwasher, however, has a noise rating of 62 decibels.

Find the value of $\dfrac{\text{Intensity of the noise from the dishwasher}}{\text{Intensity of the noise from the hot water pump}}$. [3]

7



The diagram shows the curve $y=(6x+2)^{\frac{1}{3}}$ and the point $A\ (1, 2)$ which lies on the curve. The tangent to the curve at A cuts the y-axis at B and the normal to the curve at A cuts the x-axis at C.

(i) Find the equation of the tangent AB and the equation of the normal AC. [4]

(ii) Find the length of BC. [2]

(iii) Find the coordinates of the point of intersection, E, of OA and BC. [4]

8 It is given that $y_1=\tan x$ and $y_2=2\cos 2x+1$.

(i) State the period, in radians, of y_1 and the amplitude of y_2. [2]

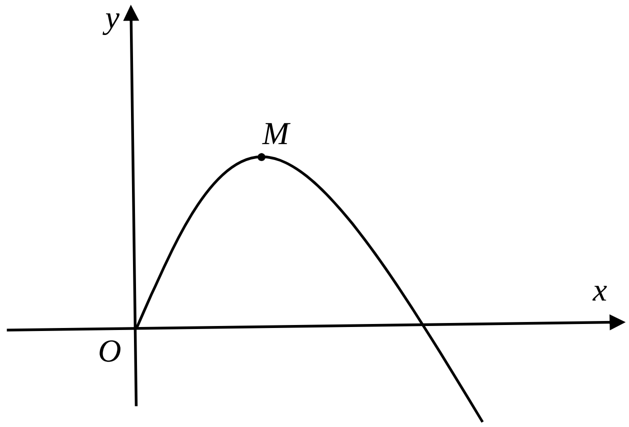
For the interval $0\le x\le 2\pi$,

(ii) sketch, on the same diagram, the graphs of y_1 and y_2, [3]

(iii) state the number of roots of the equation $|\tan x|-2\cos 2x=1$, [1]

(iv) find the range(s) of values of x for which y_1 and y_2 are both increasing as x increases. [2]

9 **(a)**



The diagram shows part of the curve,

$$y = \tan x \cos 2x,$$

and its maximum point M.

(i) Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 4\cos^2 x - \sec^2 x - 2$. [5]

(ii) Hence find the x-coordinate of M. [3]

(b) A particle moves along the line $y = \ln\sqrt{\dfrac{5x}{x-2}}$ in such a way that the x-coordinate is increasing at a constant rate of 0.4 units per second.
Find the rate at which the y-coordinate of the particle is increasing at the instant when $x = 2.5$. [3]

10 **(a)** The function f is defined for all real values of x by $f(x) = e^{2x} - 3e^{-2x}$.

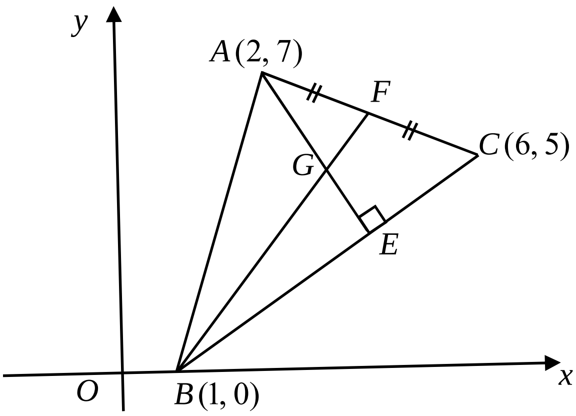
(i) Show that $f'(x) > 0$ for all values of x. [2]

(ii) Show that $f''(x) = h f(x)$, where h is an integer. [2]

(iii) Find the value of x for which $f''(x) = 0$ in the form $x = p \ln q$, where p and q are rational numbers. [2]

(b) The function g is defined for all real values of x by $g(x) = e^{2x} + 3e^{-2x}$.
The curve $y = g(x)$ and the line $x = \frac{1}{4}\ln 3$ intersect at point Q.
Show that the y-coordinate of Q is $k\sqrt{3}$, where k is an integer. [2]

11 **Solutions to this question by accurate drawing will not be accepted.**



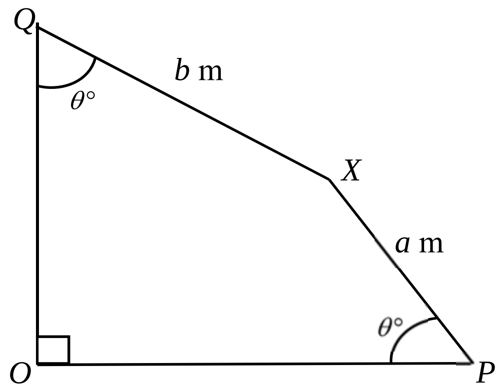
The diagram, which is not drawn to scale, shows a triangle ABC with vertices A (2, 7), B (1, 0) and C (6, 5) respectively. E and F are points on BC and AC respectively for which AE is perpendicular to BC and BF bisects AC.
G is the point of intersection of lines AE and BF.
Find

(i) the coordinates of G, [4]

(ii) the coordinates of the point D such that $ABCD$ is a parallelogram, [2]

(iii) the area of $ABCD$. [2]

12



The diagram above shows a quadrilateral in which $PX = a$ m and $QX = b$ m.
Angle OQX = Angle OPX = $\theta°$ and OQ is perpendicular to OP.

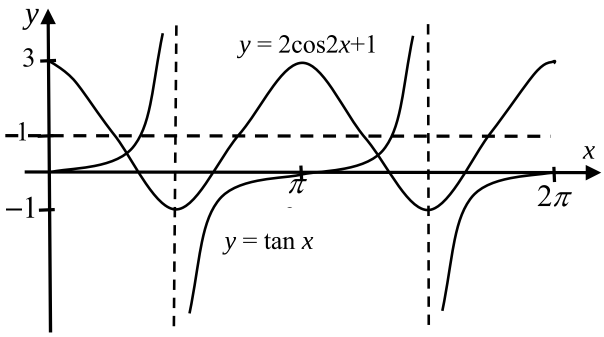
(i) Show that $OP = a\cos\theta + b\sin\theta$. [3]

(ii) It is given that the maximum length of OP is $\sqrt{5}$ m and the corresponding value of θ is $63.43°$.
By using $OP = R\cos(\theta - \alpha)$, where $R > 0$ and θ is acute, find the value of a and of b. [5]

(iii) Given that $OP = 2.15$ m, find the value of θ . [2]

End of Paper

CEDAR GIRLS' SECONDARY SCHOOL
SECONDARY 4 ADDITIONAL MATHEMATICS
Answer Key for Prelim Examination 2018

PAPER 4047/02			
1a	$y=\frac{10}{\sqrt{x}}$	**8(i)**	Period of $y_1=\pi$ radians
1bi	$-p$		Amplitude of $y_2=2$
bii	$2+3p$	**8(ii)**	
2ii	$a=\frac{1}{4}, b=\frac{1}{8}$		
3(i)	$m=-3$		
3(ii)	$x=\frac{1}{3}$		
3(iii)	$y=\frac{x}{10x-4}$		
4(i)	$x=-\frac{2p}{3}$		
4(ii)	$(0, 0)$ is a minimum point.	**8(iii)**	4
	maximum point at $x=-\frac{2p}{3}$	**8(iv)**	$\frac{\pi}{2}<x<\pi$, $\frac{3\pi}{2}<x<2\pi$
4(iii)	$\{p:0<p<3\}$	**9a(ii)**	0.452 or 25.9°
5(i)	$k=\frac{25}{16}$	**9b**	-0.32 units per second
5(ii)	$k<1$	**10a(iii)**	$x=\frac{1}{4}\ln 3$
5(iii)	$k\le\frac{47}{56}$	**10b**	$2\sqrt{3}$
6ai	17.3 years	**11(i)**	$G\left(3\frac{2}{3}, 5\frac{1}{3}\right)$
6aii	$t=38.0$	**11(ii)**	$(7,12)$
6b	15.8	**11(iii)**	30 sq units
7(i)	Eqn of AB: $y=\frac{1}{2}x+\frac{3}{2}$	**12 (ii)**	$a=1.00$, $b=2.00$
	Eqn of AC: $y=-2x+4$	**12(iii)**	$\theta=79.4$ or 47.5
7(ii)	2.5 units		
7(iii)	Coordinates of $E=\left(\frac{6}{11}, 1\frac{1}{11}\right)$		

2018 Preliminary Examination 2
Additional Mathematics 4047/2
Solutions

Qn	Working	Marks	Total	Remarks
1a	$3\lg\left(x\sqrt[3]{y}\right)=2+2\lg x-\lg y$ $3\lg x+\lg y=2+2\lg x-\lg y$ $\lg x+2\lg y=2$ $\lg\left(xy^2\right)=2$ $xy^2=10^2=100$ $y=\sqrt{\dfrac{100}{x}}=\dfrac{10}{\sqrt{x}}=\dfrac{10\sqrt{x}}{x}$		[3]	
b(i)	$\log_8\dfrac{1}{q}=\log_8 1-\log_8 q$ $=0-p=-p$		[2]	
b(ii)	$\log_2 4q=\log_2 4+\log_2 q$ $=2+\dfrac{\log_8 q}{\log_8 2}$ $=2+3p$		[2]	
		Total	[7]	
2(i)	$\dfrac{d}{dx}(\sin x\cos x)$ $=\sin x(-\sin x)+\cos x(\cos x)$ $=\cos^2 x-\sin^2 x$ $=\cos^2 x-\left(1-\cos^2 x\right)$ $=2\cos^2 x-1$		[2]	
(ii)	$\int_0^{\frac{\pi}{4}}\left(2\cos^2 x-1\right)dx=\left[\sin x\cos x\right]_0^{\frac{\pi}{4}}$ $\int_0^{\frac{\pi}{4}}\left(2\cos^2 x\right)dx-\int_0^{\frac{\pi}{4}}1\,dx=\left[\sin x\cos x\right]_0^{\frac{\pi}{4}}$ $=\dfrac{\sqrt{2}}{2}\times\dfrac{\sqrt{2}}{2}=\dfrac{1}{2}$ $2\int_0^{\frac{\pi}{4}}\left(2\cos^2 x\right)dx=\dfrac{1}{2}+\left[x\right]_0^{\frac{\pi}{4}}$ $\int_0^{\frac{\pi}{4}}\left(\cos^2 x\right)dx=\dfrac{1}{4}+\dfrac{\pi}{8}\Rightarrow a=\dfrac{1}{4}, b=\dfrac{1}{8}$		[4]	
		Total	[6]	

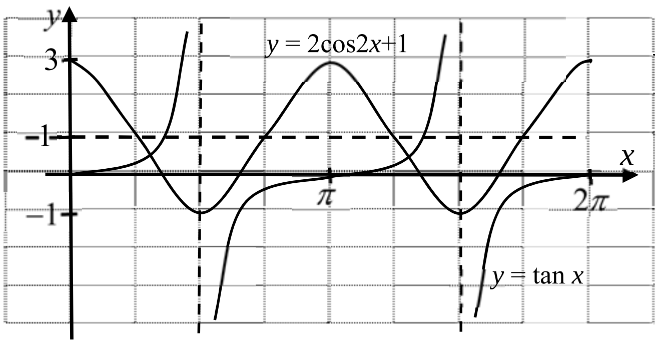
Qn	Working	Marks	Total	Remarks
3(i)	The linear equation is $\frac{1}{y}+\frac{1}{x}=m\left(\frac{1}{x}\right)+c$ Subst $y=\frac{1}{6}$ and $x=1$, $6+1=m+c \Rightarrow m+c=7$ Subst $y=\frac{1}{2}$ and $x=\frac{1}{2}$ $2+2=2m+c \Rightarrow 2m+c=4$ $m=-3$ and $c=10$		[4]	
(ii)	Since $\frac{3}{y}+\frac{3}{x}=3 \Rightarrow \frac{1}{y}+\frac{1}{x}=1$, $1=\frac{-3}{x}+10 \Rightarrow x=\frac{1}{3}$		[2]	
(iii)	$\frac{1}{y}+\frac{1}{x}=-3\left(\frac{1}{x}\right)+10$ $\frac{x+y}{xy}=\frac{-3+10x}{x}$ $y=\frac{x}{10x-4}$		[2]	
		Total	[8]	

Qn	Working	Marks	Total	Remarks
4(i)	$y = x^3 + px^2$ $\frac{dy}{dx} = 3x^2 + 2px = x(3x+2p)$ For stationary point, $\frac{dy}{dx} = 0$ $\therefore x = 0$ or $x = -\frac{2p}{3}$ When $x = 0$, $y = 0$. Therefore, $(0, 0)$ is a stationary point. The other x-coordinate of stationary point is $x = -\frac{2p}{3}$		[3]	
(ii)	$\frac{d^2y}{dx^2} = 6x + 2p$ When $x = 0$, $\frac{d^2y}{dx^2} = 2p > 0$ as $p > 0$ Therefore, $(0, 0)$ is a minimum point. When $x = -\frac{2p}{3}$, $\frac{d^2y}{dx^2} = 6\left(-\frac{2p}{3}\right) + 2p = -2p < 0$ as $p > 0$ Therefore, there is a maximum point at $x = -\frac{2p}{3}$		[3]	
(iii)	$y = x^3 + px^2 + px$ $\frac{dy}{dx} = 3x^2 + 2px + p$ Since $\frac{dy}{dx} \neq 0$, $b^2 - 4ac < 0$ $(2p)^2 - 4(3)(p) < 0$ $4p^2 - 12p < 0$ $4p(p-3) < 0$ The set is $\{p : 0 < p < 3\}$		[3]	
		Total	[9]	

Qn	Working	Marks	Total	Remarks
5(i)	$f(x) = k(x-2)^2 - (x-3)(x+2)$ $= k(x^2 - 4x + 4) - (x^2 - x - 6)$ $= kx^2 - 4kx + 4k - x^2 + x + 6$ $= (k-1)x^2 + (1-4k)x + 4k + 6$ Since it touches the x-axis at one point, $b^2 - 4ac = 0$ $(1-4k)^2 - 4(k-1)(4k+6) = 0$ $25 - 16k = 0$ $k = \frac{25}{16}$		**[3]**	
(ii)	$k < 1$		**[1]**	
(iii)	$(k-1)x^2 + (1-4k)x + 4k + 6 \le 18$ $(k-1)x^2 + (1-4k)x + 4k - 12 \le 0$ $b^2 - 4ac \le 0$ and $k < 1$ $(1-4k)^2 - 4(k-1)(4k-12) \le 0$ and $k < 1$ $56k - 47 \le 0$ and $k < 1$ $k \le \frac{47}{56}$ and $k < 1$ The solution is $k \le \frac{47}{56}$		**[3]**	
		Total	**[7]**	

Qn	Working	Marks	Total	Remarks
6a(i)	When $t = 0,\ m = 240$ When $240e^{-0.04t} = 120$ $e^{-0.04t} = 0.5$ $t = \frac{\ln 0.5}{-0.04}$ $t = 17.3$ No. of years = 17.3		[2]	
a(ii)	$\frac{dm}{dt} = 240(-0.04)e^{-0.04t} = -9.6e^{-0.04t}$ $-9.6e^{-0.04t} = -2.1$ $t = \frac{\ln\left(\frac{2.1}{9.6}\right)}{-0.04} = 38.0$		[3]	
b	$10\lg\left(\frac{I_P}{k}\right) = 50 \Rightarrow \left(\frac{I_P}{k}\right) = 10^5$ where I_P = intensity of pump $\lg\frac{I_D}{k} = \frac{62}{10} = 6.2 \Rightarrow \left(\frac{I_D}{k}\right) = 10^{6.2}$ where I_D = intensity of dishwasher $\frac{I_D}{I_P} = \frac{10^{6.2}k}{10^5 k} = 15.8$		[3]	
		Total	[8]	

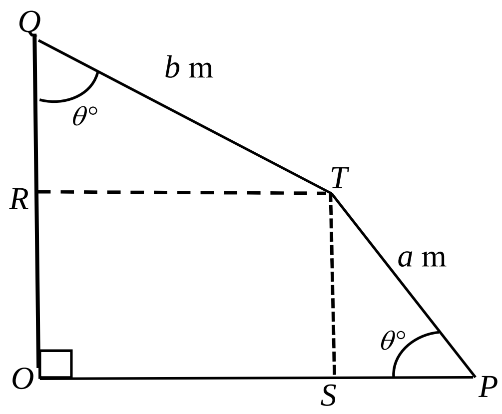
Qn	Working	Marks	Total	Remarks
7(i)	$y=(6x+2)^{\frac{1}{3}}$ $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{3}(6x+2)^{-\frac{2}{3}}\bullet 6=\frac{2}{(6x+2)^{\frac{2}{3}}}$ When $x=1$, $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2}{(6(1)+2)^{\frac{2}{3}}}=\frac{1}{2}$ Eqn of AB: $y-2=\frac{1}{2}(x-1)\Rightarrow y=\frac{1}{2}x+\frac{3}{2}$ Eqn of AC: $y-2=-2(x-1)\Rightarrow y=-2x+4$		[4]	Use of chain rule Correct substitution
7ii	When $x=0$, $y=1.5$ Coordinates of $B=(0,1.5)$ When $y=0$, $-2x+4=0\Rightarrow x=2$ Coordinates of $C=(2,0)$ $BC=\sqrt{1.5^2+2^2}=2.5$ units		[2]	
7iii	Gradient of $OA=\frac{2-0}{1-0}=2$ Therefore, eqn of OA: $y=2x$ Gradient of $BC=\frac{1.5}{-2}=-\frac{3}{4}$ Therefore, eqn of BC: $y=-\frac{3}{4}x+\frac{3}{2}$ At E, $2x=-\frac{3}{4}x+\frac{3}{2}$ $\frac{11x}{4}=\frac{3}{2}\Rightarrow x=\frac{6}{11}$ $y=2\left(\frac{6}{11}\right)=\frac{12}{11}=1\frac{1}{11}$ Coordinates of $E=\left(\frac{6}{11},1\frac{1}{11}\right)$		[4]	
		Total	**[10]**	

Qn	Working	Marks	Total	Remarks
8i	Period of $y_1 = \pi$ radians Amplitude of $y_2 = 2$		[2]	
ii				
iv	$\frac{\pi}{2} < x < \pi$, $\frac{3\pi}{2} < x < 2\pi$		[2]	

Qn	Working	Marks	Total	Remarks
9a(i)	$y=\tan x\cos 2x$ $\frac{dy}{dx}=\tan x(-2\sin 2x)+\cos 2x(\sec^2 x)$ $=\frac{\sin x}{\cos x}(-2\times 2\sin x\cos x)+(2\cos^2 x-1)\left(\frac{1}{\cos^2 x}\right)$ $=-4\sin^2 x+2-\sec^2 x$ $=-4(1-\cos^2 x)+2-\sec^2 x$ $=4\cos^2 x-\sec^2 x-2$		**[5]**	
(ii)	When $\frac{dy}{dx}=0$, $4\cos^2 x-\sec^2 x-2=0$ $4\cos^4 x-2\cos^2 x-1=0$ $\cos^2 x=\frac{2\pm\sqrt{4-4(4)(-1)}}{8}$ $=0.80902$ $\cos x=0.89945$ $x=0.452$ or $25.9°$ The x-coordinate of M is 0.452.		**[3]**	
b	$y=\frac{1}{2}[\ln 5x-\ln(x-2)]$ $\frac{dy}{dx}=\frac{1}{2}\left(\frac{5}{5x}\right)-\frac{1}{2}\left(\frac{1}{x-2}\right)$ $=\frac{1}{2x}-\frac{1}{2(x-2)}$ $\frac{dy}{dt}=\frac{dy}{dx}\times\frac{dx}{dt}$ When $x=2.5$, $\frac{dy}{dt}=\left(\frac{1}{5}-\frac{1}{2(0.5)}\right)\times 0.4=-\frac{8}{25}=-0.32$ The rate is -0.32 units per second.		**[3]**	
		Total	**[11]**	

Qn	Working	Marks	Total	Remarks
10(a)(i)	$\mathrm{f}(x)=e^{2x}-3e^{-2x}$ $\mathrm{f}'(x)=2e^{2x}+6e^{-2x}$ Since $e^{2x}>0$ and $e^{-2x}>0$, $\mathrm{f}'(x)>0$		[2]	
(ii)	$\mathrm{f}''(x)=4e^{2x}-12e^{-2x}=4(e^{2x}-3e^{-2x})$ Therefore $\mathrm{f}''(x)=4\mathrm{f}(x)$		[2]	
(iii)	$e^{2x}-3e^{-2x}=0$ $e^{2x}=\dfrac{3}{e^{2x}}$ $e^{4x}=3$ $4x\ln e=\ln 3$ $x=\dfrac{1}{4}\ln 3$		[2]	
(b)	$g(x)=e^{2x}+3e^{-2x}$, When $x=\dfrac{1}{4}\ln 3$, $g(x)=e^{2(\frac{1}{4}\ln 3)}+ke^{-2(\frac{1}{4}\ln 3)}=e^{\frac{1}{2}\ln 3}+ke^{-\frac{1}{2}\ln 3}$ $=\sqrt{3}+\dfrac{3}{\sqrt{3}}=2\sqrt{3}$ Therefore the y-coordinate is $2\sqrt{3}$.		[2]	
		Total	[8]	

Qn	Working	Marks	Total	Remarks
11i	Mid-point of AC, $F = \left(\frac{2+6}{2}, \frac{7+5}{2}\right) = (4, 6)$ Gradient of $BF = \frac{6-0}{4-1} = 2$ Eqn of BF: $y - 0 = 2(x-1) \Rightarrow y = 2x - 2$ Gradient of $BC = \frac{5-0}{6-1} = 1$ Gradient of $AE = -1$ Eqn of AE: $y - 7 = -1(x-2) \Rightarrow y = -x + 9$ $-x + 9 = 2x - 2$ $x = 3\frac{2}{3}$ $\therefore y = -3\frac{2}{3} + 9 = 5\frac{1}{3}$ $G\left(3\frac{2}{3}, 5\frac{1}{3}\right)$		[4]	
(ii)	Let (x, y) be coordinates of D. $\left(\frac{1+x}{2}, \frac{0+y}{2}\right) = (4, 6)$ $\Rightarrow x = 7, y = 12$ Coordinates of $D = (7, 12)$		[2]	
(iii)	Area of $ABCD = \frac{1}{2}\begin{vmatrix} 2 & 1 & 6 & 7 & 2 \\ 7 & 0 & 5 & 12 & 7 \end{vmatrix} = 30$ sq units		[2]	
		Total	[8]	

Qn	Working	Marks	Total	Remarks
12				
(i)	$\cos\theta = \frac{SP}{a} \Rightarrow SP = a\cos\theta$ $\sin\theta = \frac{OS}{b} \Rightarrow OS = b\sin\theta$ $OP = SP + OS$ $OP = a\cos\theta + b\sin\theta.$		**[3]**	
(ii)	$\sqrt{R} = \sqrt{a^2+b^2} \Rightarrow a^2+b^2 = 5$ Max. value of OP occurs at $\theta = 63.43°$. $\cos(\theta-\alpha)=1 \Rightarrow \theta-\alpha=0 \Rightarrow \alpha=\theta=63.43$ $\tan\alpha = \frac{b}{a} \Rightarrow \frac{b}{a} = \tan 63.43 = 1.9996 \Rightarrow b = 1.9996a$ Subst $b = 1.9996a$ in $a^2+b^2=5$ $a^2+(1.9996a)^2 = 5 \Rightarrow a = 1.00$ $\therefore b = 2.00$		**[5]**	
(iii)	$\cos\theta + 2\sin\theta = 2.15$ $\sqrt{5}\cos(\theta-63.43) = 2.15$ $(\theta-63.43) = \cos^{-1}\left(\frac{2.15}{\sqrt{5}}\right)$ $\theta = 79.4$ or 47.5		**[2]**	
		Total	**[10]**	

Name: Mark Scheme	Register No.:	Class:

CRESCENT GIRLS' SCHOOL
SECONDARY FOUR
PRELIMINARY EXAMINATION

ADDITIONAL MATHEMATICS

4047/01

Paper 1

16 August 2018

2 hours

Additional Materials: Answer Paper
Mark Sheet

READ THESE INSTRUCTIONS FIRST

Write your name, register number and class in the spaces provided at the top of this page and on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use paper clips, highlighter, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work and mark sheet securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$\Delta = \frac{1}{2}ab \sin C$$

1 The straight line $y-1=2m$ does not intersect the curve $y=x+\dfrac{m^2}{x}$.

Find the largest integer value of m. [5]

Solutions

$y=2m+1 \quad --(1)$

$y=x+\dfrac{m^2}{x} \quad --(2)$

(1) = (2): $x+\dfrac{m^2}{x}=2m+1$ [M1]

$x^2-2mx-x+m^2=0$

$x^2-(2m+1)x+m^2=0$ [M1] -- simplification

Line does not intersect curve, $b^2-4ac<0$

$[-(2m+1)]^2-4(1)(m^2)<0$ [M1]

$(2m+1+2m)(2m+1-2m)<0$

$4m+1<0$

$m<-\dfrac{1}{4}$ [A1]

The largest integer value of m is -1. [A1]

2 The line $2y+x=5$ intersects the curve $y^2=6-xy$ at the points P and Q.
Determine, with explanation, if the point $(1, 2)$ lies on the line joining the midpoint of PQ and $(3,1)$. [5]

Solutions

$x=5-2y$ ---- (1)

Sub (1) into $y^2=6-xy$

$y^2=6-(5-2y)y$ [M1] – Substitution

$y^2-5y+6=0$

$(y-3)(y-2)=0$

Hence $y=3$ or $y=2$ [A1]

Correspondingly, $x=5-2(3)$ or $x=5-2(2)$

$x=-1$ or $x=1$

The coordinates of P and Q are $(-1, 3)$ and $(1, 2)$.

Midpoint of $PQ=\left(\dfrac{-1+1}{2}, \dfrac{3+2}{2}\right)=(0, 2.5)$ [A1]

Equation of line joining midpoint of PQ and (3, 1) is $\dfrac{y-1}{2.5-1}=\dfrac{x-3}{0-3}$ [M1]

$y=-\dfrac{1}{2}x+\dfrac{5}{2}$

When $x=1$, $y=-\dfrac{1}{2}(1)+\dfrac{5}{2}=2$

Therefore, the point (1, 2) lies on the line joining midpoint of PQ and (3, 1) [A1] – *conclusion*

Alternative method

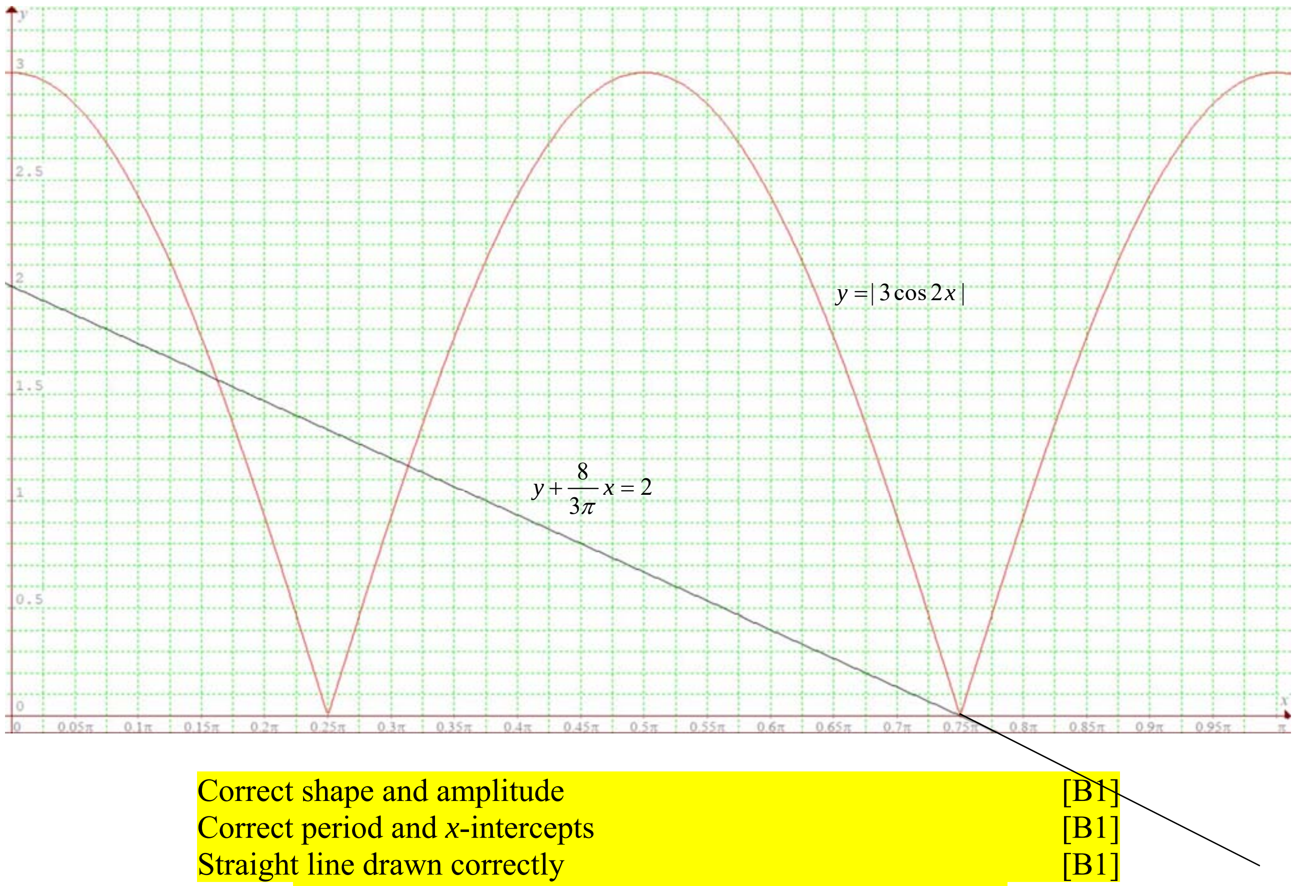
Let R be the coordinates of the midpoint of PQ, S be the point (3, 1) and T be the point (1, 2). Find gradient of RT and gradient of RS and conclude that point T lies on RS due to collinearity.

3 **(i)** Sketch on the same graph $y = |3\cos 2x|$ and $y + \frac{8}{3\pi}x = 2$ for $0 \le x \le \pi$. [3]

(ii) Hence, showing your working clearly, deduce the number of solutions in $|\cos 2x| - \frac{2}{3} + \frac{8x}{3\pi} = 0$ in the interval $0 \le x \le \pi$. [2]

Solutions

(i)



Correct shape and amplitude [B1]
Correct period and x-intercepts [B1]
Straight line drawn correctly [B1]

Minus 1m if eqn of graphs and/or axes are not labelled.

(ii) $|\cos 2x| - \frac{2}{3} + \frac{8x}{3\pi} = 0$

$|3\cos 2x| = 2 - \frac{8}{\pi}x$

$y = 2 - \frac{8}{\pi}x$ (y-intercept = 2; x-intercept = $2 \div \frac{8}{\pi} = \frac{\pi}{4}$) [M1] – St line NOT required

There is one solution. [A1]

4 **(i)** Find the value of a and of b if the curve $f(x) = ax + \frac{b}{x}$ where $x \neq 0$ has a stationary point at $(-2, -8)$. [4]

(ii) By considering the sign of $f'(x)$, determine the nature of the stationary point. [2]

Solutions

(i) $f(x) = ax + \frac{b}{x}$ Sub $x = -2$, $f(x) = -8$

$-8 = -2a - \frac{b}{2}$

$4a + b = 16$ ---- (1) [B1]

$f'(x) = a - \frac{b}{x^2}$. When $x = -2$, $f'(x) = 0$ [M1] – for $f'(x)$

$0 = a - \frac{b}{4} \Rightarrow b = 4a$ ---- (2)

Sub (2) into (1): $4a + 4a = 16$ [M1] – solve simultaneous equations

$a = 2$

Hence $b = 2(4) = 8$ [A1] – both correct

(ii) $f'(x) = 2 - \frac{8}{x^2}$

x	-2^-	-2	-2^+
Sign of $f'(x)$	+	0	–
Sketch of tangent	/	–	\

[M1] – First derivative test

$(-2, -8)$ is a maximum point. [A1] – Awarded only with correct first derivative test

5 It is given that $\int f'(x)\,dx = \frac{x}{2} - \frac{\sin kx}{8} + c$ where c is a constant of integration, and that $\int_0^{\frac{\pi}{8}} f'(x)\,dx = \frac{\pi}{16} - \frac{1}{8}$.

(i) Show that $k = 4$. [2]

(ii) Hence find $f'(x)$, expressing your answer in $\sin^2 px$, where p is a constant. [2]

(iii) Find the equation of the curve $y = f(x)$ given that the point $\left(\frac{\pi}{4}, 0\right)$ lies on the curve. [2]

Solutions

(i) $\int_0^{\frac{\pi}{8}} f'(x)\,dx = \frac{\pi}{16} - \frac{1}{8}$

$\frac{\frac{\pi}{8}}{2} - \frac{\sin k\left(\frac{\pi}{8}\right)}{8} = \frac{\pi}{16} - \frac{1}{8}$ [M1]

$\sin\left(\frac{k\pi}{8}\right) = 1$

$\frac{k\pi}{8} = \frac{\pi}{2}$ [A1]

$k = 4$ (shown)

(ii) $\int f'(x)\,dx = \frac{x}{2} - \frac{\sin 4x}{8} + c$

$f'(x) = \frac{1}{2} - \frac{1}{8}(4\cos 4x)$ [M1] -- Differentiation

$= \frac{1}{2} - \frac{1}{2}\cos 4x$

$= \frac{1}{2} - \frac{1}{2}(1 - 2\sin^2 2x)$

$= \sin^2 2x$ [A1] – Upon correct application of double angle formula

(iii) $\int f'(x)\,dx = f(x) = \frac{x}{2} - \frac{\sin 4x}{8} + c$

At $\left(\frac{\pi}{4}, 0\right)$, $0 = \frac{\pi}{8} - 0 + c$ [M1]

$c = -\frac{\pi}{8}$

$f(x) = \frac{x}{2} - \frac{\sin 4x}{8} - \frac{\pi}{8}$ [A1]

6 **(a)** The length of each side of a square of area $\left(49 + 20\sqrt{6}\right)$ m^2 can be expressed in the form $\left(\sqrt{c} + \sqrt{d}\right)$ m where c and d are integers and $c < d$.
Find the value of c and of d. [3]

(b) A parallelogram with base equals to $\left(4 - \sqrt{12}\right)$ m has an area of $\left(22 - \sqrt{48}\right)$ m^2.
Find, without using a calculator, the height of the parallelogram in the form $\left(p + q\sqrt{3}\right)$ m. [3]

Solutions

(a) $\left(\sqrt{c} + \sqrt{d}\right)^2 = 49 + 20\sqrt{6}$

$c + d + 2\sqrt{cd} = 49 + 20\sqrt{6}$ [M1] – correct expansion

$c + d = 49$

$d = 49 - c$ ----- (1)

$2\sqrt{cd} = 20\sqrt{6} \Rightarrow cd = 600$ ---- (2) [M1] – compare rational and irrational terms

Sub (1) into (2),

$c(49 - c) = 600$

$c^2 - 49c + 600 = 0$

$(c - 25)(c - 24) = 0$

Since $c < d$, $c = 24$, $d = 25$ [A1] – Both correct

(b) Height $= \dfrac{22 - 4\sqrt{3}}{4 - 2\sqrt{3}} \cdot \dfrac{4 + 2\sqrt{3}}{4 + 2\sqrt{3}}$ [M1] – Rationalise denominator

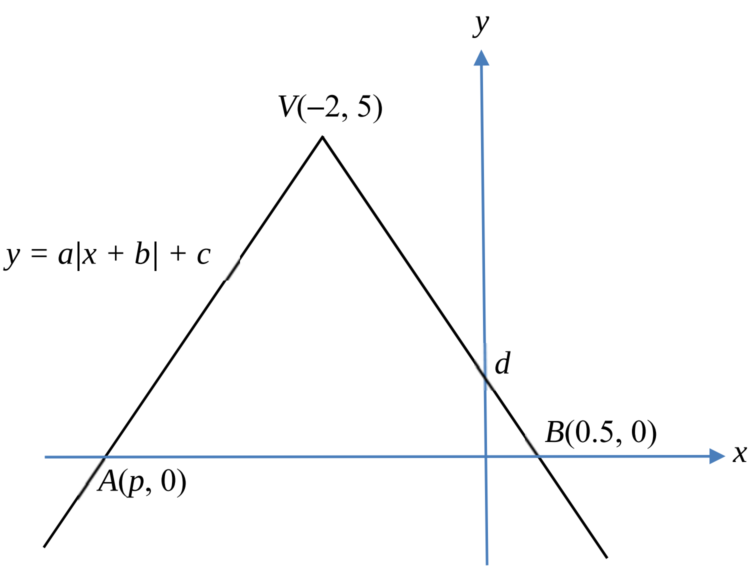
$= \dfrac{(22 - 4\sqrt{3})(4 + 2\sqrt{3})}{4^2 - 4(3)}$

$= \dfrac{1}{4}\left(88 + 44\sqrt{3} - 16\sqrt{3} - 24\right)$ [M1] – correct expansion

$= \dfrac{1}{4}\left(64 + 28\sqrt{3}\right)$

$= (16 + 7\sqrt{3})$ m [A1]

7 The diagram shows part of the graph $y = a|x + b| + c$. The graph cuts the x-axis at A $(p, 0)$ and at B $(0.5, 0)$. The graph has a vertex point at V $(-2, 5)$ and y-intercept, d.



(i) Explain why $p = -4.5$. [1]

(ii) Determine the value of each of a, b and c. [4]

(iii) State the set of values of k for which the line $y = kx + d$ intersects the graph at two distinct points. [2]

Solutions

(i) $\frac{p+0.5}{2}=-2$ [B1]

$p=-4.5$

(ii) y-coordinate of vertex point, c = 5 [B1]

$b=2$ [B1]

$y=a|x+b|+c$

$y=a|x+2|+5$

At B, $0=a|0.5+2|+5$ [M1]

$a=-2$ [A1]

(iii) Gradient of $AV=\frac{5-0}{-2+4.5}=2$

Gradient of $VB=-2$ [B1] – Any one

Hence $-2<k<2$ [B1]

8 **(i)** Differentiate $x^3 \ln x$ with respect to x. [2]

(ii) Hence find $\int \frac{x^2 \ln x}{2}\,dx$. [4]

Solutions

(i) $\frac{d}{dx}\left(x^3 \ln x\right)=x^3\left(\frac{1}{x}\right)+(\ln x)(3x^2)$ [M1] – Product Rule

$=x^2+3x^2 \ln x$ [A1]

(ii) $\frac{d}{dx}\left(x^3 \ln x\right)=x^2+3x^2 \ln x$

$\frac{x^2 \ln x}{2}=\frac{1}{6}\frac{d}{dx}(x^3 \ln x)-\frac{x^2}{6}$ [M1]

$\int \frac{x^2 \ln x}{2}\,dx=\frac{1}{6}x^3 \ln x-\frac{1}{6}\int x^2\,dx$

[M1] [M1]

$=\frac{1}{6}x^3 \ln x-\frac{1}{18}x^3+c$ [A1]

9 **(a)** If $32^y \times 5^{4y}=2^{4y+4}\times 5^{3y-1}$, determine the value of 10^y. [3]

(b) **(i)** Sketch on the same axes, the graphs of $y=x^{-2}$ and $y=\sqrt{3x}$. [2]

(ii) Find the point of intersection between the graphs. [3]

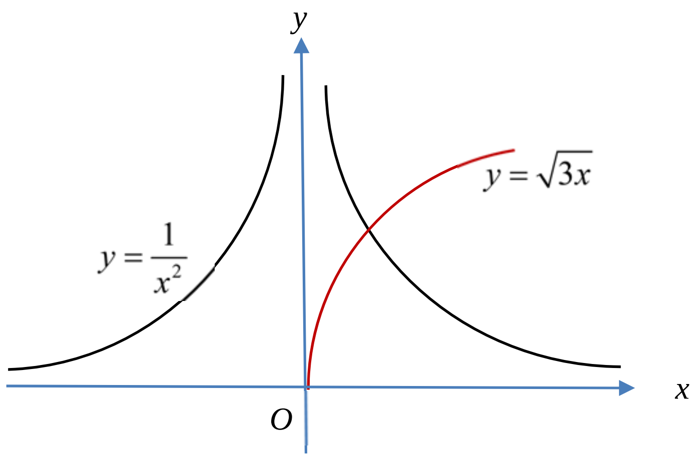
Solutions

$32^y \times 5^{4y} = 2^{4y+4} \times 5^{3y-1}$

$2^{5y} \times 5^{4y} = 2^{4y}(2^4) \times 5^{3y}\left(\frac{1}{5}\right)$ [M1] -- splitting

$\frac{2^{5y}}{2^{4y}} \times \frac{5^{4y}}{5^{3y}} = (2^4) \times \left(\frac{1}{5}\right)$ [M1] – using Laws of Indices

$2^y \times 5^y = 10^y = \frac{16}{5}$ [A1]



[B1][B1] – axes and eqns must be labelled.
Graph does not level off for $y = \sqrt{3x}$.

$\frac{1}{x^2} = \sqrt{3x}$

$\frac{1}{x^4} = 3x$ [M1] – square both sides

$x = \sqrt[5]{\frac{1}{3}} = 0.80274$ [A1]

$y = \frac{1}{0.80274^2} = 1.55$

The point of intersection is (0.803, 1.55). [A1] – 3 s.f.

10 **(i)** Express $\frac{x+1}{x(x+3)^2-(x+3)^2}$ in partial fractions. [5]

(ii) Hence find the value of $\int_2^3 \frac{x+1}{x(x+3)^2-(x+3)^2}\,dx$ giving your answer to 2 decimal places. [3]

Solutions

(i) $\frac{x+1}{x(x+3)^2-(x+3)^2} = \frac{x+1}{(x-1)(x+3)^2} = \frac{A}{x-1} + \frac{B}{x+3} + \frac{C}{(x+3)^2}$ [M1]

$x+1 = A(x+3)^2 + B(x-1)(x+3) + C(x-1)$

Sub $x=-3$: $-2=C(-4) \Rightarrow C=\frac{1}{2}$ [A1]

Sub $x=1$: $2=A(16) \Rightarrow A=\frac{1}{8}$ [A1]

Sub $x=0$: $1=\left(\frac{1}{8}\right)(9)+B(-1)(3)+\left(\frac{1}{2}\right)(-1) \Rightarrow B=-\frac{1}{8}$ [A1]

$$\frac{x+1}{x(x+3)^2-(x+3)^2}=\frac{x+1}{(x-1)(x+3)^2}=\frac{1}{8(x-1)}-\frac{1}{8(x+3)}+\frac{1}{2(x+3)^2}$$ [A1]

(ii) $$\int_2^3 \frac{x+1}{x(x+3)^2-(x+3)^2}\,dx=\int_2^3 \frac{1}{8(x-1)}-\frac{1}{8(x+3)}+\frac{1}{2(x+3)^2}\,dx$$

$$=\left[\frac{1}{8}\ln(x-1)-\frac{1}{8}\ln(x+3)+\frac{1}{2}\cdot\frac{(x+3)^{-1}}{-1}\right]_2^3$$

[M1] –Any [M1]

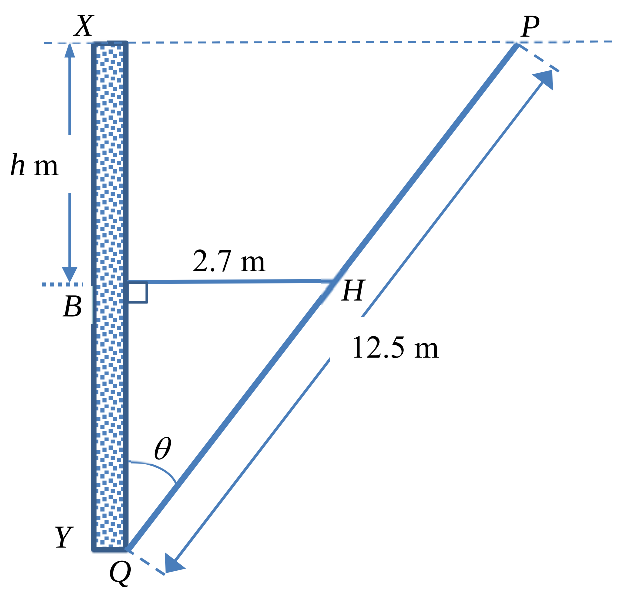
$$=\left[\frac{1}{8}\ln\left(\frac{x-1}{x+3}\right)-\frac{1}{2}\cdot\frac{1}{(x+3)}\right]_2^3$$

$$=\frac{1}{8}\ln\frac{1}{3}-\frac{1}{12}-\left(\frac{1}{8}\ln\frac{1}{5}-\frac{1}{10}\right)$$

$= 0.08$ (to 2 d.p.) [A1]

11 **(a)** Show that $\frac{d}{d\theta}(\cot\theta)=-\frac{1}{\sin^2\theta}$. [2]

(b) In the diagram below, a straight wooden plank *PQ*, of length 12.5 m is supported at an angle θ to a vertical wall *XY* by a taut rope fixed to a hook at *H*.
The length of the rope *BH* from the wall is 2.7 m. The end *P* of the plank is at a vertical height *h* m above *H*.



(i) Show that $h = 12.5\cos\theta - \dfrac{2.7\cos\theta}{\sin\theta}$. [2]

(ii) Using part (a), determine the value of $\sin\theta$ for which $\dfrac{dh}{d\theta} = 0$. [2]

(iii) Hence or otherwise, show that as θ varies, h attains a maximum value and find this value. [3]

Solutions

(a) $\dfrac{d}{d\theta}(\cot\theta) = \dfrac{d}{d\theta}\left(\dfrac{\cos\theta}{\sin\theta}\right)$

$= \dfrac{\sin\theta(-\sin\theta) - \cos\theta(\cos\theta)}{\sin^2\theta}$ [M1] – Quotient Rule

$= \dfrac{-(\sin^2\theta + \cos^2\theta)}{\sin^2\theta}$ [A1] -- $\sin^2\theta + \cos^2\theta = 1$

$= -\dfrac{1}{\sin^2\theta}$ (shown)

Method 2

$\dfrac{d}{d\theta}(\cot\theta) = \dfrac{d}{d\theta}(\tan\theta)^{-1}$

$= (-1)(\tan\theta)^{-2}(\sec^2\theta)$ [M1] – Chain Rule

$= (-1)\left(\dfrac{\cos^2\theta}{\sin^2\theta}\right)\left(\dfrac{1}{\cos^2\theta}\right)$

$= -\dfrac{1}{\sin^2\theta}$ [A1]

(b)(i) $\cos\theta = \dfrac{XY}{12.5} \Rightarrow XY = 12.5\cos\theta$

$\tan\theta = \dfrac{2.7}{BY} \Rightarrow BY = \dfrac{2.7}{\tan\theta} = \dfrac{2.7\cos\theta}{\sin\theta}$ [M1] -- either XY or BY

$h = XY - BY$

$= 12.5\cos\theta - \dfrac{2.7\cos\theta}{\sin\theta}$ [A1] – clear working above

(ii) $\dfrac{dh}{d\theta} = -12.5\sin\theta - 2.7\left(\dfrac{d}{d\theta}\left(\dfrac{\cos\theta}{\sin\theta}\right)\right)$

$= -12.5\sin\theta + \dfrac{2.7}{\sin^2\theta}$ [M1]

$\dfrac{dh}{d\theta} = 0 \Rightarrow -12.5\sin\theta + \dfrac{2.7}{\sin^2\theta} = 0$

$\sin\theta = \sqrt[3]{\frac{2.7}{12.5}}$
$= 0.6$ [A1]

(iii)
$\sin\theta = \frac{3}{5}$ giving rise to $\cos\theta = \frac{4}{5}$ [M1]

$\frac{d^2h}{d\theta^2} = -12.5\cos\theta - \frac{5.4\cos\theta}{\sin^3\theta}$

$= -12.5\left(\frac{4}{5}\right) - \frac{5.4\left(\frac{4}{5}\right)}{\left(\frac{3}{5}\right)^3} = -30 < 0$ [M1] – verify max

Max $h = 12.5(0.8) - \frac{2.7(0.8)}{(0.6)} = 6.4$ m [A1]

Alternative method

$\theta = 36.870^0$

$\frac{d^2h}{d\theta^2} = -12.5\cos\theta + 2.7(-2)(\sin\theta)^{-3}(\cos\theta)$

$= -12.5\cos\theta - \frac{5.4\cos\theta}{\sin^3\theta}$ [M1] – first or second derivative test

When $\theta = 36.870^0$,

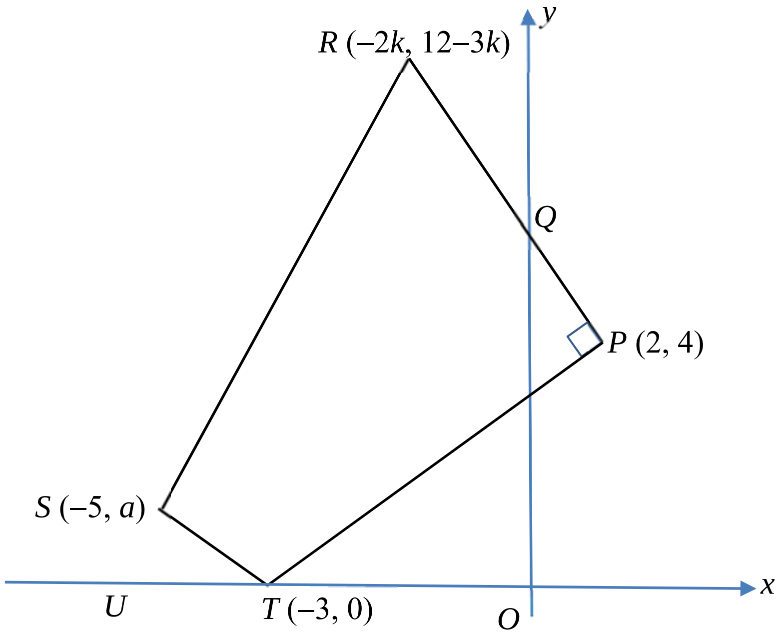
$\frac{d^2h}{d\theta^2} = -12.5\cos 36.870^o - \frac{5.4\cos 36.870^o}{\sin^3 36.870^o} = -30.0 < 0$ [M1] – verify max

h is maximum when θ is 36.870°.

Maximum $h = 12.5\cos 36.870^o - \frac{2.7\cos 36.870^o}{\sin 36.870^o}$
$= 6.40$ m [A1]

Solutions to this question by accurate drawing will not be accepted.

12 The figure shows a quadrilateral *PTSR* for which *P* is (2, 4), *T* is (–3, 0), *S* is (–5, *a*), *R* is (–2*k*, 12–3*k*) and angle *QPT* is a right angle. *RQP* is a straight line with point *Q* lying on the *y*-axis.



(i) Find the value of *k*. [2]

(ii) Given that angle $STU = 45^\circ$, determine the value of *a*. [2]

(iii) A line passing through *Q* and is perpendicular to *TS* cuts the *x*-axis at *V*. Find the value of VR^2. [5]

Solutions

(i) Gradient of $PT = \frac{4}{5}$

Gradient of *PR*, $\frac{12-3k-4}{-2k-2} = -\frac{5}{4}$ [M1]

$4(8-3k) = 5(2k+2)$

$-22k = -22$

$k = 1$ [A1]

(ii) angle $STU = 45^\circ \Rightarrow$ gradient of $ST = -1$ [M1]

$\frac{a-0}{-5+3} = -1$ [A1]

$a = 2$

(iii) Equation of PR is $y-4=-\frac{5}{4}(x-2)$ [M1]

$$-4(y-4)=5(x-2)$$
$$4y+5x=26$$

At Q, $x=0$

$4y=26 \Rightarrow y=6.5$

$Q(0, 6.5)$ [A1]

Equation of line passing through Q and perpendicular to TS is

$y-6.5=\frac{-1}{-1}(x-0)$

$y=x+6.5$ [M1]

At V, $y=0$. Hence $x=-6.5$

$V(-6.5, 0)$ [A1]

$VR^2=(-2+6.5)^2+9^2$

$=101.25$ [A1]

END OF PAPER

Name:	Register No.:	Class:

CRESCENT GIRLS' SCHOOL
SECONDARY FOUR
PRELIMINARY EXAMINATION

ADDITIONAL MATHEMATICS — 4047/01

Paper 1

16 August 2018

2 hours

Additional Materials: Answer Paper, Mark Sheet

READ THESE INSTRUCTIONS FIRST

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Answer **all** the questions.
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The total number of marks for this paper is 80.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n ,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

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$$\sin 2A = 2 \sin A \cos A$$

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Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

1 The straight line $y-1=2m$ does not intersect the curve $y=x+\frac{m^2}{x}$.
Find the largest integer value of m. [5]

2 The line $2y+x=5$ intersects the curve $y^2=6-xy$ at the points P and Q.
Determine, with explanation, if the point $(1, 2)$ lies on the line joining the midpoint of PQ and $(3,1)$. [5]

3 **(i)** Sketch on the same graph $y=|3\cos 2x|$ and $y+\frac{8}{3\pi}x=2$ for $0\le x\le \pi$. [3]

(ii) Hence, showing your working clearly, deduce the number of solutions in $|\cos 2x|-\frac{2}{3}+\frac{8x}{3\pi}=0$ in the interval $0\le x\le \pi$. [2]

4 **(i)** Find the value of a and of b if the curve $f(x)=ax+\frac{b}{x}$ where $x\neq 0$ has a stationary point at $(-2, -8)$. [4]

(ii) By considering the sign of $f'(x)$, determine the nature of the stationary point. [2]

5 It is given that $\int f'(x)\,dx=\frac{x}{2}-\frac{\sin kx}{8}+c$ where c is a constant of integration, and that $\int_0^{\frac{\pi}{8}} f'(x)\,dx=\frac{\pi}{16}-\frac{1}{8}$.

(i) Show that $k=4$. [2]

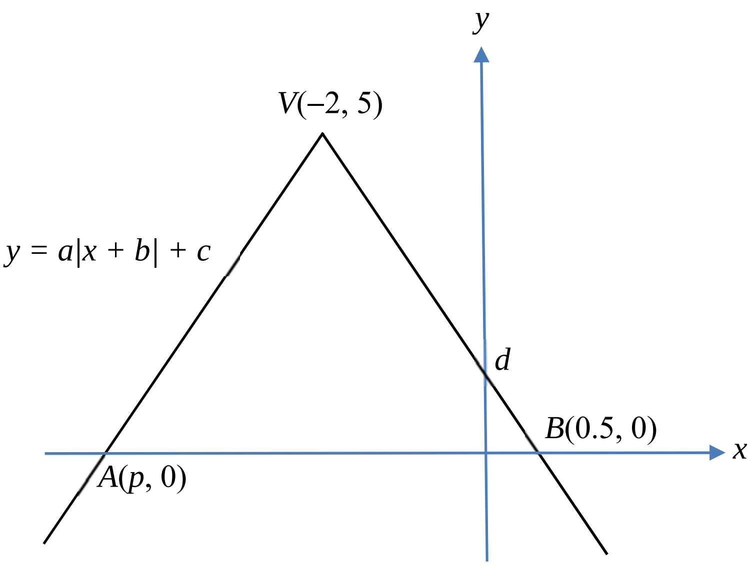
(ii) Hence find $f'(x)$, expressing your answer in $\sin^2 px$, where p is a constant. [2]

(iii) Find the equation of the curve $y=f(x)$ given that the point $\left(\frac{\pi}{4},0\right)$ lies on the curve. [2]

6 **(a)** The length of each side of a square of area $\left(49+20\sqrt{6}\right)$ m^2 can be expressed in the form $\left(\sqrt{c}+\sqrt{d}\right)$ m where c and d are integers and $c < d$.
Find the value of c and of d. [3]

(b) A parallelogram with base equals to $\left(4-\sqrt{12}\right)$ m has an area of $\left(22-\sqrt{48}\right)$ m^2.
Find, without using a calculator, the height of the parallelogram in the form $\left(p+q\sqrt{3}\right)$ m. [3]

7 The diagram shows part of the graph $y = a\,|\,x+b\,|+c$. The graph cuts the x-axis at A (p, 0) and at B (0.5, 0). The graph has a vertex point at V (–2, 5) and y-intercept, d.



(i) Explain why $p = -4.5$. [1]

(ii) Determine the value of each of a, b and c. [4]

(iii) State the set of values of k for which the line $y = kx + d$ intersects the graph at two distinct points. [2]

8 **(i)** Differentiate $x^3 \ln x$ with respect to x. [2]

(ii) Hence find $\int \frac{x^2 \ln x}{2}\,dx$. [4]

9 **(a)** If $32^y \times 5^{4y} = 2^{4y+4} \times 5^{3y-1}$, determine the value of 10^y. [3]

(b) **(i)** Sketch on the same axes, the graphs of $y = x^{-2}$ and $y = \sqrt{3x}$. [2]

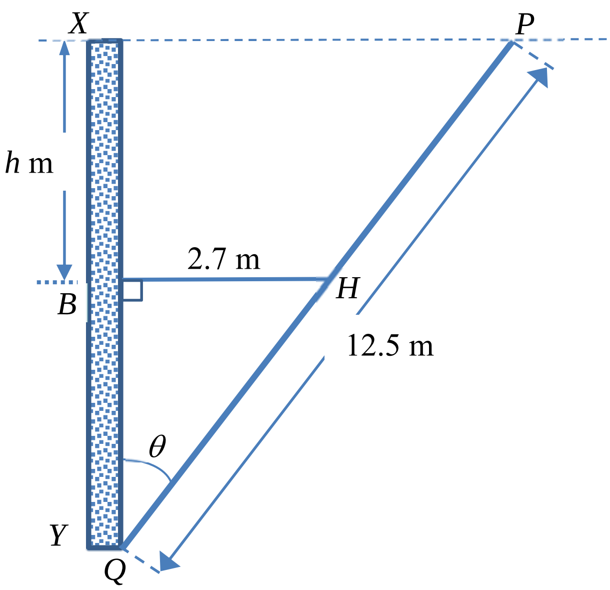
(ii) Find the point of intersection between the graphs. [3]

10 **(i)** Express $\dfrac{x+1}{x(x+3)^2 - (x+3)^2}$ in partial fractions. [5]

(ii) Hence find the value of $\int_2^3 \dfrac{x+1}{x(x+3)^2 - (x+3)^2}\,dx$ giving your answer to 2 decimal places. [3]

11 **(a)** Show that $\dfrac{d}{d\theta}(\cot\theta) = -\dfrac{1}{\sin^2\theta}$. [2]

(b) In the diagram below, a straight wooden plank *PQ*, of length 12.5 m is supported at an angle θ to a vertical wall *XY* by a taut rope fixed to a hook at *H*.
The length of the rope *BH* from the wall is 2.7 m. The end *P* of the plank is at a vertical height *h* m above *H*.



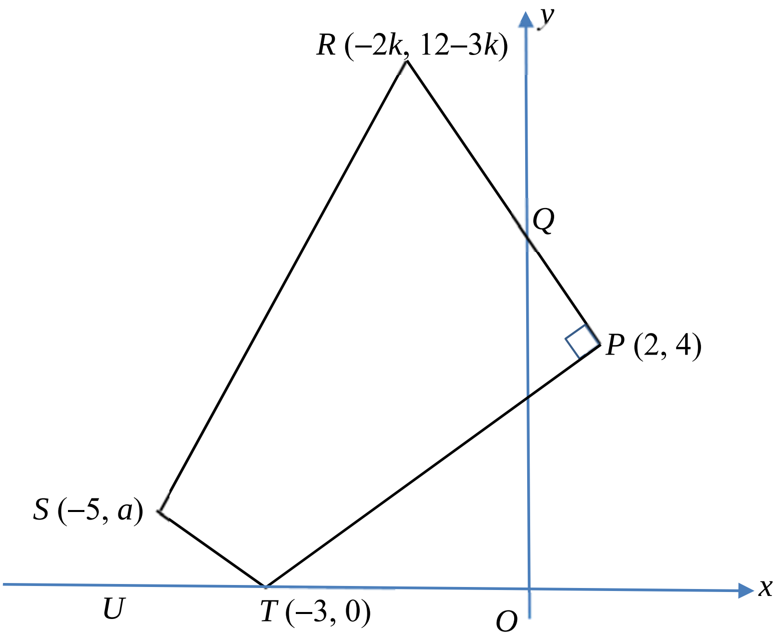
(i) Show that $h = 12.5\cos\theta - \dfrac{2.7\cos\theta}{\sin\theta}$. [2]

(ii) Using part (a), determine the value of $\sin\theta$ for which $\dfrac{dh}{d\theta} = 0$. [2]

(iii) Hence or otherwise, show that as θ varies, *h* attains a maximum value and find this value. [3]

Solutions to this question by accurate drawing will not be accepted.

12 The figure shows a quadrilateral $PTSR$ for which P is (2, 4), T is (−3, 0), S is (−5, a), R is (−2k, 12−3k) and angle QPT is a right angle. RQP is a straight line with point Q lying on the y-axis.



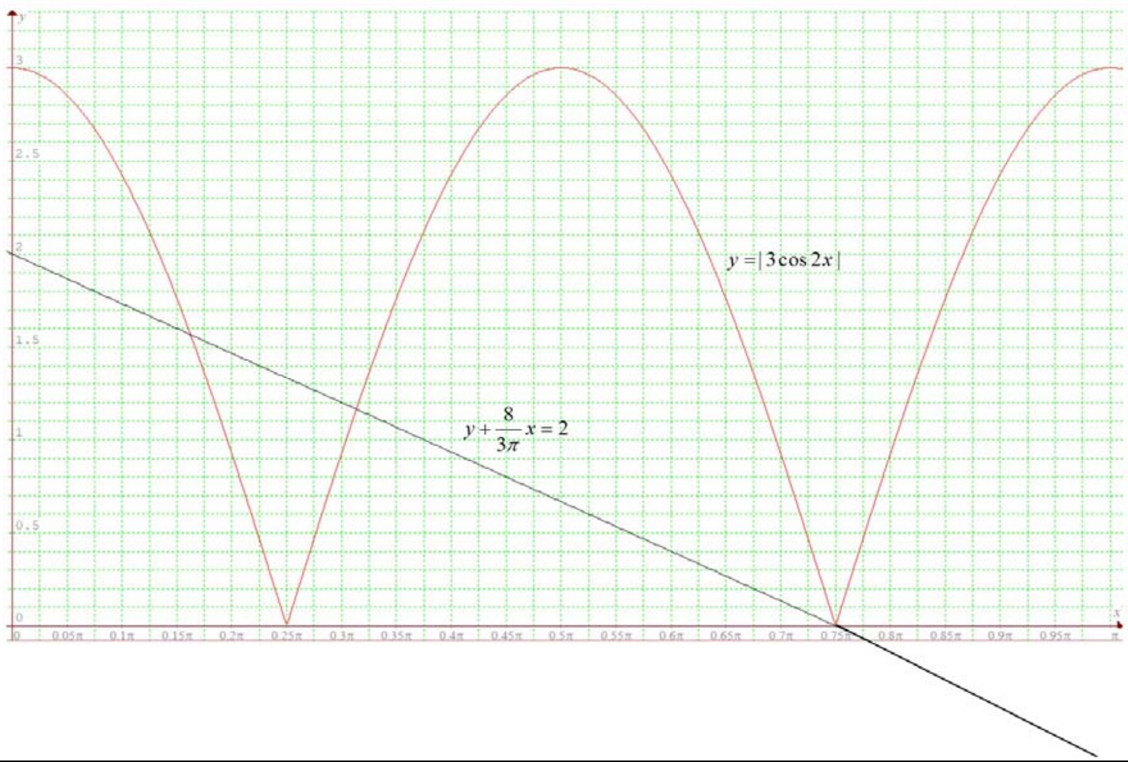
(i) Find the value of k. [2]

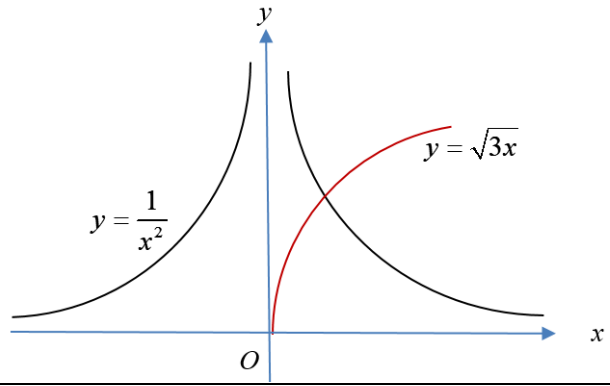
(ii) Given that angle $STU = 45°$, determine the value of a. [2]

(iii) A line passing through Q and is perpendicular to TS cuts the x-axis at V. Find the value of VR^2. [5]

END OF PAPER

2018 CGS A Math Prelim Paper 1 Answer Key

Qn	Ans Key
1	$m=-1$
2	Yes
3(i)	
3(ii)	1 solution
4(i)	$a=2$; $b=8$
4(ii)	Maximum point
5(ii)	$\sin^2 2x$
5(iii)	$f(x)=\frac{x}{2}-\frac{\sin 4x}{8}-\frac{\pi}{8}$
6(a)	$c=24$; $d=25$
6(b)	$h=(16+7\sqrt{3})$ m
7(ii)	$a=-2$; $b=2$; $c=5$
7(iii)	$-2<k<2$
8(i)	$x^2+3x^2\ln x$
8(ii)	$\frac{1}{6}x^3\ln x-\frac{1}{18}x^3+c$
9(a)	3.2

9(b)(i)	
9(b)(ii)	(0.803, 1.55)
10(i)	$\frac{1}{8(x-1)}-\frac{1}{8(x+3)}+\frac{1}{2(x+3)^2}$
10(ii)	0.08
11(b)(ii)	$\sin\theta=\frac{3}{5}$
11(b)(iii)	$h=6.4$ m
12(i)	$k=1$
12(ii)	$a=2$
12(iii)	101.25

Name:	Register No.:	Class:

CRESCENT GIRLS' SCHOOL
SECONDARY FOUR
PRELIMINARY EXAMINATION

ADDITIONAL MATHEMATICS — 4047/02

Paper 2

17 August 2018

2 hours 30 minutes

Additional Materials: Answer Paper, Mark Sheet

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Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

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$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

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1 **(i)** Write down and simplify the first four terms in the expansion $\left(2x - \frac{p}{x^2}\right)^5$ in descending powers of *x*, where *p* is a non-zero constant. [3]

(ii) Given that the coefficient of x^{-1} in the expansion $\left(4x^3 - 1\right)\left(2x - \frac{p}{x^2}\right)^5$ is $-160p^2$, find the value of *p*. [4]

2 Variables *x* and *y* are related by the equation $y = ax^b + 3$ where *a* and *b* are constants. When $\lg(y-3)$ is plotted against $\lg x$, a straight line is obtained. The straight line passes through $(-2.5,\ 8)$ and $(3.5,\ -4)$. Find

(i) the value of *a* and of *b*, [5]

(ii) the coordinates of the point on the line when $x = 10^6$. [3]

3 **(a)** Given that $x = \log_3 a$ and $y = \log_3 b$, express $\log_3 \frac{\sqrt{b^5}}{27a^4}$ in terms of *x* and *y*. [3]

(b) Solve the equation $\log_2 (5x+3)^2 - \log_{5x+3} 2 = 1$. [5]

4 **(i)** The roots of the equation $2x^2 + px - 8 = 0$, where *p* is a constant, are α and β. The roots of the equation $4x^2 - 24x + q = 0$, where *q* is a constant, are $\alpha + 2\beta$ and $2\alpha + \beta$. Find the values of *p* and *q*. [6]

(ii) Hence form the quadratic equation whose roots are α^3 and β^3. [3]

5 The equation of a circle C is $x^2 + y^2 - 12x - 8y - 13 = 0$.

(i) Find the centre and radius of *C*. [3]

(ii) Find the equation of the line which passes through the centre of *C* and is perpendicular to the line $4x + 7y = 117$. [3]

(iii) Show that the line $4x + 7y = 117$ is a tangent to *C* and state the coordinates of the point where the line touches *C*. [5]

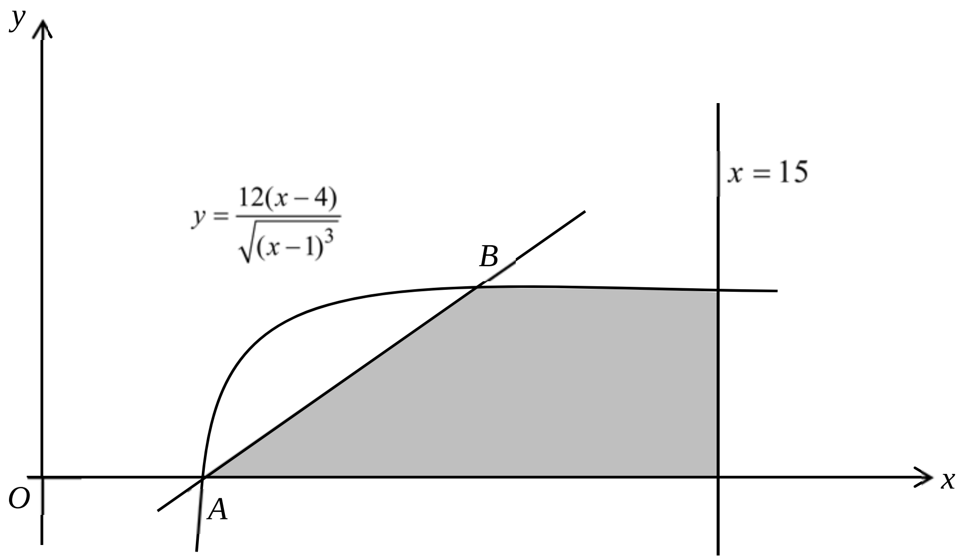
6 **(a)** A car travelling on a straight road passes through a traffic light X with speed of 90 m/s. The acceleration, a m/s^2 of the car, t seconds after passing X, is given by $a = 20 - 8t$. Determine with working whether the car is travelling towards or away from X when it is travelling at maximum speed. [4]

(b) A particle moving in a straight line such that its displacement, s m, from the fixed point O is given by $s = 7\sin t - 2\cos 2t$, where t is the time in seconds, after passing through a point A.

(i) Find the value of t when the particle first comes to instantaneous rest. [5]

(ii) Find the total distance travelled by the particle during the first 4 seconds of its motion. [3]

7 **(i)** Show that $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\dfrac{x+2}{\sqrt{x-1}}\right) = \dfrac{x-4}{2\sqrt{(x-1)^3}}$. [3]



The diagram shows the line $x = 15$ and part of the curve $y = \dfrac{12(x-4)}{\sqrt{(x-1)^3}}$. The curve intersect the x-axis at the point A. The line through A with gradient $\dfrac{4}{9}$ intersects the curve again at the point B.

(ii) Verify that the y-coordinate of B is $2\dfrac{2}{3}$. [4]

(iii) Determine the area of the region bounded by the curve, the x-axis, the line $x = 15$ and the line AB. [4]

8 A curve has equation given by $y = \dfrac{e^{4x-3}}{8e^{2x}}$.

(i) Show that $\dfrac{dy}{dx} = \dfrac{e^{2x-3}}{4}$. [2]

(ii) Given that x is decreasing at a rate of $4e^2$ units per second, find the exact rate of change of y when $x = 1$. [3]

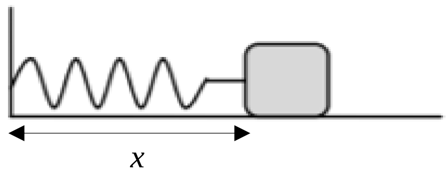
(iii) The curve passes through the y-axis at P. Find the equations of the tangent and normal to the curve at point P. [4]

(iv) The tangent and normal to the curve at point P meets the x-axis at Q and R respectively. Show that the area of the triangle PQR is $\dfrac{1+16e^6}{512e^9}$ units^2. [3]

9 **(a)** Prove that $\operatorname{cosec}^4 x - \cot^4 x = 2\operatorname{cosec}^2 x - 1$. [3]

(b) Solve the equation $6\tan 2x + 1 = \cot 2x$, for the interval $0 \le x \le 180°$. [5]

(c)

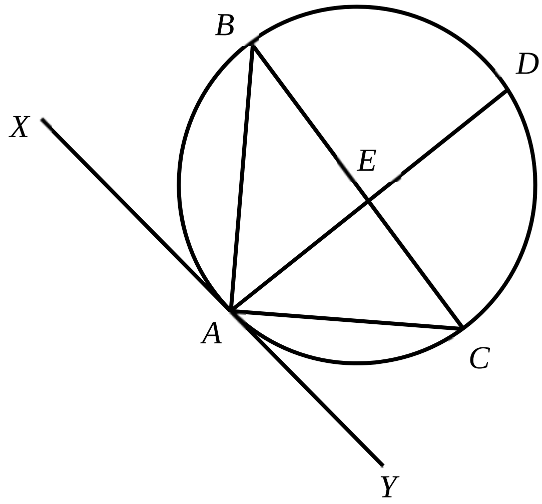


An object is connected to the wall with a spring that has a original horizontal length of 20 cm. The object is pulled back 8 cm past the original length and released. The object completes 4 cycles per second.

(i) Given that the function $x = 8\cos(a\pi t) + b$, where x is the horizontal distance, in centimetres, of the object from the wall and t is the time in seconds after releasing the object, find the values of a and b. [2]

(ii) Find the duration of time for each cycle such that the object is more than 27 cm from the wall. [3]

10



Given that AD and BC are straight lines, AC bisects angle DAY and AB bisects angle DAX, show that

(i) $AC^2 = EC \times BC$, [3]

(ii) BC is a diameter of the circle, [3]

(iii) AD and BC are perpendicular to each other. [3]

END OF PAPER

Answer Key for Paper 2

1(i)	$32x^5 - 80px^2 + \frac{80p^2}{x} - \frac{40p^3}{x^4} + ...$
(ii)	$p = 0.5$
2(i)	$a = 1000,\ b = -2$
(ii)	$(6, -9)$
3(a)	$\frac{5}{2}y - 4x - 3$
(b)	$x = -0.459$ or -0.2
4(i)	$p = -4$, $q = 16$
(ii)	$x^2 - 32x - 64 = 0$
5(i)	Centre $= (6,\ 4)$, Radius $= \sqrt{65}$ units
(ii)	$4y = 7x - 26$
(iii)	$(10,\ 11)$
6(a)	Travelling away from X
(b)(i)	$\frac{\pi}{2}$ s
(b)(ii)	25.0 m
7(iii)	21.0 units^2
8 (ii)	$-e$ units/s
(iii)	$y = \frac{x}{4e^3} + \frac{1}{8e^3}$, $y = -4e^3x + \frac{1}{8e^3}$
9(b)	$x = 9.2°,\ 76.7°,\ 99.2°,\ 166.7°$
(c)(i)	$a = 8$, $b = 20$
(c)(ii)	0.0402 s

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CRESCENT GIRLS' SCHOOL
SECONDARY FOUR
PRELIMINARY EXAMINATION

ADDITIONAL MATHEMATICS **4047/02**

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17 August 2018

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(ii) Given that the coefficient of x^{-1} in the expansion $\left(4x^3-1\right)\left(2x-\frac{p}{x^2}\right)^5$ is $-160p^2$, find the value of *p*. [4]

Solution:

(i)

$$\left(2x-\frac{p}{x^2}\right)^5=\left(2x\right)^5+5\left(2x\right)^4\left(-\frac{p}{x^2}\right)+10\left(2x\right)^3\left(-\frac{p}{x^2}\right)^2+10\left(2x\right)^2\left(-\frac{p}{x^2}\right)^3+\ldots \quad \text{[M1]}$$

$$=32x^5-80px^2+\frac{80p^2}{x}-\frac{40p^3}{x^4}+\ldots \quad \text{[A2]}$$

(ii)

$$\left(4x^3-1\right)\left(2x-\frac{p}{x^2}\right)^5=\left(4x^3-1\right)\left(32x^5-80px^2+\frac{80p^2}{x}-\frac{40p^3}{x^4}+\ldots\right) \quad \text{[M1]}$$

Coefficient of $x^{-1}=4\left(-40p^3\right)+\left(-1\right)\left(80p^2\right)$ [M1]

$$=-160p^3-80p^2$$

$$-160p^3-80p^2=-160p^2 \quad \text{[M1]}$$

$$80p^2\left(2p-1\right)=0$$

$$p=0\ (NA) \quad \text{or} \quad p=0.5 \quad \text{[A1]}$$

2 Variables x and y are related by the equation $y = ax^b + 3$ where a and b are constants. When $\lg(y-3)$ is plotted against $\lg x$, a straight line is obtained. The straight line passes through $(-2.5,\ 8)$ and $(3.5,\ -4)$. Find

(i) the value of a and of b, [5]

(ii) the coordinates of the point on the line when $x = 10^6$. [3]

Solution:

(i) $y = ax^b + 3$

$y - 3 = ax^b$

$\lg(y-3) = \lg a + b\lg x$ [M1]

Gradient $= \dfrac{8-(-4)}{-2.5-3.5}$ [M1]

$= -2$

$b = -2$ [A1]

Sub $\lg x = -2.5$, $\lg(y-3) = 8$ and $b = -2$,

$8 = -2(-2.5) + \lg a$ [M1]

$\lg a = 3$

$a = 10^3 = 1000$ [A1]

(ii) $\lg(y-3) = -2\lg x + 3$

$x = 10^6$

$\lg x = 6$ [M1]

$\lg(y-3) = -2(6) + 3 = -9$ [M1]

Coordinates $= (6,\ -9)$ [A1]

3 **(a)** Given that $x = \log_3 a$ and $y = \log_3 b$, express $\log_3 \frac{\sqrt{b^5}}{27a^4}$ in terms of x and y. [3]

(b) Solve the equation $\log_2 (5x+3)^2 - \log_{5x+3} 2 = 1$. [5]

Solution

(a)

$$\log_3 \frac{\sqrt{b^5}}{27a^4} = \log_3 \sqrt{b^5} - \log_3 27 - \log_3 a^4$$ [M1]

$$= \frac{5}{2}\log_3 b - 3 - 4\log_3 a$$ [M1]

$$= \frac{5}{2}y - 4x - 3$$ [A1]

(b)

$$\log_2 (5x+3)^2 - \log_{5x+3} 2 = 1$$

$$2\log_2 (5x+3) - \frac{\log_2 2}{\log_2 (5x+3)} = 1$$ [M1]

$$2\left[\log_2 (5x+3)\right]^2 - 1 = \log_2 (5x+3)$$

$$2\left[\log_2 (5x+3)\right]^2 - \log_2 (5x+3) - 1 = 0$$ [M1]

Let $y = \log_2 (5x+3)$.

$$2y^2 - y - 1 = 0$$

$$(2y+1)(y-1) = 0$$ [M1]

$y = -0.5$ or $y = 1$

$\log_2 (5x+3) = -0.5$ $\qquad$ $\log_2 (5x+3) = 1$ [M1]

$5x+3 = 2^{-0.5}$ $\qquad$ $5x+3 = 2$

$x = -0.459$ $\qquad$ $x = -0.2$ [A1]

4 **(i)** The roots of the equation $2x^2 + px - 8 = 0$, where *p* is a constant, are α and β. The roots of the equation $4x^2 - 24x + q = 0$, where *q* is a constant, are $\alpha + 2\beta$ and $2\alpha + \beta$. Find the values of *p* and *q*. [6]

(ii) Hence form the quadratic equation whose roots are α^3 and β^3. [3]

Solution:

(i) $2x^2 + px - 8 = 0$

$\alpha + \beta = -\frac{p}{2}, \quad \alpha\beta = -4$ [B1]

$4x^2 - 24x + q = 0$

$\alpha + 2\beta + 2\alpha + \beta = 6$ [M1]

$3(\alpha + \beta) = 6$

Sub $\alpha + \beta = -\frac{p}{2}$,

$-\frac{p}{2} = 2 \quad \Rightarrow \quad p = -4$ [A1]

$(\alpha + 2\beta)(2\alpha + \beta) = \frac{q}{4}$ [M1]

$2(\alpha^2 + \beta^2) + 5\alpha\beta = \frac{q}{4}$

$2\left[(\alpha + \beta)^2 - 2\alpha\beta\right] + 5\alpha\beta = \frac{q}{4}$ [M1]

$2(\alpha + \beta)^2 + \alpha\beta = \frac{q}{4}$

Sub $\alpha + \beta = 2,\ \alpha\beta = -4$,

$2(2)^2 - 4 = \frac{q}{4}$

$q = 16$ [A1]

(ii) $\alpha^3 + \beta^3 = (\alpha + \beta)(\alpha^2 - \alpha\beta + \beta^2)$ [M1]

$= (\alpha + \beta)\left[(\alpha + \beta)^2 - 3\alpha\beta\right]$

$= 2\left[2^2 - 3(-4)\right]$

$= 32$

$(\alpha\beta)^3 = (-4)^3 = -64$ [M1]

$\therefore x^2 - 32x - 64 = 0$ [A1]

5 The equation of a circle C is $x^2+y^2-12x-8y-13=0$.

(i) Find the centre and radius of *C*. [3]

(ii) Find the equation of the line which passes through the centre of *C* and is perpendicular to the line $4x+7y=117$. [3]

(iii) Show that the line $4x+7y=117$ is a tangent to *C* and state the coordinates of the point where the line touches *C*. [5]

Solution:

(i) $x^2+y^2-12x-8y-13=0$

$(x-6)^2+(y-4)^2-36-16-13=0$ [M1]

$(x-6)^2+(y-4)^2=65$

Centre $=(6,\ 4)$ [A1]

Radius $=\sqrt{65}$ units [A1]

(ii) For $4x+7y=117$,

Gradient of the line $=-\frac{4}{7}$

Gradient of the line passing through $C=\frac{7}{4}$ [M1]

Equation of the line:

$y-4=\frac{7}{4}(x-6)$ [M1]

$4y-16=7x-42$

$4y=7x-26$ [A1]

(iii) $4x+7y=117$ ----- (1)

$4y=7x-26 \quad \Rightarrow \quad y=\frac{7}{4}x-\frac{26}{4}$ ----- (2)

Sub (2) into (1):

$4x+7\left(\frac{7}{4}x-\frac{26}{4}\right)=117$ [M1]

$16.25x=162.5$

$x=10$ [M1]

$y=11$

Distance between $(10,\ 11)$ and centre of circle $=\sqrt{(10-6)^2+(11-4)^2}$ [M1]

$=\sqrt{65}$ units

Since distance from the point and the centre of circle equals to the radius, the line is a tangent to the circle. [A1]

Coordinates of the point $=(10,\ 11)$ [A1]

Alternative Solution:

$4x+7y=117 \quad \Rightarrow \; x=\dfrac{117-7y}{4}$ ----- (1)

$x^2+y^2-12x-8y-13=0$ ----- (2)

Sub (1) into (2):

$$\left(\frac{117-7y}{4}\right)^2+y^2-12\left(\frac{117-7y}{4}\right)-8y-13=0$$ [M1]

$$\frac{13689-1638y+49y^2}{16}+y^2-351+21y-8y-13=0$$

$$13689-1638y+49y^2+16y^2-5616+336y-128y-208=0$$

$$65y^2-1430y+7865=0$$ [M1]

$$y^2-22y+121=0$$

$$b^2-4ac=(-22)^2-4(1)(121)$$ [M1]

$$=0$$

Since $b^2-4ac=0$, the line is a tangent to C. [A1]

$$y^2-22y+121=0$$

$$(y-11)^2=0$$

$$y=11$$

$$x=10$$

Coordinate of the point = $(10,\ 11)$ [A1]

6 **(a)** A car travelling on a straight road passes through a traffic light *X* with speed of 90 m/s. The acceleration, a m/s^2 of the car, *t* seconds after passing *X*, is given by $a = 20 - 8t$. Determine with working whether the car is travelling towards or away from *X* when it is travelling at maximum speed. [4]

(b) A particle moving in a straight line such that its displacement, *s* m, from the fixed point *O* is given by $s = 7\sin t - 2\cos 2t$, where *t* is the time in seconds, after passing through at a point *A*.

(i) Find the value of *t* when the particle first comes to instantaneous rest. [5]

(ii) Find the total distance travelled by the particle during the first 4 seconds of its motion. [3]

Solution:

(a) $a = 20 - 8t$

$v = \int 20 - 8t \text{ dt} = 20t - 4t^2 + c$, where c is a constant [M1]

When $t = 0,\ v = 90,\ c = 90.$

$\therefore v = 20t - 4t^2 + 90$

When car is travelling at max speed, $a = 0$.

$a = 20 - 8t \quad \Rightarrow \quad t = 2.5$ [M1]

$v = 20(2.5) - 4(2.5)^2 + 90 = 115$

$s = \int 20t - 4t^2 + 90 \text{ dt} = 10t^2 - \frac{4}{3}t^3 + 90t + d$, where d is a constant [M1]

When $t = 0,\ s = 0,\ d = 0.$

$\therefore s = 10t^2 - \frac{4}{3}t^3 + 90t$

When $t = 2.5$, $s = 10(2.5)^2 - \frac{4}{3}(2.5)^3 + 90(2.5) = 266\frac{2}{3}$

Since $s > 0$ and $v > 0$, the car is travelling away from X at maximum speed. [A1]

Alternative Solution:

When the car is at instantaneous rest, $v = 0$.

$20t - 4t^2 + 90 = 0$

$t = \frac{-20 \pm \sqrt{(-20)^2 - 4(-4)(90)}}{2(-4)} = -2.8619 \text{ or } 7.8619$ [M1]

Since there is no change of direction from $t = 0$ to $t = 7.86$ s, [B1]

the car is travelling away from X at maximum speed.

(b)(i) $s = 7\sin t - 2\cos 2t$

$v = 7\cos t + 4\sin 2t$ [M1]

When the particle is at instantaneous rest, $v = 0$.

$7\cos t + 4\sin 2t = 0$ [M1]

$7\cos t + 8\sin t\cos t = 0$

$\cos t(7 + 8\sin t) = 0$ [M1]

$\cos t = 0$ or $\sin t = -\frac{7}{8}$

$t = \frac{\pi}{2},\ \frac{3\pi}{2}$ $t = 4.2069,\ 5.2177$ [A1]

Time when particle first comes to instantaneous rest $= \frac{\pi}{2}$ s [A1]

(b)(ii) When $t = 0,\ s = -2$.

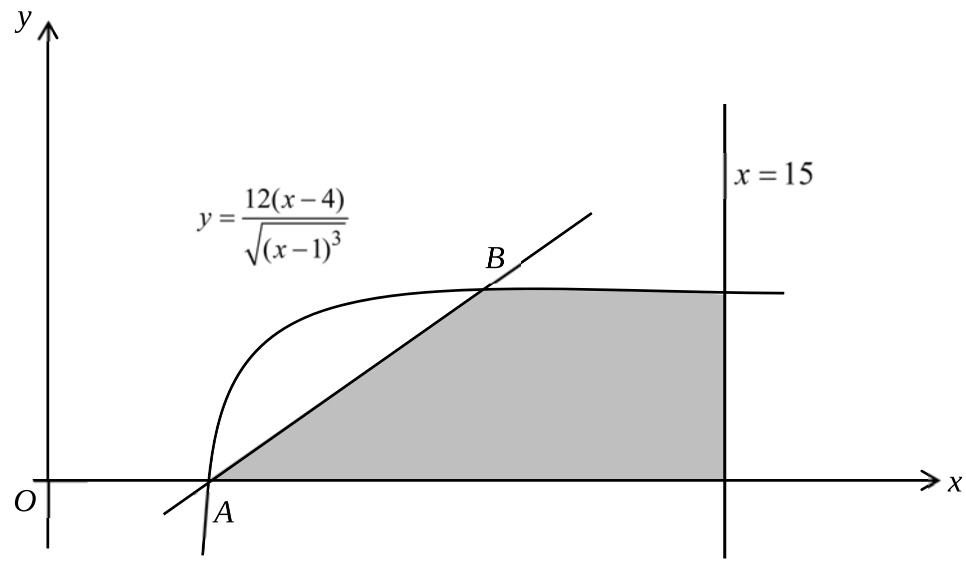
When $t = \frac{\pi}{2},\ s = 9$.

When $t = 4,\ s = -5.0066$. [M1]

Total distance travelled $= 2 + 2(9) + 5.0066$ [M1]

$= 25.0$ m [A1]

7 **(i)** Show that $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{x+2}{\sqrt{x-1}}\right)=\frac{x-4}{2\sqrt{(x-1)^3}}$. [3]



The diagram shows the line $x=15$ and part of the curve $y=\frac{12(x-4)}{\sqrt{(x-1)^3}}$. The curve intersect the *x*-axis at the point *A*. The line through *A* with gradient $\frac{4}{9}$ intersects the curve again at the point *B*.

(ii) Verify that the *y*-coordinate of *B* is $2\frac{2}{3}$. [4]

(iii) Determine the area of the region bounded by the curve, the *x*-axis, the line $x=15$ and the line *AB*. [4]

Solution:

(i)

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{x+2}{\sqrt{x-1}}\right)=\frac{\sqrt{x-1}-(x+2)\left[\frac{1}{2}(x-1)^{-\frac{1}{2}}\right]}{x-1}$$ [M1]

$$=\frac{\left[\frac{1}{2}(x-1)^{-\frac{1}{2}}\right][2x-2-x-2]}{x-1}$$ [M1]

$$=\frac{x-4}{2\sqrt{(x-1)^3}}$$ [A1]

(ii) $A = (4,\ 0)$

Equation of AB: $y = \frac{4}{9}(x-4)$ ---- (1) [M1]

$y = \frac{12(x-4)}{\sqrt{(x-1)^3}}$ ---- (2)

(1) = (2):

$\frac{4}{9}(x-4) = \frac{12(x-4)}{\sqrt{(x-1)^3}}$ [M1]

$(x-4)(x-1)^{\frac{3}{2}} = 27(x-4)$

$(x-4)\left[(x-1)^{\frac{3}{2}} - 27\right] = 0$ [M1]

$x = 4$ or $(x-1)^{\frac{3}{2}} = 27$

$x = 10$

Sub $x = 10$ in (1):

$y = \frac{4}{9}(10-4) = 2\frac{2}{3}$ [A1]

$y-$coordinate of $B = 2\frac{2}{3}$ (shown)

(iii) Area $= \frac{1}{2}\left(2\frac{2}{3}\right)(10-4) + \int_{10}^{15} \frac{12(x-4)}{\sqrt{(x-1)^3}}\ \mathrm{d}x$ [M1]

$= 8 + 24\int_{10}^{15} \frac{x-4}{2\sqrt{(x-1)^3}}\ \mathrm{d}x$ [M1]

$= 8 + 24\left[\frac{x+2}{\sqrt{x-1}}\right]_{10}^{15}$ [M1]

$= 8 + 24\left(\frac{17}{\sqrt{14}} - \frac{12}{\sqrt{9}}\right)$

$= 21.0$ units^2 [A1]

8 A curve has equation given by $y = \frac{e^{4x-3}}{8e^{2x}}$.

(i) Show that $\frac{dy}{dx} = \frac{e^{2x-3}}{4}$. [2]

(ii) Given that x is decreasing at a rate of $4e^2$ units per second, find the exact rate of change of y when $x = 1$. [3]

(iii) The curve passes through the y-axis at P. Find the equations of the tangent and normal to the curve at point P. [4]

(iv) The tangent and normal to the curve at point P meets the x-axis at Q and R respectively. Show that the area of the triangle PQR is $\frac{1+16e^6}{512e^9}$ units^2. [3]

Solution:

(i)

$y = \frac{e^{2x-3}}{8}$ [M1]

$\frac{dy}{dx} = \frac{e^{2x-3}}{4}$ [A1]

(ii)

$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$

$= \frac{e^{2x-3}}{4} \times \left(-4e^2\right)$ [M1]

$= -e^{2x-1}$ [M1]

When $x = 1$, $\frac{dy}{dt} = -e$ units/s [A1]

(iii)

When $x = 0$, $y = \frac{1}{8e^3}$

Gradient of tangent at $P = \frac{1}{4e^3}$ [M1]

Equation of tangent at P:

$y - \frac{1}{8e^3} = \frac{1}{4e^3}(x) \quad \Rightarrow \quad y = \frac{x}{4e^3} + \frac{1}{8e^3}$ [M1]

Gradient of normal at $P = -4e^3$ [M1]

Equation of normal at P:

$y - \frac{1}{8e^3} = -4e^3(x) \quad \Rightarrow \quad y = -4e^3 x + \frac{1}{8e^3}$ [A1]

(iv) Equation of tangent at P: $y = \frac{x}{4e^3} + \frac{1}{8e^3}$

When $y = 0$, $x = -\frac{1}{2}$. $\quad \therefore Q = \left(-\frac{1}{2},\ 0\right)$

Equation of normal at P: $y = -4e^3 x + \frac{1}{8e^3}$

When $y = 0$, $x = \frac{1}{32e^6}$. $\quad \therefore R = \left(\frac{1}{32e^6},\ 0\right)$ [M1]

Area of triangle $PQR = \frac{1}{2}\left(\frac{1}{8e^3}\right)\left[\frac{1}{32e^6} - \left(-\frac{1}{2}\right)\right]$ [M1]

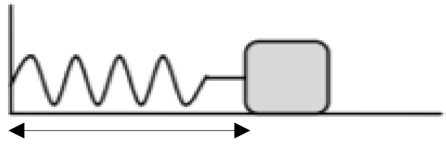
$= \frac{1}{16e^3}\left(\frac{1+16e^6}{32e^6}\right)$ [A1]

$= \frac{1+16e^6}{512e^9}$ units^2

9 **(a)** Prove that $\operatorname{cosec}^4 x - \cot^4 x = 2\operatorname{cosec}^2 x - 1$. [3]

(b) Solve the equation $6\tan 2x + 1 = \cot 2x$, for the interval $0 \le x \le 180°$. [5]

(c)



An object is connected to the wall with a spring that has a original horizontal length of 20 cm. The object is pulled back 8 cm past the original length and released. The object completes 4 cycles per second.

(i) Given that the function $x = 8\cos(a\pi t) + b$, where x is the horizontal distance, in centimetres, of the object from the wall and t is the time in seconds after releasing the object, find the values of *a* and *b*. [2]

(ii) Find the duration of time for each cycle such that the object is more than 27 cm from the wall. [3]

Solution:

(a)
$$\text{LHS} = (\operatorname{cosec}^2 x - \cot^2 x)(\operatorname{cosec}^2 x + \cot^2 x) \quad \text{[B1]}$$
$$= \operatorname{cosec}^2 x + \cot^2 x \quad \text{[B1]}$$
$$= \operatorname{cosec}^2 x + \operatorname{cosec}^2 x - 1 \quad \text{[B1]}$$
$$= 2\operatorname{cosec}^2 x - 1$$
$$= \text{RHS}$$

(b)
$$6\tan 2x + 1 = \cot 2x$$
$$6\tan^2 2x + \tan 2x - 1 = 0 \quad \text{[M1]}$$
$$(3\tan 2x - 1)(2\tan 2x + 1) = 0 \quad \text{[M1]}$$
$$0 \le x \le 360° \quad \Rightarrow \quad 0 \le 2x \le 720°$$

$\tan 2x = \frac{1}{3}$ or $\tan 2x = -\frac{1}{2}$ [M1]

α=18.435° $\quad$ α=26.565°

$2x = 18.435°, 198.43°$ $\quad$ $2x = 153.43°, 333.43°$

$x = 9.2°, 99.2°$ (1 dp) $\quad$ $x = 76.7°, 166.7°$ (1 dp) [A2]

(c)(i) $b = 20$ [B1]

$$\text{Period} = \frac{2\pi}{a\pi}$$
$$\frac{1}{4} = \frac{2\pi}{a\pi} \quad \Rightarrow \quad a = 8 \quad \text{[B1]}$$

(c)(ii) $27 = 8\cos(8\pi t) + 20$

$\cos(8\pi t) = \frac{7}{8}$ [M1]

$\alpha = 0.50536$

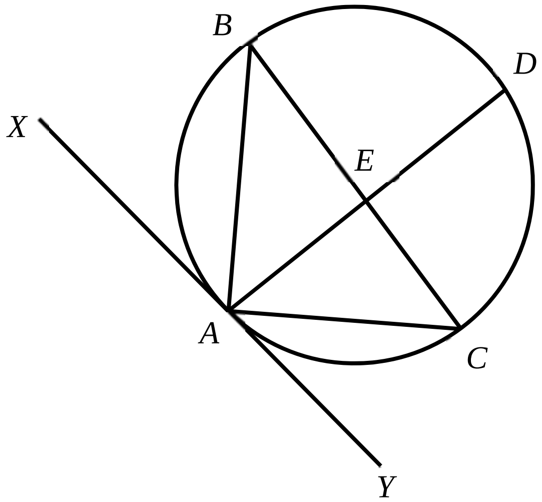
$8\pi t = 0.50536$ [M1]

$t = 0.020107$

Duration of time $= 0.020107 \times 2$

$= 0.0402$ s [A1]

10



Given that *AD* and *BC* are straight lines, *AC* bisect angle *DAY* and *AB* bisects angle *DAX*, show that

(i) $AC^2 = EC \times BC$, [3]

(ii) *BC* is a diameter of the circle, [3]

(iii) *AD* and *BC* are perpendicular to each other. [3]

Solution:

(i) $\angle BCA = \angle ACE$ (Common angle)

$\angle ABC = \angle CAY$ (Angles in the alternate segments) [B1]

$= \angle EAC$ (AC bisects $\angle DAY$)

$\therefore \Delta BAC$ and ΔAEC are similar. [B1]

$\frac{AC}{EC} = \frac{BC}{AC}$ (corresponding sides of similar triangles) [B1]

$AC^2 = EC \times BC$ (shown)

(ii) $\angle CAY = \angle EAC$ (AC bisects $\angle DAY$)

$\angle BAX = \angle EAB$ (AB bisects $\angle BAX$) [B1]

$\angle BAX + \angle EAB + \angle EAC + \angle CAY = 180°$ (angles on a straight line) [B1]

$2\angle EAB + 2\angle EAC = 180°$

$\angle EAB + \angle EAC = \angle BAC = 90°$

Since $\angle BAC = 90°$, BC is a diameter of the circle. [B1]

(iii) $\angle ABE = \angle CAY$ (Angles in the alternate segments)

$\angle CAY = \angle EAC$ (AC bisects $\angle BAY$)

$\therefore \angle ABE = \angle EAC$ [B1]

$\angle EAB + \angle EAC = \angle EAB + \angle ABE = 90°$ (from (ii))

$\angle AEB = 90°$ (sum of $\angle$s in a triangle) [B1]

$\therefore$ AD and BC are perpendicular. [B1]

END OF PAPER

Name ______________________________( ) Class:______

CHIJ KATONG CONVENT
PRELIMINARY EXAMINATION 2018
SECONDARY 4 EXPRESS / 5 NORMAL (ACADEMIC)

ADDITIONAL MATHEMATICS 4047/01
PAPER 1

Duration: 2 hours

Classes: 403, 405, 406, 502

READ THESE INSTRUCTIONS FIRST

Write your name, class and registration number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid/tape.

Answer **all** the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.
Omission of essential working will result in loss of marks.

There are two sections in this paper.
At the end of the examination, fasten sections *A* and *B* separately.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots\ldots. + \binom{n}{r} a^{n-r} b^r + \ldots\ldots. + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots\ldots.(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

Answer **all** questions.

Section A

1 A metal cube with sides $2x$ mm is heated. The sides are expanding at a rate of 0.05 mm/s. Calculate the rate of change of the total surface area of the cube when $x = 0.57$ mm. [3]

2 Without using a calculator, find the integer value of a and of b for which the solution of the equation $2x\sqrt{5} = x\sqrt{2} + \sqrt{18}$ is $\dfrac{\sqrt{a}+b}{3}$. [4]

3 The equation of a curve is $y = \dfrac{3x^2}{\sqrt{4x-h}}$.
Given that the x-coordinate of the stationary point is 1, find the value of h. [4]

4 The roots of the quadratic equation $8x^2 - 49x + c = 0$ are $\dfrac{2\alpha}{\beta}$ and $\dfrac{2\beta}{\alpha}$.

(i) Show that $c = 32$. [1]

(ii) Given that $\alpha\beta = 4$, find two distinct quadratic equations whose roots are α and β. [4]

5 Given that $y = \dfrac{2 - 3\sec^2 2x}{\tan^2 2x + 1}$,

(i) express y in the form $\cos 4x + k$, [2]

(ii) sketch the graph of $|y|$ for $-\dfrac{\pi}{2} \le x \le \pi$ and state the value of n when $|y| = n$ has four solutions. [3]

6 The polynomial $\mathrm{f}(x) = px^3 + 3x^2 + qx - 6$ is divisible by $x^2 + x - 6$.

(i) Find the value of p and of q. [4]

(ii) Find the remainder in terms of x when $\mathrm{f}(x)$ is divided by $x^2 - 1$. [2]

7 Given the equation $\dfrac{2}{\sin^2\theta} = 5 - \cot\theta$ where $0^\circ < \theta < 360^\circ$, find

(i) the values of θ, [4]

(ii) the exact values of cos θ. [2]

8 **(i)** Express $\dfrac{2x-1}{x^2(x+1)}$ in partial fractions. [4]

(ii) Hence, determine $\displaystyle\int \frac{2x-1}{x^2(x+1)}\,\mathrm{d}x$. [2]

Section B

Begin this section on a new sheet of writing paper.

9 Given the curve $y = (m+1)x^2 - 8x + 3m$ has a minimum value, find the range of values of m

(i) for which the line $y = m - 4mx$ meets the curve, [5]

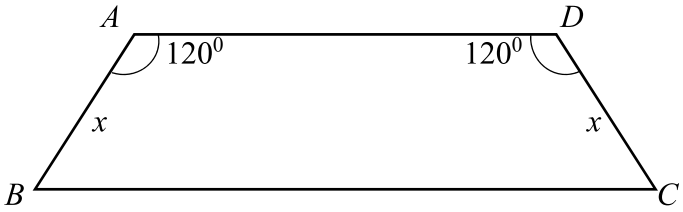
(ii) for which the y-intercept of the curve is greater than $-\dfrac{5}{2}$. [2]

10 **(i)** Solve the equation $3\log_{27}\left[\log_{1000}(x^2+9) - \log_{1000} x\right] = -1$. [3]

(ii) **(a)** On the same axes, sketch the graphs of $y = \log_{\frac{1}{2}} x - 1$ and $y = \log_2 x + 1$. [2]

(b) Explain why the two graphs are symmetrical about the x-axis. [2]

11



A piece of wire of length 80 cm is bent into the shape of a trapezium *ABCD*.
$AB = CD = x$ cm and angle BAD = angle $ADC = 120°$.

(i) Show that the area of the trapezium *ABCD* is given by $\frac{\sqrt{3}}{2}x(40-x)$ cm^2. [4]

(ii) Given that *x* can vary, find the value of *x* for which the area has a stationary value. [2]

(iii) Determine whether this stationary value is a maximum or a minimum. [2]

12 A particle moves in a straight line so that its velocity, v m/s, is given by $v = 2 - \frac{18}{(t+2)^2}$ where *t* is the time in seconds, after leaving a fixed point *O*.
Its displacement from *O* is 9 m when it is at instantaneous rest.

Find

(i) the value of t when it is at instantaneous rest, [2]

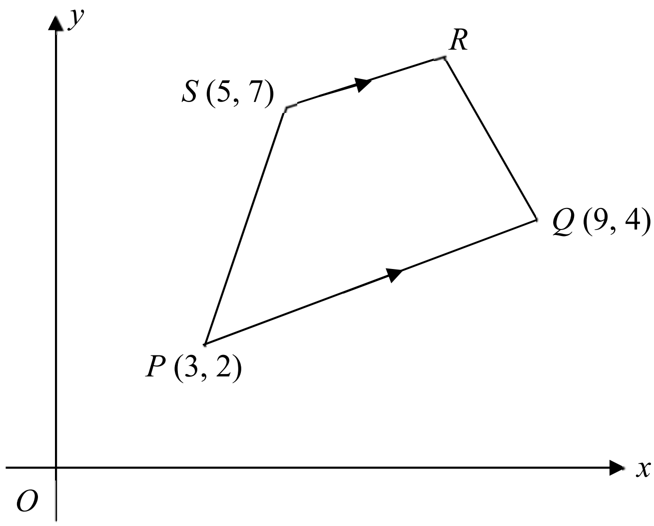
(ii) the distance travelled during the first 4 seconds. [4]

At $t = 7$, the particle starts with a new velocity, *V* m/s, given by $V = -h(t^2 - 7t) + k$.

(iii) Find the value of *k*. [1]

(iv) Given that the deceleration is 0.9 m/s^2 when $t = 8$, find the value of *h*. [2]

13 Solutions to this question by accurate drawing will not be accepted.



In the diagram, *PQ* is parallel to *SR* and the coordinates of *P*, *Q* and *S* are (3, 2), (9, 4) and (5, 7) respectively.

The gradient of the line *OR* is 1.

Find

(i) the coordinates of *R*, [4]

(ii) the area of the quadrilateral *PQRS*, [2]

(iii) the coordinates of the point *H* on the line $y = 1$ which is equidistant from *P* and *Q*. [4]

End of Paper

CHIJ KATONG CONVENT
PRELIMINARY EXAMINATION 2018
SECONDARY 4 EXPRESS /
5 NORMAL (ACADEMIC)

ADDITIONAL MATHEMATICS **4047/01**
PAPER 1
Classes: 403, 405, 406, 502

Solutions for students

1 A metal cube with sides $2x$ mm is heated. The sides are expanding at a rate of 0.05 mm/s.

Calculate the rate of change of the total surface area of the cube when $x = 0.57$ mm. [3]

Solution

Let $l = 2x$

Area $A = 6l^2$

$\frac{dA}{dt} = \frac{dA}{dl} \times \frac{dl}{dt}$

$= 12l \times 0.05$ chain rule

$= 12(2(0.57)) \times 0.05$

$= 0.684$

Answer: 0.684 mm^2/s.

OR

Most students applied this method but used 0.05 wrongly for $\frac{dx}{dt}$.

Some students used wrong formula for SA.

Area $A = 6(2x)^2 = 24x^2$

$\frac{dA}{dt} = \frac{dA}{dx} \times \frac{dx}{dt}$

$= 48x \times 0.025$ chain rule

$= 48(0.57) \times 0.025$

$= 0.684$

Answer: 0.684 mm^2/s.

2 Without using a calculator, find the integer value of a and of b for which the solution of the equation $2x\sqrt{5} = x\sqrt{2} + \sqrt{18}$ is $\frac{\sqrt{a}+b}{3}$. [4]

Solution

$$x\left(2\sqrt{5}-\sqrt{2}\right)=\sqrt{18}$$

$$x=\frac{\sqrt{18}}{2\sqrt{5}-\sqrt{2}}\times\frac{2\sqrt{5}+\sqrt{2}}{2\sqrt{5}+\sqrt{2}}$$

conjugate surds

$$=\frac{2\sqrt{90}+6}{18}$$

$$=\frac{6\sqrt{10}+6}{18}$$

$$=\frac{\sqrt{10}+1}{3}$$

$a=10$, $b=1$

OR

A handful used this method but did not reject one answer/ did not know why one of the answers is not acceptable.

$$\left(2x\sqrt{5}\right)^2=\left(x\sqrt{2}+\sqrt{18}\right)^2$$

$$20x^2=2x^2+2\sqrt{36}x+18$$

$$18x^2-12x-18=0$$

$$3x^2-2x-3=0$$

$$x=\frac{2+\sqrt{4-4(2)(-3)}}{2(3)}$$

$$=\frac{1+\sqrt{10}}{3} \text{ or } \frac{1-\sqrt{10}}{3}\text{(reject)}$$

$a=10, b=1$

3 The equation of a curve is $y=\frac{3x^2}{\sqrt{4x-h}}$.

Given that the x-coordinate of the stationary point is 1, find the value of h. [4]

Solution

$$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\sqrt{4x-h}\,(6x)-3x^2\left(\frac{1}{2}\right)(4x-h)^{-\frac{1}{2}}(4)}{4x-h}$$

quotient OR product rule

$$=\frac{(4x-h)^{-\frac{1}{2}}\left[(6x)(4x-h)-6x^2\right]}{4x-h}$$

$$=\frac{18x^2-6hx}{(4x-h)^{\frac{3}{2}}}$$

At stationary point, $\frac{\mathrm{d}y}{\mathrm{d}x}=0$.

When $x=1$, $\frac{18(1)^2-6h(1)}{(4(1)-h)^{\frac{3}{2}}}=0$

$h=3$

4 The roots of the quadratic equation $8x^2 - 49x + c = 0$ are $\frac{2\alpha}{\beta}$ and $\frac{2\beta}{\alpha}$.

(i) Show that $c = 32$. [1]

(ii) Given that $\alpha\beta = 4$, find two distinct quadratic equations whose roots are α and β. [4]

Solution

(i)

$$\left(\frac{2\alpha}{\beta}\right)\left(\frac{2\beta}{\alpha}\right) = \frac{c}{8}$$

$$4 = \frac{c}{8}$$

$$c = 32$$

(ii)

$$\frac{2\alpha}{\beta} + \frac{2\beta}{\alpha} = \frac{49}{8}$$ SOR

$$\frac{2\alpha^2 + 2\beta^2}{\alpha\beta} = \frac{49}{8}$$

$$\frac{2\alpha^2 + 2\beta^2}{4} = \frac{49}{8}$$

$$\alpha^2 + \beta^2 = \frac{49}{4}$$

$$(\alpha + \beta)^2 - 2\alpha\beta = \frac{49}{4}$$ apply perfect square

$$(\alpha + \beta)^2 - 8 = \frac{49}{4}$$

$$(\alpha + \beta)^2 = \frac{81}{4}$$

$$\alpha + \beta = \pm\frac{9}{2}$$

Eqns are $2x^2 - 9x + 8 = 0$, $\quad 2x + 9x + 8 = 0$.

both eqns, accept fractional coefficients

5 Given that $y = \dfrac{2 - 3\sec^2 2x}{\tan^2 2x + 1}$,

(i) express y in the form $\cos 4x + k$, [2]

(ii) sketch the graph of $|y|$ for $-\frac{\pi}{2} \le x \le \pi$ and state the value of n when $|y| = n$ has four solutions. [3]

Solution

(i) $\frac{2-3\sec^2 2x}{\tan^2 2x+1} = \frac{2-3\sec^2 2x}{\sec^2 2x}$

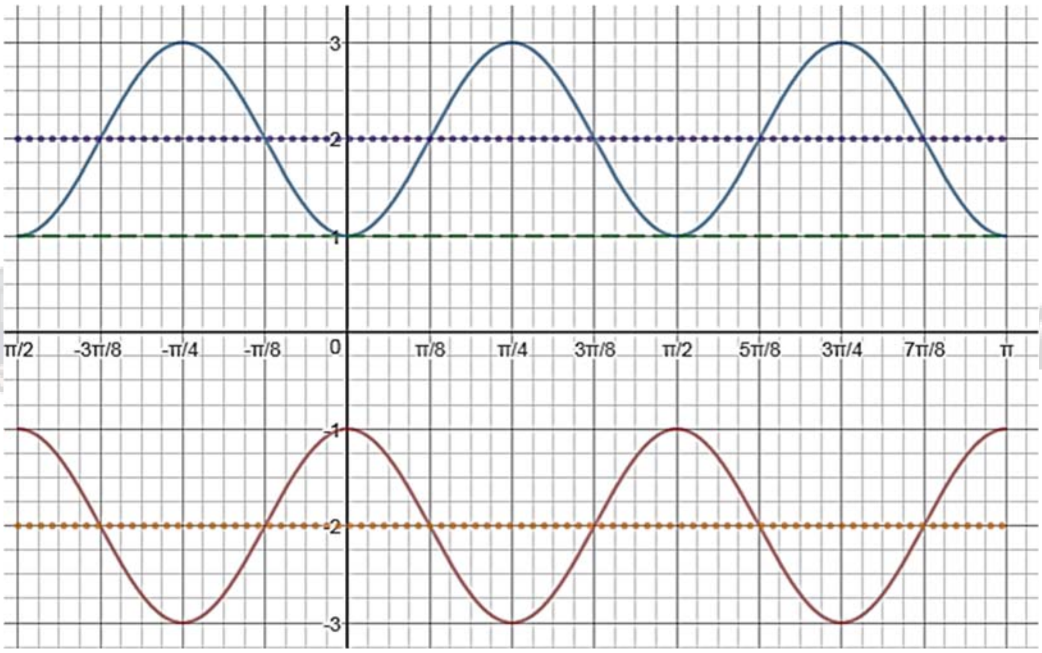
$= 2\cos^2 2x - 3$

$= 2\cos^2 2x - 1 - 2$

$= \cos 4x - 2$

(iii) graph

$n = 1$



6 The polynomial $f(x) = px^3 + 3x^2 + qx - 6$ is divisible by $x^2 + x - 6$.

(i) Find the value of p and of q. [4]

(ii) Find the remainder in terms of x when f(x) is divided by x^2-1. [2]

Solution

(i) $x^2+x-6=(x-2)(x+3)$

By the factor thm, $f(2)=0$

$p(2)^3+3(2)^2+q(2)-6=0$ factor thm

$8p+2q+6=0$

$4p+q=-3$ ....(1)

$f(-3)=0$

$p(-3)^3+3(-3)^2+q(-3)-6=0$ factor thm

$-27p-3q+21=0$

$9p+q=7$ ....(2)

Solve (1) and (2); $p=2$, $q=-11$

OR

$px^3+3x^2+qx-6=(x-2)(x+3)(px+1)$

(ii) Using $x^2=1$,

$f(x)=2x^3+3x^2-11x-6$

$=2x^2(x)+3x^2-11x-6$

$=2x+3-11x-6$

$=-9x-3$

OR long division (ecf)

Many used this method.

$$
\begin{array}{r|l}
 & 2x + 3 \\
x^2-1 & 2x^3 + 3x^2 - 11x - 6 \\
 & 2x^3 \qquad\quad - 2x \\
\hline
 & \qquad 3x^2 - 9x - 6 \\
 & \qquad 3x^2 \qquad - 3 \\
\hline
 & \qquad\qquad -9x - 3
\end{array}
$$

7 Given the equation $\dfrac{2}{\sin^2\theta}=5-\cot\theta$ where $0^0<\theta<360^0$, find

(i) the values of θ. [4]

(ii) the exact values of cos θ. [2]

Solution

(i) $2\operatorname{cosec}^2\theta=5-\cot\theta$

$2(1+\cot^2\theta)-5+\cot\theta=0$ identity

$2\cot^2\theta+\cot\theta-3=0$

$(2\cot\theta+3)(\cot\theta-1)=0$ factorisation

$\cot\theta=-\dfrac{3}{2}$ or $\cot\theta=1$

$\tan\theta=-\dfrac{2}{3}$ or $\tan\theta=1$

Basic angle = 33.69^0 , 45^0

$\theta=146.3^0, 326.3^0, 45^o, 225^0$

OR

$5\sin^2\theta-\sin\theta\cos\theta-2=0$

which is common to many but at the same time spells the end of qn 7.

$5\sin^2\theta-\sin\theta\cos\theta-2(\cos^2\theta+\sin^2\theta)=0$

$3\sin^2\theta-\sin\theta\cos\theta-2\cos^2\theta=0$

$(3\sin\theta+2\cos\theta)(\sin\theta-\cos\theta)=0$

$\tan\theta=-\dfrac{2}{3}$ or $\tan\theta=1$

(ii) $\tan\theta=-\dfrac{2}{3}$ (quadrants 2, 4) or $\tan\theta=1$ (quadrants 1, 3)

$\cos\theta = \pm\frac{3}{\sqrt{13}}$, $\cos\theta = \pm\frac{1}{\sqrt{2}}$

8 **(i)** Express $\frac{2x-1}{x^2(x+1)}$ in partial fractions. [4]

(ii) Hence, determine $\int \frac{2x-1}{x^2(x+1)}\,dx$. [2]

Solution

(i) $\frac{2x-1}{x^2(x+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+1}$ correct factors

$2x-1 = Ax(x+1) + B(x+1) + Cx^2$ or comparing coeff.

Let $x = -1$, $-3 = C(-1)^2 \Longrightarrow C = -3$

Let $x = 0$, $B = -1$

Let $x = 1$, $1 = 2A - (2) - 3(1)^2 \Longrightarrow A = 3$

Hence, $\frac{2x-1}{x^2(x+1)} = \frac{3}{x} - \frac{1}{x^2} - \frac{3}{x+1}$

(ii) $\int \frac{2x-1}{x^2(x+1)}\,dx = \int\left(\frac{3}{x} - \frac{1}{x^2} - \frac{3}{x+1}\right)dx$

9 Given the curve $y = (m+1)x^2 - 8x + 3m$ has a minimum value, find the range of values of m

(i) for which the line $y = m - 4mx$ meets the curve, [5]

(ii) for which y – intercept of the curve is greater than $-\frac{5}{2}$. [2]

Solution

(i) $(m+1)x^2 - 8x + 3m = m - 4mx$

$(m+1)x^2 + 4mx - 8x + 2m = 0$ quadratic eqn

$b^2 - 4ac \geq 0$

$(4m-8)^2 - 4(m+1)(2m) \geq 0$ discriminant, inequality

$(4(m-2))^2 - 8m(m+1) \geq 0$

$2(m^2 - 4m + 4) - m^2 - m \geq 0$ expansion, simplify

$m^2 - 9m + 8 \geq 0$

$(m-1)(m-8) \geq 0$ factorisation

$m \leq 1$ or $m \geq 8$

Since it is a minimum graph, $m + 1 > 0$, ie $m > -1$

So $-1 < m \leq 1$ or $m \geq 8$

(ii) At y – intercept, $x = 0$,

$$(m+1)x^2 - 8x + 3m > -\frac{5}{2}$$

$$m > -\frac{5}{6}$$

10 **(i)** Solve the equation $3\log_{27}\left[\log_{1000}(x^2+9) - \log_{1000} x\right] = -1$. [3]

Solution

$$\log_{1000}\frac{x^2+9}{x} = 27^{-\frac{1}{3}}$$

$$\log_{1000}\frac{x^2+9}{x} = \frac{1}{3} \qquad \text{index form}$$

$$\frac{x^2+9}{x} = 1000^{\frac{1}{3}}$$

$$x^2 + 9 = 10x \qquad \text{index form}$$

$$x^2 - 10x + 9 = 0$$

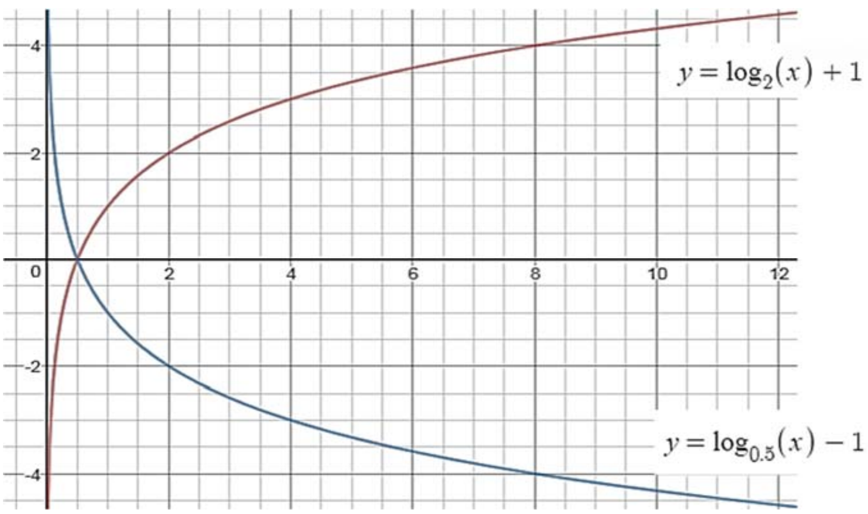
$$(x-1)(x-9) = 0$$

$$x = 1 \text{ or } 9$$

(ii) **(a)** On the same axes, sketch the graphs of $y = \log_{\frac{1}{2}} x - 1$ and $y = \log_2 x + 1$. [2]

(b) Explain why the two graphs are symmetrical about the x-axis. [2]

Solution



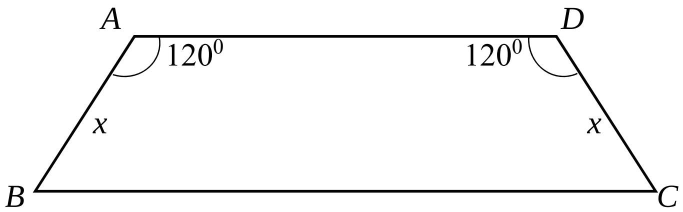
(ii) $-(\log_{\frac{1}{2}} x - 1) = -\dfrac{\log_2 x}{\log_2 \frac{1}{2}} + 1$ [M1]

$= -\dfrac{\log_2 x}{\log_2 2^{-1}} + 1$

$= \log_2 x + 1$

The functions are negative of each other. [A1]

11

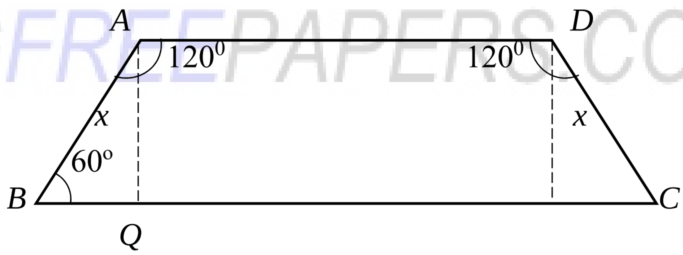


A piece of wire of length 80 cm is bent into the shape of a trapezium *ABCD*.
$AB = CD = x$ cm and angle BAD = angle $ADC = 120°$.

(i) Show that the area of the trapezium *ABCD* is given by $\dfrac{\sqrt{3}}{2}x(40-x)$ cm^2. [4]

(ii) Given that *x* can vary, find the value of *x* for which the area has a stationary value. [2]

(iii) Determine whether this stationary value is a maximum or a minimum. [2]



Solution

$\angle ABC = 180 - 120 (\text{int.}\angle s, AD // BC)$

$= 60^0$

(i) $\cos \angle ABC = \dfrac{BQ}{x}$

$BQ = \dfrac{x}{2}$

Perimeter $= BC + 2x + AD$

$80 = \dfrac{x}{2} + AD + \dfrac{x}{2} + x + AD + x$

$AD = \dfrac{80-3x}{2}$

$\sin 60° = \dfrac{AQ}{x}$ ➔ $AQ = \dfrac{\sqrt{3}}{2}x$

Area $= \dfrac{1}{2}(AD + BC)\left(\dfrac{\sqrt{3}}{2}x\right)$

OR (Most used this method)

$AD + BC = 80 - 2x$

$\angle ABC = 180 - \angle BAD$ (int.angles, $AD//BC$)

$= 60°$

AQ = height of the trapezium

$\sin 60 = \dfrac{AQ}{x}$

$AQ = \dfrac{\sqrt{3}}{2}x$

Area $= \dfrac{1}{2}\left(\dfrac{\sqrt{3}}{2}x\right)(AD + BC)$

$= \dfrac{1}{2}\left(\dfrac{\sqrt{3}}{2}x\right)(80 - 2x)$

$= \dfrac{\sqrt{3}}{2}x(40 - x)$ (shown)

$= \frac{1}{4}(80 - 3x + x)\sqrt{3}x$

$= \frac{\sqrt{3}}{4}x(80 - 2x)$

$= \frac{\sqrt{3}}{2}x(40 - x)$ (Shown)

(ii) $\frac{dA}{dx} = 0$ when the area has a stationary value

$20\sqrt{3} - \frac{\sqrt{3}}{2}(2x) = 0$ differentiation

$x = 20$

(iii) $\frac{d^2A}{dx^2} = -\sqrt{3} < 0$. second derivative or using first derivative

Area is a maximum

12 A particle moves in a straight line so that its velocity, v m/s, is given by $v = 2 - \frac{18}{(t+2)^2}$ where t is the time in seconds, after leaving a fixed point O.

Its displacement from O is 9 m when it is at instantaneous rest.

(i) the value of t when it is at instantaneous rest, [2]

(ii) the distance travelled during the first 4 seconds. [4]

At $t = 7$, the particle starts with a new velocity, V ms^{-1}, given by $V = -h(t^2 - 7t) + k$.

(iii) Find the value of k. [1]

(iv) Given that the deceleration is 1.9 m/s^2 when $t = 8$, find the value of h. [2]

Solution

(i) At turning pt, $v = 0$

$2 - \frac{18}{(t+2)^2} = 0$

$t = 1$ or -5 (NA)

(ii)

$s = \int \frac{dv}{dt} dt \quad = 2t + \frac{18}{t+2} + c$

When $t = 1$, $s = 9$

$2(1) + \frac{18}{1+2} + c = 9$

$c = 1$, so $\quad s = 2t + \frac{18}{t+2} + 1$

When $t = 0$, s = 10 m
When $t = 1$, $s = 9$ m
When $t = 4$, $s = 12$ m

Total distance travelled = 10 - 9 +12 – 9 = 4 m

(iii) When $t = 7$, $v = 2 - \frac{18}{(7+2)^2} = \frac{16}{9}$

$V = -h(t-7) + k = \frac{16}{9}$, hence $k = \frac{16}{9}$

(iv) $V = -h(t^2 - 7t) + k = -ht^2 + 7ht + k$

$a = \frac{dV}{dt} = -2ht + 7h$

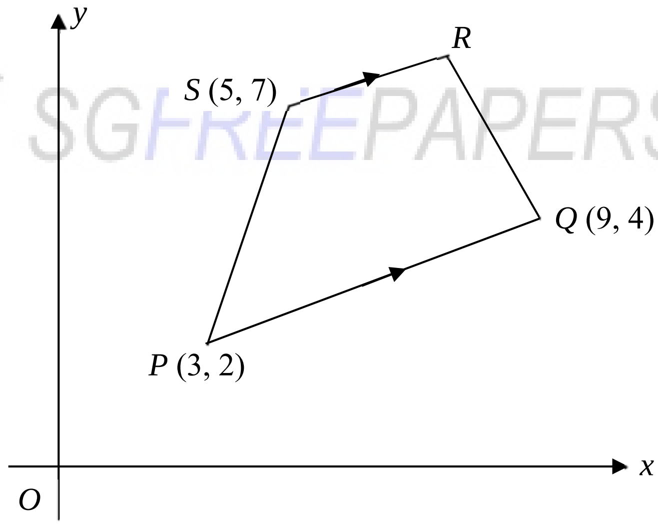
$-2h(8) + 7h = -0.9$

$-16h + 7h = -0.9$

$-9h = -0.9$

$h = 0.1$

13 **Solutions to this question by accurate drawing will not be accepted.**



In the diagram, PQ is parallel to SR and the coordinates of P, Q and S are (3, 2), (9, 4) and (5, 7) respectively. The gradient of the line OR is 1.

Find

(i) the coordinates of R, [4]

(ii) the area of the quadrilateral $PQRS$, [2]

(iii) the coordinates of the point H on the line $y = 1$ which is equidistant from P and Q. [4]

Solution

(i) $m_{PQ} = \frac{1}{3}$

Since $PQ \parallel SR$, $m_{SR} = \frac{1}{3}$

Eqn of SR, $(y-7)=\frac{1}{3}(x-5)$ $==>$ $y=\frac{x}{3}+\frac{16}{3}$

Sub. $R(a, a)$ into $y=\frac{x}{3}+\frac{16}{3}$, $a=8$ OR use eqn of OR as $y=x$

$\therefore R=(8,\ 8)$

(ii) Area of $PQRS = \frac{1}{2}\begin{vmatrix} 3 & 9 & 8 & 5 & 3 \\ 2 & 4 & 8 & 7 & 2 \end{vmatrix}$ [M1]

$=\frac{1}{2}(39)=19.5 \text{ units}^2$ [A1]

(iii) Since the point H lies on the line $y=1$ and is equidistant from P and Q, H must lie on the $\perp$ bisector of PQ.

Mid-point of $PQ=(6, 3)$

gradient of $\perp$ bisector $=-3$.

Equation, $(y-3)=-3(x-6)$

$y=-3x+21$

Since $y=1$,

$1=-3x+21,\quad x=6\frac{2}{3}$

$\therefore H(6\frac{2}{3},\ 1)$

OR

$PH=QH$

$\sqrt{(2-1)^2+(3-x)^2}=\sqrt{(4-1)^2+(9-x)^2}$ using length

$1+9-6x+x^2=9+81-18x+x^2$ expansion

$12x=80$

$x=\frac{20}{3}$

$H=(\frac{20}{3},1)$

Name ____________________________( ) Class:______

CHIJ KATONG CONVENT
PRELIMINARY EXAMINATION 2018
SECONDARY 4 EXPRESS/
5 NORMAL (ACADEMIC)

ADDITIONAL MATHEMATICS 4047/02
PAPER 2

Duration: 2 hours 30 minutes

Classes: 403, 405, 406, 502

READ THESE INSTRUCTIONS FIRST

Write your name, class and registration number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid/tape.

Answer **all** the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.
Omission of essential working will result in loss of marks.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 100.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots\ldots. + \binom{n}{r}a^{n-r}b^r + \ldots\ldots. + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots\ldots.(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2}bc \sin A$$

Name: ___________________________ (   ) Class: ______

1 A rectangular garden, with length x m and breadth y m, has an area of 270 m^2. It has a path of width 2.5 m all round it. Given that the outer perimeter of the path is 87 m, find the length and breadth of the garden. [5]

2 **(a)** Solve $2\left(9^{x-1}\right)-5\left(3^{x}\right)=27$. [4]

(b) Given that $f(x)=\ln(5x-2)^3$,

(i) State the range of x for $f(x)$ to be defined. [1]

(ii) Show that $5f'(x)+(5x-2)f''(x)=0$. [4]

3 **(a)** **(i)** Write down the first four terms in the expansion of $(1+x)^{50}$ and $(1-x)^{50}$.

Hence, write down the first two terms for $(1+x)^{50}-(1-x)^{50}$. [3]

(ii) Without the use of calculator, deduce if 1.01^{50} or $1^{50}+0.99^{50}$ is larger. [3]

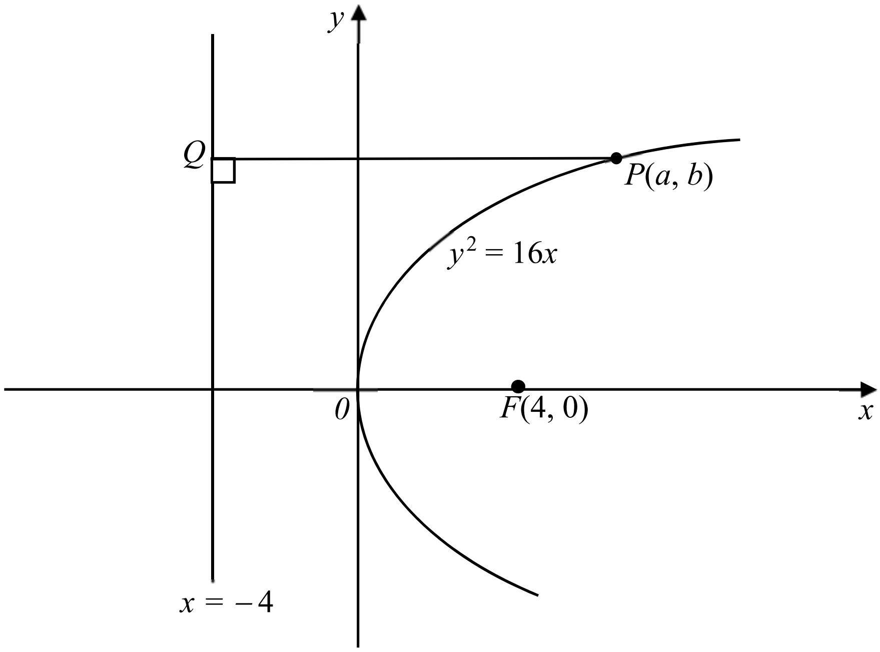
(b) The term independent of x in $x^{11}\left(2x+\frac{k}{x^2}\right)^7$ is 896.

Find the two possible values of k. [4]

4 **(i)** Prove that $\tan A+\cot A=\frac{2}{\sin 2A}$. [4]

(ii) Hence, or otherwise, solve $\tan A+\cot A=2.5$ for $0°<A<270°$. [4]

5 In the diagram, not drawn to scale, $P(a, b)$ is a point on the graph $y^2 = 16x$, and Q is a point on the line $x = -4$. PQ is the perpendicular distance from P to this line. $F(4, 0)$ is a point on the x-axis.



(i) Find the length PF in terms of a. [3]

(ii) Given that the tangent to the curve at P cuts the y-axis at G, find the coordinates of G in terms of a. [4]

(iii) Show that G is the the mid-point of QF. [2]

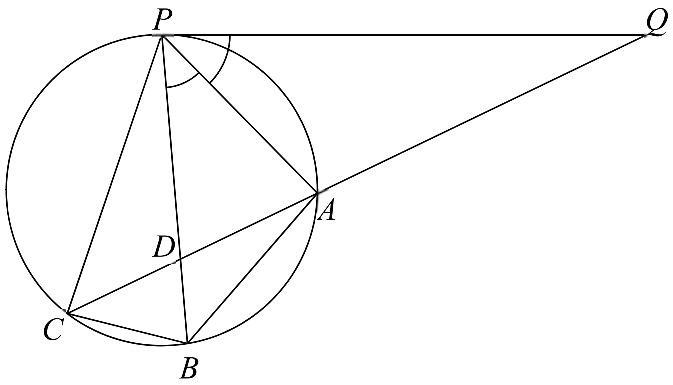
(iv) Find the equation of the normal at P in terms of a. [2]

6 **(a)** Evaluate $\int_0^{\frac{\pi}{6}} \sin\left(2x + \frac{\pi}{6}\right) \mathrm{d}x$, leaving your answer in surd form. [3]

(b) **(i)** Find $\frac{\mathrm{d}}{\mathrm{d}x}\left[e^{2x}\left(\cos 3x + \frac{3}{2}\sin 3x\right)\right]$. [4]

(ii) Hence find $\int e^{2x} \cos 3x \ \mathrm{d}x$. [2]

7 The diagram shows a point P on a circle and PQ is a tangent to the circle. Points A, B and C lie on the circle such that PA bisects angle QPB and QAC is a straight line. The lines QC and PB intersect at D.



(i) Prove that $AP = AB$. [4]

(ii) Prove that CD bisects angle PCB. [4]

(iii) Prove that triangles CDP and CBA are similar. [2]

8 The table below shows experimental values of two variables x and y obtained from an experiment.

x	1	2	3	4	5	6
y	5.1	17.5	37.5	60.5	98	137

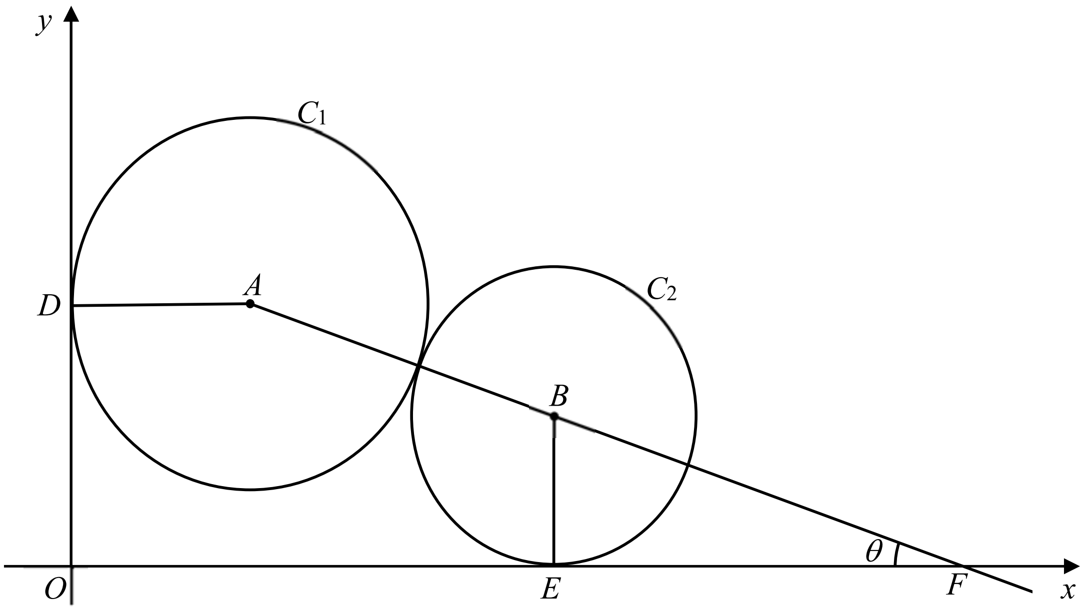
It is also given that x and y are related by the equation $y = ax + bx^2$, where a and b are constants.

(i) Plot $\frac{y}{x}$ against x and draw a straight line graph. Use 2 cm to represent 1 unit on the horizontal axis and 4 cm to represent 10 units on the vertical axis. [4]

(ii) Use the graph to estimate the value of a and of b. [2]

(iii) By drawing a suitable straight line, estimate the value of x for which $(b+5)x = 38 - a$. [4]

9 The figure below shows two circles, C_1 and C_2, touching each other in the first quadrant of the Cartesian plane. C_1 has radius 5 and touches the y-axis at D. C_2 has radius 4 and touches the x-axis at E. The line AB joining the centre of C_1 and C_2, meets the x-axis at F. Angle BFO is θ.



(i) Find expressions for OD and OE in terms of θ and show that

$$DE^2 = 122 + 90\cos\theta + 72\sin\theta.$$ [3]

(ii) Hence express DE^2 in the form $122 + R\cos(\theta - \alpha)$, where R > 0 and α is acute. [3]

(iii) Calculate the greatest possible length of DE and state the corresponding value of θ. [3]

10 The population of a town is estimated to increase by k % per year. The population at the end of 2017 was 20000. The population, y, after x years can be modelled by $y = A(1.11)^x$.

(i) Deduce the value of A and of k with the information provided. [2]

(ii) Sketch the graph of y. [1]

(iii) Find the value of x when $y = 9600$.
Explain the meaning of this value of x. [3]

(iv) Calculate the population of the town at the end of 2027. [2]

11 Given that $y = 2x^3 + 3x^2 + 11x + 5$,

(i) show that

(a) y is an increasing function for all values of x, [2]

(b) y has only one real root at $x = -\frac{1}{2}$. [3]

(ii) sketch the graph of y, [2]

(iii) hence, calculate the area bounded by $y = 2x^3 + 3x^2 + 11x + 5$, the x-axis and the lines $x = -1$ and $x = 1$. [4]

End of paper

1	$xy = 270$ $y = \frac{270}{x}$ .... (1) $2(x+5+y+5) = 87$ $x + y = \frac{67}{2}$ .... (2) Substitute (1) into (2), $x + \frac{270}{x} = \frac{67}{2}$ $2x^2 - 67x + 540 = 0$ $(2x-27)(x-20) = 0$ $2x - 27 = 0$ or $x - 20 = 0$ $x = 13.5$ or $x = 20$ When $x = 13.5$, $y = 20$ When $x = 20$, $y = 13.5$ Since x is the length, then $x = 20$ m and $y = 13.5$ m.
2a	$2\left(3^{2x} \bullet \frac{1}{9}\right) - 5\left(3^x\right) = 27$ Let 3^x, $\frac{2}{9}y^2 - 5y - 27 = 0$ $2y^2 - 45y - 243 = 0$ $(2y+9)(y-27) = 0$ $y = -\frac{9}{2}$ or $y = 27$ $3^x = -\frac{9}{2}$ (rejected) or $3^x = 3^3$ $\therefore x = 3$
2bi	$5x - 2 > 0$ $x > \frac{2}{5}$
	$f'(x) = \frac{3(5x-2)^2 \bullet 5}{(5x-2)^3}$ $= \frac{15}{5x-2}$

	OR $f'(x)=\frac{3\bullet 5}{(5x-2)}$ $=\frac{15}{5x-2}$ $f''(x)=-\frac{15}{(5x-2)^2}\bullet 5$ $=-\frac{75}{(5x-2)^2}$ $\therefore 5f'(x)+(5x-2)f''(x)$ $=\frac{75}{5x-2}-\frac{75}{5x-2}$ $=0$ (shown)
3ai	$(1+x)^{50}=1^{50}+50x+{}^{50}C_2x^2+{}^{50}C_3x^3+...+x^{50}$ $=1+50x+1225x^2+19600x^3+...+x^{50}$ $(1-x)^{50}=1-50x+1225x^2-19600x^3+...-x^{50}$ $(1+x)^{50}-(1-x)^{50}=100x+39200x^3$
ii	Let $x = 0.01$, $1.01^{50}-0.99^{50}=100(0.01)+39200(0.01)^3$ $=1+0.0392$ $1.01^{50}=1+0.0392+0.99^{50}$ $>1+0.99^{50}$ Hence, 1.01^{50} is larger.
3b	$T_{r+1}={}^7C_r(2x)^{7-r}\left(\frac{k}{x^2}\right)^r$ $={}^7C_r2^{7-r}k^rx^{7-3r}$ For $7-3r=-11$ $r=6$

	OR $x^{11}T_{r+1} = {}^7C_r\,(2x)^{7-r}\left(\dfrac{k}{x^2}\right)^r x^{11}$ $= {}^7C_r\,2^{7-r}k^r x^{18-3r}$ For $18 - 3r = 0$ $r = 6$ Term independent of $x = 896$ $x^{11}\left(2x+\dfrac{k}{x^2}\right)^7 = 896$ ${}^7C_6\,2^{7-6}k^6 = 896$ $k^6 = 64$ $k = \pm 2$ Alternative method: $x^{11}\left(2x+\dfrac{k}{x^2}\right)^7$ $= x^{11}\begin{pmatrix} 2^7x^7 + 7(2x)^6\left(\dfrac{k}{x^2}\right) + {}^7C_2\,(2x)^5\left(\dfrac{k}{x^2}\right)^2 \\ + {}^7C_3\,(2x)^4\left(\dfrac{k}{x^2}\right)^3 + {}^7C_4\,(2x)^3\left(\dfrac{k}{x^2}\right)^4 \\ + {}^7C_5\,(2x)^5\left(\dfrac{k}{x^2}\right)^2 + {}^7C_6\,(2x)^6\left(\dfrac{k}{x^2}\right) \\ + \left(\dfrac{k}{x^2}\right)^7 \end{pmatrix}$ $= x^{11}\begin{pmatrix} 2^7x^7 + 7(2^6)kx^4 + {}^7C_2 2^5k^2x + {}^7C_3 2^4k^3x^{-2} \\ + {}^7C_4 2^3k^4x^{-5} + {}^7C_5 2^2k^5x^{-5} + {}^7C_6 2k^6x^{-11} + k^7x^{-15} \end{pmatrix}$ Term independent term of $x = 896$ $896 = x^{11}\left({}^7C_6 2k^6x^{-11}\right)$ $896 = 14k^6$ $k^6 = 64$ $k = \pm 2$

4i	$\text{LHS} = \tan A + \cot A$ $= \frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}$ $= \frac{\sin^2 A + \cos^2 A}{\sin A \cos A}$ $= \frac{1}{\frac{1}{2}(2\sin A\cos A)}$ $= \frac{2}{\sin 2A}$ $= \text{RHS} \quad \text{(shown)}$ OR $\text{LHS} = \tan A + \cot A$ $= \tan A + \frac{1}{\tan A}$ $= \frac{\tan^2 A + 1}{\tan A}$ $= \frac{\sec^2 A}{\tan A}$ $= \frac{1}{\cos^2 A} \bullet \frac{\cos}{\sin A}$ $= \frac{1}{\sin A\cos A}$ $= \frac{2}{2\sin A\cos A}$ $= \frac{2}{\sin 2A}$ $= \text{RHS} \quad \text{(shown)}$
4ii	$\frac{2}{\sin 2A} = \frac{5}{2}$ $\sin 2A = \frac{4}{5}$ $\alpha = 53.13°$ $2A = 53.13°, 126.87°, 413.13°, 486.67°$ $A = 26.6°, 63.4°, 206.6°, 243.4°$

5i	$y^2 = 16x$ At P, $b^2 = 16a$ $\text{PF} = \sqrt{(a-4)^2 + b^2}$ $= \sqrt{a^2 - 8a + 16 + 16a}$ $= \sqrt{(a+4)^2}$ $= a + 4$
ii	$y^2 = 16x$ $y = 4\sqrt{x}$ $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2}{\sqrt{x}}$ At P, $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2}{\sqrt{a}}$ Equation of tangent at P, $y - b = \frac{2}{\sqrt{a}}(x - a)$ $y = \frac{2x}{\sqrt{a}} - 2\sqrt{a} + 4\sqrt{a}$ $= \frac{2x}{\sqrt{a}} + 2\sqrt{a}$ When $x = 0$, $y = 2\sqrt{a}$ $\therefore$ G (0, $2\sqrt{a}$)
iii	Mid-point of QF $= \left(\frac{-4+4}{2}, \frac{b+0}{2}\right)$ $= \left(0, \frac{4\sqrt{a}}{2}\right)$ $= \left(0, 2\sqrt{a}\right)$ Hence, G lies in the centre of QF. OR find lengths of QG and GP.

iv	Gradient of normal at P = $-\frac{\sqrt{a}}{2}$ Equation of normal at P: $$y-b=-\frac{\sqrt{a}}{2}(x-a)$$ $$y=-\frac{\sqrt{a}}{2}x+\frac{a\sqrt{a}}{2}+4a$$
6a	$\int_0^{\frac{\pi}{6}}\sin\left(2x+\frac{\pi}{6}\right)dx$ $=\left[-\frac{\cos\left(2x+\frac{\pi}{6}\right)}{2}\right]_0^{\frac{\pi}{6}}$ $=-\frac{\cos\frac{\pi}{2}}{2}-\left(-\frac{\cos\frac{\pi}{6}}{2}\right)$ $=0+\frac{\sqrt{3}}{4}$ $=\frac{\sqrt{3}}{4}$
6bi	$\frac{d}{dx}\left[e^{2x}\left(\cos 3x+\frac{3}{2}\sin 3x\right)\right]$ $=2e^{2x}\left(\cos 3x+\frac{3}{2}\sin 3x\right)+e^{2x}\left(-3\sin 3x+\frac{9}{2}\cos 3x\right)$ $=e^{2x}\left(2\cos 3x+3\sin 3x-3\sin 3x+\frac{9}{2}\cos 3x\right)$ $=\frac{13}{2}e^{2x}\cos 3x$
6bii	$\int e^{2x}\cos 3x\ dx=\frac{2}{13}\int\frac{13}{2}e^{2x}\cos 3x\ dx$ $=\frac{2}{13}e^{2x}\left(\cos 3x+\frac{3}{2}\sin 3x\right)+C$
7i	$\measuredangle ABP=\measuredangle APQ$ (alt. segment theorem) Since PA bisects $\measuredangle$QPB, $\measuredangle APQ=\measuredangle APB$ $\therefore\measuredangle ABP=\measuredangle APB$ (base $\measuredangle$s of isosceles triangle APB) Hence, AP = AB.

7ii

$\measuredangle ACB = \measuredangle APB$ ($\measuredangle$s in the same segemnt)

$\measuredangle ACP = \measuredangle ABP$ ($\measuredangle$s in the same segemnt)

$= \measuredangle APB$ (shown)

$\measuredangle$ACB=$\measuredangle$ACP

Hence, CD bisects $\measuredangle PCB$.

7iii

$\measuredangle ACB = \measuredangle ACP$ (from ii)

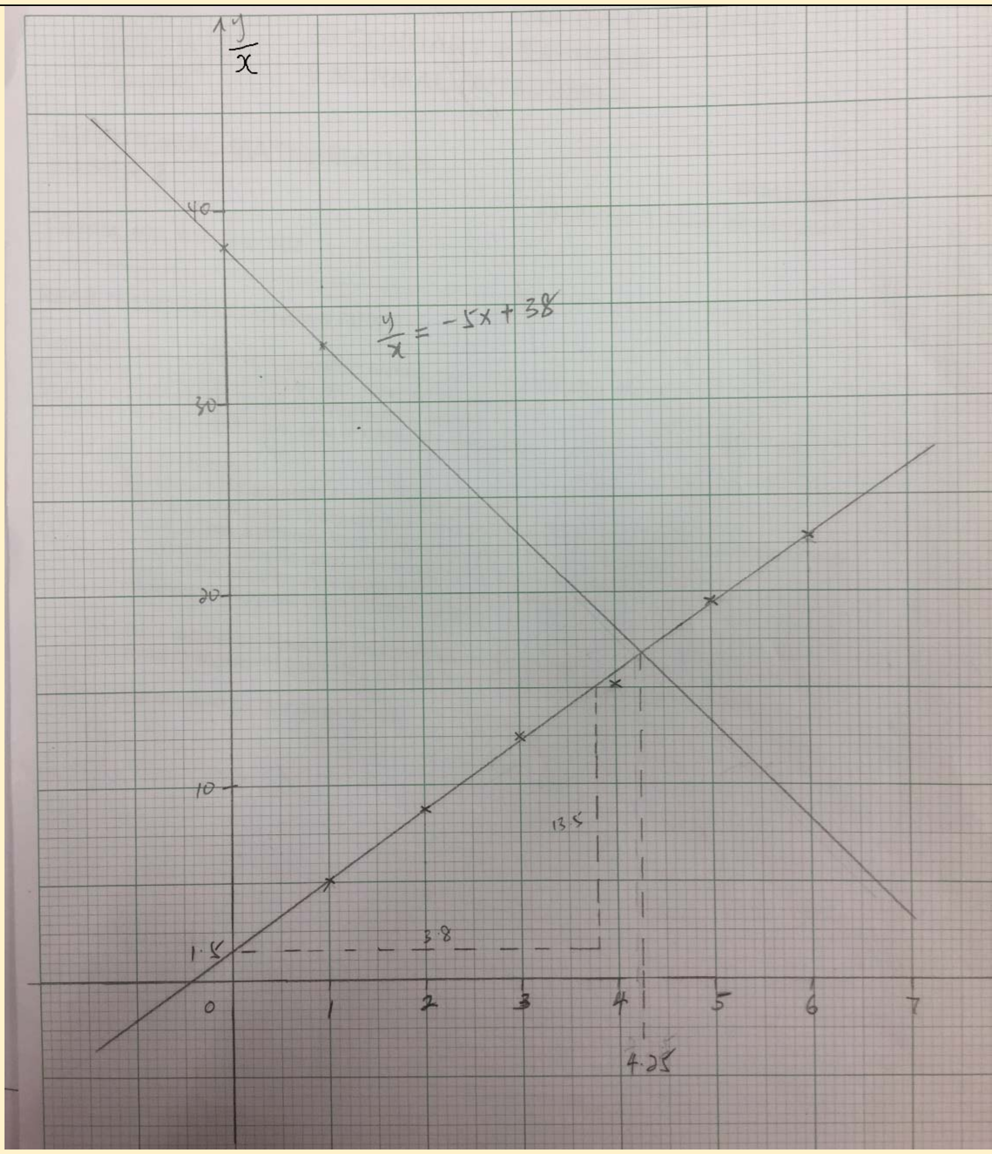
$\measuredangle CPD = \measuredangle CAB$ ($\measuredangle$s in the same segemnt)

Hence, ΔCDX and ΔCBA are similar.

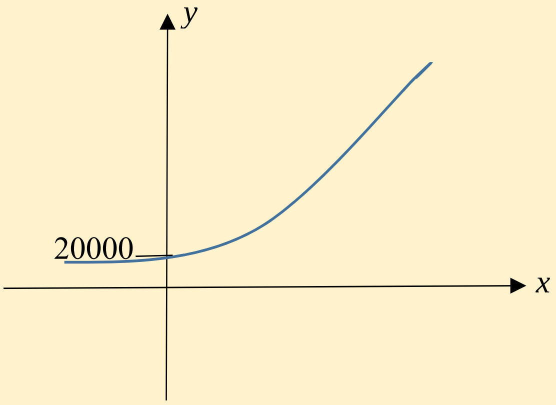
8i

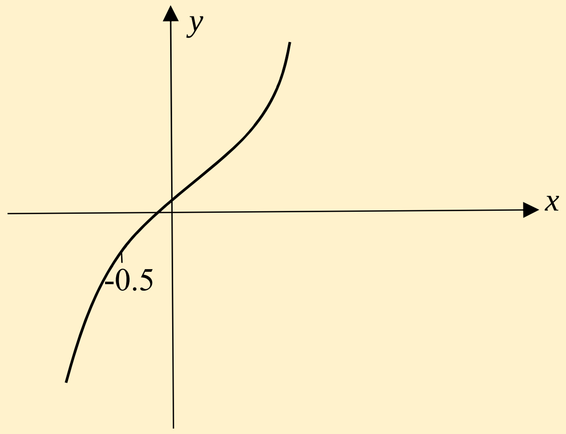
$\frac{y}{x} = bx + a$

x	1	2	3	4	5	6
y/x	5.1	8.75	12.5	15.13	19.6	22.83



ii	$a = \frac{y}{x} - \text{intercept}$ $= 1.5$ $b = gradient$ $= \frac{13.5}{3.8}$ $= 3.55$
iii	$(b+5)x = 38 - a$ $bx + 5x = 38 - a$ $bx + a = 38 - 5x$ Draw $\frac{y}{x} = 38 - 5x$, at point of intersection, x = 4.25
9i	$OE = 5 + 9\cos\theta$ $OD = 4 + 9\sin\theta$ $DE^2 = OE^2 + OD^2$ $= (5 + 9\cos\theta)^2 + (4 + 9\sin\theta)^2$ $= 25 + 90\cos\theta + 81\cos^2\theta$ $+16 + 72\sin\theta + 81\sin^2\theta$ $= 41 + 81 + 90\cos\theta + 72\sin\theta$ $= 122 + 90\cos\theta + 72\sin\theta$
ii	Let $90\cos\theta + 72\sin\theta = R\cos(\theta - \alpha)$. $R = \sqrt{90^2 + 72^2}$ $= \sqrt{13284}$ $= 115$ (3 s.f.) $\theta = \tan^{-1}\frac{72}{90}$ $= 38.65°$ $DE^2 = 122 + 115\cos(\theta - 38.7°)$ OR $122 + \sqrt{13284}\cos(\theta - 38.7°)$

iii	DE is greatest when $\cos(\theta - 38.7°) = 1$ $DE = \sqrt{122 + 115}$ $= 15.4$ units (3 s.f.) Corresponding θ is $38.7°$.
10i	A = 20000, k = 11
ii	
iii	When y = 9600, $9600 = 20000(1.11)^x$ $x = \lg \frac{9600}{20000} \div \lg 1.11$ $= -7.03$ (3 s.f.) The population of the town was 9600 approximately 7 years ago.
iv	When x = 10, $y = 20000(1.11)^{10}$ $= 56788$ The population of the town would be 56788 (or 56800) at the end of 2027.
11i	$y = 2x^3 + 3x^2 + 11x + 5$ $\frac{dy}{dx} = 6x^2 + 6x + 11$ $= 6\left(x + \frac{1}{2}\right)^2 + \frac{19}{2}$ $\frac{dy}{dx} > 0$ as $\left(x + \frac{1}{2}\right)^2 \geq 0$ for all values of x, hence y is an increasing function for all values of x.

ii	Using long division, $y=\left(2x+1\right)\left(x^2+x+5\right)$ But for x^2+x+5 , discriminant = -19 < 0, hence x^2+x+5 has no real roots. Therefore, *y* has only one real root at $x=-\frac{1}{2}$.
iii	
iv	Area required $=\int_{-1}^{1} y \ \mathrm{dx}$ $=\left\lvert\int_{-1}^{-0.5} 2x^3+3x^2+11x+5 \ \ \mathrm{dx}\right\rvert$ $+\int_{-0.5}^{1} 2x^3+3x^2+11x+5 \ \ \mathrm{dx}$ $=\left\lvert\left[\frac{x^4}{2}+x^3+\frac{11}{2}+5x\right]_{-1}^{-0.5}\right\rvert+\left[\frac{x^4}{2}+x^3+\frac{11}{2}+5x\right]_{-0.5}^{1}$ $=\left\lvert-\frac{39}{32}\right\rvert+\left[12-\left(-\frac{39}{32}\right)\right]$ $=14\frac{7}{16}$ *or* $=14.4$ sq. units (3 s.f.)

COMMONWEALTH SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2018

ADDITIONAL MATHEMATICS
PAPER 1

Name: ____________________________________ (   )   Class: _______________

SECONDARY FOUR EXPRESS **Wednesday 12 September 2018**
SECONDARY FIVE NORMAL ACADEMIC **11 00 – 13 00**
4047/1 **2 h**

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate writing paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

Name of setter: Mrs Margaret Loh

This paper consists of **7** printed pages including the cover page.

[Turn over

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots\ldots + \binom{n}{r}a^{n-r}b^r + \ldots\ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2}bc \sin A$$

1. The slope at any point (x, y) of a curve is given by $\frac{dy}{dx} = \frac{k}{(2x+3)^2} - 1$ where k is a constant. If the tangent to the curve at $(-1, 0)$ is perpendicular to the line $3y = x + 1$, find

(i) the value of k, [3]

(ii) the equation of the curve. [3]

2. (i) On the same axes, sketch the curves $y = -8x^{-\frac{1}{2}}$ and $y^2 = \frac{1}{4}x$. [2]

(ii) Find the equation of the line passing through the origin and the point of intersection of the two curves. [3]

3. The equation $y = \frac{x+c}{x+d}$, where c and d are constants, can be represented by a straight line when $xy - x$ is plotted against y. The line passes through the points $(0, 4)$ and $(0.2, 0)$.

(i) Find the value of c and of d, [4]

(ii) If $(2.5, a)$ is a point on the straight line, find the value of a. [1]

4. The roots of a quadratic equation are α and β, where $\alpha^3 + \beta^3 = 0$, $\alpha\beta = \frac{27}{64}$, $\alpha + \beta > 0$.

(i) Find this quadratic equation with integral coefficient. [4]

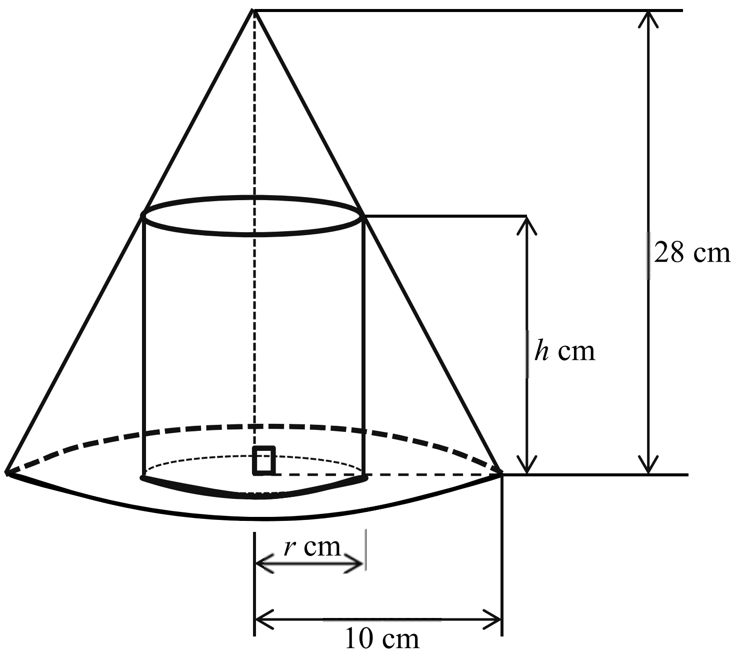
The roots of another quadratic equation $x^2 + px + q = 0$ are $\alpha - \beta$ and $\beta - \alpha$.

(ii) Find the value of p and of q. [3]

5. (i) Prove the identity $\sin^2 2x(\cot^2 x - \tan^2 x) = 4\cos 2x$. [4]

(ii) Hence find, for $0 \le x \le 2\pi$, the values of x for which $\sin^2 2x = \frac{e}{\cot^2 x - \tan^2 x}$. [3]

6.



(a) The diagram shows a cylinder of height h cm and base radius r cm inscribed in a cone of height 28 cm and base radius 10 cm. Show that

(i) the height, h cm, of the cylinder is given by

$$h = 28 - \frac{14}{5}r\,.$$ [1]

(ii) the volume, V cm^3, of the cylinder is given by

$$V = 14\pi r^2 (2 - \frac{r}{5}).$$ [1]

(b) (i) Given that r can vary, find the maximum volume of the cylinder. [5]

(ii) Show that, in this case, the cylinder occupies $\frac{4}{9}$ of the volume of the cone. [2]

7. (a) A circle with centre P lies in the first quadrant of the Cartesian plane. It is tangential to the x-axis and the y-axis, and passes through the points $A(4, 18)$ and $B(18, 16)$.

Find

(i) the equation of the perpendicular bisector of the line segment AB, [3]

(ii) the coordinates of the centre P, [2]

(iii) the equation of the circle, [1]

The tangent at A touches the x-axis at R. The line joining A and P is produced to touch the x-axis at S.

(b) Find the area of triangle ARS. [4]

8. Use the result $(\sqrt{x}+\sqrt{y})^2 \equiv x+y+2\sqrt{(xy)}$, or otherwise, find the square root of $12+\sqrt{140}$ in the form $\sqrt{a}+\sqrt{b}$, where a and b are constants to be determined. [5]

9. Given that $P(x)=2x^4-5x^3+5x^2-x-10$,

(i) find the quotient when $P(x)$ is divided by $(2x-1)(x^2+3)$, [2]

(ii) hence express $\dfrac{P(x)}{(2x-1)(x^2+3)}$ in partial fractions. [5]

10. The velocity, v ms^{-1}, of a particle travelling in a straight line at time t seconds after leaving a fixed point O, is given by

$$v=2t^2+(1-3k)t+8k-1,$$

where k is a constant. The velocity is a minimum at $t = 5$.

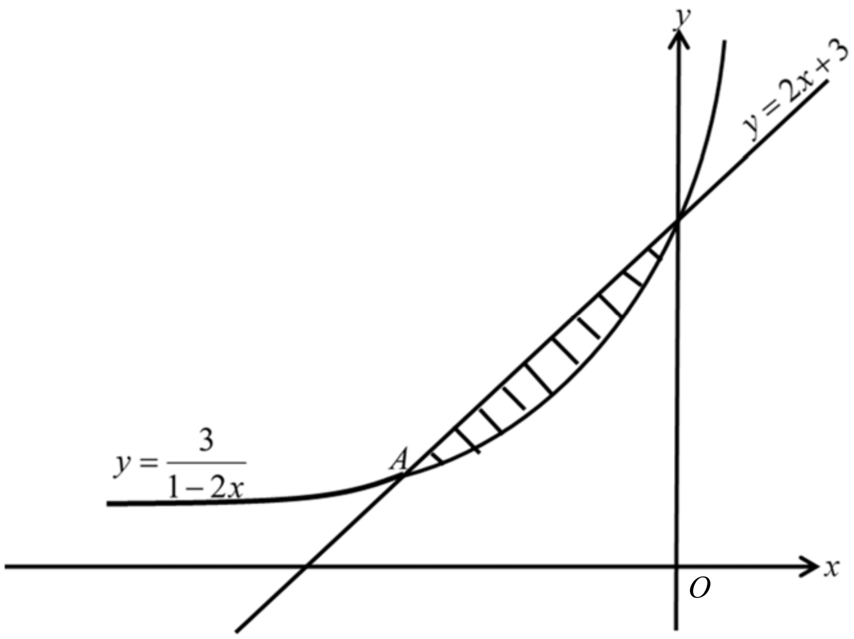
(i) Show that $k=7$. [2]

(ii) Show that the particle will never return to O with time. [2]

(iii) Find the duration when its velocity is less than 13 ms^{-1}. [2]

(iv) Find the distance travelled by the particle during the third second. [2]

11.

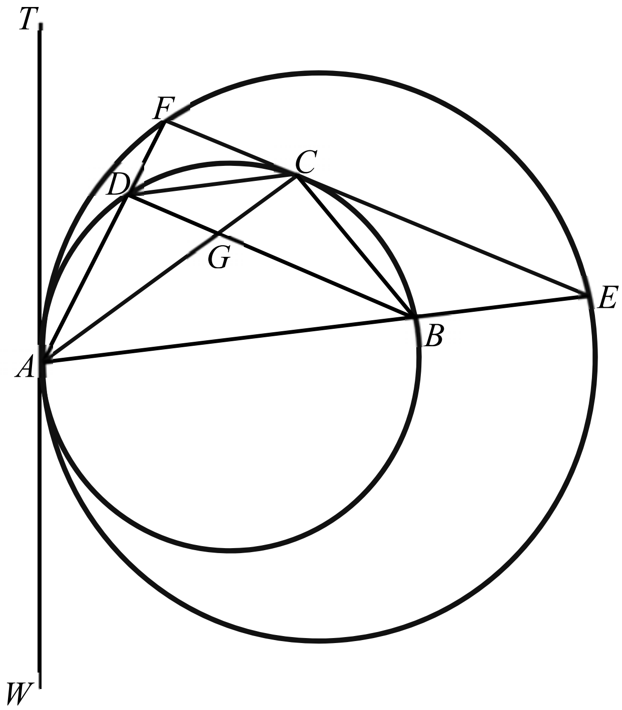


The diagram shows part of curve $y=\frac{3}{1-2x}$ intersecting with a straight line $y=2x+3$ at the point A. Find

(i) the coordinates of A. [2]

(ii) the area of the shaded region bounded by the line and the curve. [4]

12.



In the diagram, two circles touch each other at A. TA is tangent to both circles at A and FE is a tangent to the smaller circle at C. Chords AE and AF intersect the smaller circle at B and D respectively. Prove that

(i) line BD is parallel to line FE, [2]

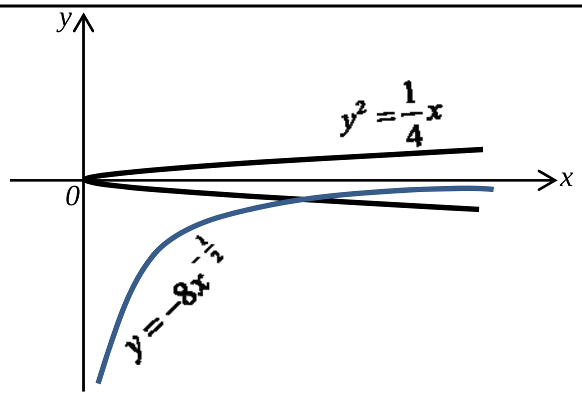
(ii) $\angle FAC = \angle CAE$, [3]

END OF PAPER

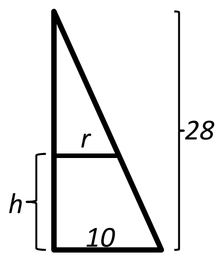
2018 CWSS Prelim AM P1 Answer Key

1.	(i) -2 (ii) $y=\frac{1}{(2x+3)}-x-2$	10.	(iii) 4s (iv) $17\frac{2}{3}$ m or 17.7 m
2.	(ii) $y=-\frac{1}{8}x$	11.	(i) $A=(-1,1)$ (ii) 0.352 units^2
3.	(i) $c=4$; $d=20$ (ii) -46	12.	(i)<u>To prove</u>: *BD* // *FE*
			<u>Proof</u>: Let $\angle TAF$ be θ.
4.	(i) $64x^2-72x+27=0$		$\angle ABD=\angle TAF=\theta$ (alt seg thm)
	(ii) $p=0$; $q=\frac{27}{64}$		$\angle AEF=\angle TAF=\theta$ (alt seg thm)
			$\therefore\ \angle ABD=\angle AEF=\theta$
5.	(ii) 0.412, 2.73, 3.55, 5.87		Using property of corresponding angles, *BD* // *EF* (shown)
6.	b(i) $\frac{11200\pi}{27}cm^3$ or $1300\,cm^3$		(ii)<u>To prove</u>: $\angle FAC=\angle CAE$
			<u>Proof</u>: Let $\angle BCE=\alpha$
7	a(i) $y=7x-60$ (ii) (10, 10)		$\angle CBD=\angle BCE=\alpha$ (alt $\angle$s, $BD//EF$)
	(iii) $(x-10)^2+(y-10)^2=100$		$\angle FAC=\angle CBD=\alpha$ ($\angle$s in same segment)
	b. 337.5 units^2		Also, $\angle CAE=\angle BCE=\alpha$ (alt seg thm)
			$\therefore\ \angle FAC=\angle CAE=\alpha$ (shown)
8.	$\sqrt{7}+\sqrt{5}$		
9.	(i) $x-2$ (ii) $x-2-\frac{3}{(2x-1)}+\frac{7}{(x^2+3)}$		

END

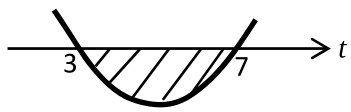
Qn No	Solutions	Marks
1	$3y = x + 1$	
	$y = \frac{1}{3}x + \frac{1}{3}$	
	$\therefore$ grad of tangent $= -3$	M1
(i)	$-3 = \frac{k}{(2x+3)^2} - 1$	M1
	$k = -2$	A1
(ii)	$\frac{dy}{dx} = \frac{-2}{(2x+1)^2} - 1$	
	$y = \int [-2(2x+1)^{-2} - 1]dx$	M1
	$= \frac{-2(2x+1)^{-1}}{(-1)(2)} - x + c$	
	$= \frac{1}{(2x+3)} - x + c$	M1
	When $y = 0,\ x = -1$ $\quad 0 = \frac{1}{-2+3} + 1 + c$	
	$c = -2$	
	$\therefore\ y = \frac{1}{(2x+3)} - x - 2$	A1
2(i)		Graphs are [B1] & [B1]
(ii)	$(-8x^{-\frac{1}{2}})^2 = \frac{1}{4}x$	M1
	$64x^{-1} = \frac{1}{4}x$	
	$256 = x^2$	
	$x = 16$ or -16 (NA)	M1
	When $x = 16,\ y = \frac{-8}{\sqrt{16}} = -2$	

	Grad of line $= \frac{-2}{16} \quad = -\frac{1}{8}$	
	$\therefore$ Eqn of line is $y = -\frac{1}{8}x$	A1
3(i)	$y(x+d) = x+c$	
	$xy - x = -yd + c$	M1
	$\therefore \quad c = 4$	B1
	Grad $= -\frac{4}{0.2} \quad = -20$	M1
	$\therefore \quad -d = -20$	
	$d = 20$	A1
(ii)	$\therefore \quad xy - x = -20y + 4$	
	$a = -20(2.5) + 4 \quad = -46$	B1
4	$\alpha^3 + \beta^3 = 0$	
	$(\alpha+\beta)[(\alpha+\beta)^2 - 3\alpha\beta] = 0$	
	$(\alpha+\beta)[(\alpha+\beta)^2 - 3\left(\frac{27}{64}\right)] = 0$	M1
	Since $\alpha \neq -\beta, \quad (\alpha+\beta)^2 = \frac{81}{64}$	
	$\alpha + \beta = \frac{9}{8}$ or $-\frac{9}{8}$ (NA)	A1
(i)	Quad eqn is $x^2 - \frac{9}{8}x + \frac{27}{64} = 0$	M1
	$64x^2 - 72x + 27 = 0$	B1
(ii)	Sum of roots $= \alpha - \beta + \beta - \alpha \quad = 0$	
	Prod of roots $= (\alpha - \beta)(\beta - \alpha)$	
	$= \alpha\beta - \alpha^2 - \beta^2 + \alpha\beta$	
	$= 2\alpha\beta - (\alpha^2 + \beta^2)$	
	$= 2\alpha\beta - [(\alpha+\beta)^2 - 2\alpha\beta]$	M1
	$= 4\alpha\beta - (\alpha+\beta)^2$	
	$= 4\left(\frac{27}{64}\right) - \left(\frac{9}{8}\right)^2$	
	$= \frac{108}{64} - \frac{81}{64} \quad = \frac{27}{64}$	
	$\therefore \quad p = 0 \quad \& \quad q = \frac{27}{64}$	B1, B1

5(i)	To prove: $\sin^2 2x(\cot^2 x - \tan^2 x) = 4\cos 2x$	
	Proof: LHS = $\sin^2 2x(\cot^2 x - \tan^2 x)$	
	$= \sin^2 2x\left(\frac{\cos^2 x}{\sin^2 x} - \frac{\sin^2 x}{\cos^2 x}\right)$	M1
	$= \sin^2 2x\left(\frac{\cos^4 x - \sin^4 x}{\sin^2 x\cos^2 x}\right)$	M1
	$= 4\sin^2 x\cos^2 x\left(\frac{(\cos^2 x - \sin^2 x)(\cos^2 x + \sin^2 x)}{\sin^2 x\cos^2 x}\right)$	M1
	$= 4(\cos^2 x - \sin^2 x)$	M1
	$= 4\cos 2x$	
	= RHS (proved)	
(ii)	$\sin^2 2x(\cot^2 x - \tan^2 x) = e$	
	$4\cos 2x = e$	M1
	$\cos 2x = \frac{e}{4}$	
	$2x \approx 0.8236,\ 5.4596,\ 7.1068,\ 11.743$	
	$x \approx 0.412,\ 2.73,\ 3.55,\ 5.87$	A1, A1
6a(i)	 Using Similar triangles, $\frac{28-h}{28} = \frac{r}{10}$ $28 - h = \frac{28}{10}r$ $h = 28 - \frac{14}{5}r$ (shown)	M1
(ii)	Vol of cylinder = $\pi r^2 h$	
	$V = \pi r^2\left(28 - \frac{14}{5}r\right)$	M1
	$V = 14\pi r^2\left(2 - \frac{1}{5}r\right)$ (shown)	
b(i)	$\frac{dV}{dr} = 56\pi r - \frac{14}{5}\pi(3r^2)$	
	$= 14\pi r(4 - \frac{3}{5}r)$	M1
	At stat pt, $\frac{dV}{dr} = 0$	
	$14\pi r(4 - \frac{3}{5}r) = 0$	M1

	$r=0$ (NA), $4-\frac{3}{5}r=0 \quad \Rightarrow \quad r=6\frac{2}{3}$	A1
	$\frac{d^2V}{dr^2}=56\pi-\frac{84}{5}\pi r$	
	$=56\pi-\frac{84}{5}\pi\left(6\frac{2}{3}\right)$	
	$=-175.93(2dp) \quad <0$	M1
	Since $\frac{d^2V}{dr^2}<0$, $\therefore$ r = $6\frac{2}{3}$ will make V a maximum.	
	Max volume = $14\pi\left(\frac{20}{3}\right)\left(\frac{20}{3}\right)\left(2-\frac{1}{5}\left[\frac{20}{3}\right]\right)$	
	$=\frac{11200}{27}\pi$ cm^3 or 1300 cm^3 $(3sf)$	B1
(ii)	<u>To show</u>: Vol of cylinder = $\frac{4}{9}$ (Vol of cone)	
	<u>Proof</u>: Vol of cone = $\frac{1}{3}\pi(10)^2(28) = \frac{2800}{3}\pi$ cm^3	M1
	$\frac{\text{Vol of cylinder}}{\text{Vol of cone}}=\frac{11200\pi}{27}\times\frac{3}{2800\pi} = \frac{4}{9}$	M1
	$\therefore$ Vol of cylinder = $\frac{4}{9}$ (Vol of cone) (shown)	
7a(i)	Mid-pt of $AB=\left(\frac{4+18}{2}, \frac{18+16}{2}\right) = (11, 17)$	M1
	Grad of $AB=\frac{18-16}{4-18} = -\frac{1}{7}$	
	Grad of perpendicular bisector = 7	M1
	Eqn of perpendicular bisector is $y-17=7(x-11)$	
	$y=7x-60$	A1
(ii)	Let the centre P be (m,m).	
	$m=7m-60$	M1
	$m=10$	
	$\therefore \quad P=(10,10)$	A1
(iii)	Eqn of circle is $(x-10)^2+(y-10)^2=100$	B1
	Or $x^2+y^2-20x-20y+100=0$	
(b)	Grad of $AP=\frac{18-10}{4-10}$	
	$=-\frac{4}{3}$	

	$\therefore$ Grad of tangent at $A=\frac{3}{4}$	
	Eqn of tangent at A is $y-18=\frac{3}{4}(x-4)$	
	$y=\frac{3}{4}x+15$	
	$\therefore R=(-20,0)$	B1
	Eqn of AP is $y-10=-\frac{4}{3}(x-10)$	
	$y=-\frac{4}{3}x+23\frac{1}{3}$	
	$\therefore S=\left(17\frac{1}{2},0\right)$	B1
	$\therefore$ Area of $\Delta ARS=\frac{1}{2}\left(20+17\frac{1}{2}\right)(18)$	M1
	$=337.5$ units^2	A1
8	$x+y=12$ ------------(1)	B1
	$4xy=140$ ------------(2)	B1
	From eqn (1): $y=12-x$ substi into eqn (2)	
	$4x(12-x)=140$	M1
	$x^2-12x+35=0$	
	$(x-7)(x-5)=0$	
	$\therefore$ $x=7$ or $x=5$	
	When $x=7$, $y=5$	A1
	When $x=5$, $y=7$	
	$\therefore$ $\sqrt{12+\sqrt{140}}=\left(\sqrt{7}+\sqrt{5}\right)$	A1
9(i)	$(2x-1)(x^2+3)=2x^3-x^2+6x-3$	
	$$\begin{array}{r} x-2 \\ 2x^3-x^2+6x-3\overline{)2x^4-5x^3+5x^2-x-10} \\ -(2x^4-x^3+6x^2-3x) \\ \hline -4x^3-x^2+2x-10 \\ -(-4x^3+2x^2-12x+6) \\ \hline -3x^2+14x-16 \end{array}$$ $\therefore$ Quotient $=x-2$	M1 A1
(ii)	$\frac{P(x)}{(2x-1)(x^2+3)}=x-2+\frac{(-3x^2+14x-16)}{(2x-1)(x^2+3)}$	

	$\dfrac{(-3x^2+14x-16)}{(2x-1)(x^2+3)} = \dfrac{A}{(2x-1)} + \dfrac{(Bx+C)}{(x^2+3)}$ where *A*, *B* and *C* are constants	
	$-3x^2+14x-16 = A(x^2+3)+(Bx+C)(2x-1)$	M1
	When $x=\dfrac{1}{2}$, $-3\left(\dfrac{1}{4}\right)+14\left(\dfrac{1}{2}\right)-16 = A\left(3\dfrac{1}{4}\right)$	
	$A=-3$	B1
	When $x=0$, $-16=3A-C$	
	$-16=-9-C$	
	$C=7$	B1
	Comparing coeff of x^2 : $-3 = A+2B$	
	$-3=-3+2B$	
	$B=0$	B1
	$\therefore \quad \dfrac{P(x)}{(2x-1)(x^2+3)} = x-2-\dfrac{3}{(2x-1)}+\dfrac{7}{(x^2+3)}$	A1
10(i)	$\dfrac{dv}{dt} = 4t+(1-3k)$	
	When vel is a minimum, $\dfrac{dv}{dt}=0$	
	$4(5)+(1-3k)=0$	M1
	$3k=21$	
	$k=7$ (shown)	A1
(ii)	When $k=7$, $v=2t^2-20t+55$	
	Discriminant = $(-20)^2-4(2)(55)$	
	$=400-440$	
	$=-40$	
	<0	M1
	$\Rightarrow$ there is no real values of t such that vel = 0, also vel > 0 hence particle will never return to *O* with time.	A1
(iii)	$2t^2-20t+55<13$	M1
	$2t^2-20t+42<0$	
	$t^2-10t+21<0$	
	$(t-7)(t-3)<0$	
	 $\therefore \quad 3<t<7$ Duration = $7-3 = 4$ s	A1

(iv)	$s=\int_2^3(2t^2-20t+55)dt$	M1
	$=\left[\frac{2t^3}{3}-10t^2+55t\right]_2^3$	
	$=[18-90+165]-\left[\frac{16}{3}-40+110\right]$	
	$=17\frac{2}{3}$ m or 17.7 m(3sf)	A1
11(i)	$\frac{3}{1-2x}=2x+3$	M1
	$3=(2x+3)(1-2x)$	
	$3=2x-4x^2+3-6x$	
	$4x^2+4x=0$	
	$4x(x+1)=0$	
	$x=0$ or $x=-1$	
	For pt A: When $x=-1$, $y=-2+3=1$	
	$\therefore\ A=(-1,1)$	A1
(ii)	Area of shaded region = $\frac{1}{2}(1+3)-\int_{-1}^{0}\frac{3}{1-2x}dx$	M1, M1
	$=2-\left[\frac{3\ln(1-2x)}{-2}\right]_{-1}^{0}$	M1
	$=2-\left[0+\frac{3}{2}\ln 3\right]$	
	$=2-1.6479\ \approx\ 0.352$ units^2	A1
12(i)	To prove: BD // FE	
	Proof: Let $\angle TAF$ be θ.	
	$\angle ABD=\angle TAF=\theta$ (alt seg thm)	M1
	$\angle AEF=\angle TAF=\theta$ (alt seg thm)	
	$\therefore\ \angle ABD=\angle AEF=\theta$	
	Using property of corresponding angles, BD // EF (shown)	A1
(ii)	To prove: $\angle FAC=\angle CAE$	
	Proof: Let $\angle BCE=\alpha$	
	$\angle CBD=\angle BCE=\alpha$ (alt $\angle$s, $BD//EF$)	B1
	$\angle FAC=\angle CBD=\alpha$ ($\angle$s in same segment)	B1
	Also, $\angle CAE=\angle BCE=\alpha$ (alt seg thm)	B1
	$\therefore\ \angle FAC=\angle CAE=\alpha$ (shown)	
	END	

1 **(i)** A particle moves along the curve $y = \ln\left(x^2 + 1\right)$ in such a way that the y-coordinate of the particle is decreasing at a constant rate of 0.2 units per second. Find the rate at which the x-coordinate of the particle is changing at the instant when $x = -0.5$. [3]

(ii) Find the x-coordinates of the point on the curve where the gradient is **stationary**. [3]

2 **(i)** Solve the equation $\log_3(2x+1) - \log_3(2x-3) = 1 + \log_3 \frac{2}{5}$. [4]

(ii) Solve the equation $\ln y + 1 = 2\log_y \mathrm{e}$, giving your answer(s) in terms of e. [5]

3 Given that $y = \mathrm{e}^x \sin x$,

(i) show that $2\frac{\mathrm{d}y}{\mathrm{d}x} - \frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 2y$. [4]

(ii) Hence, or otherwise, find the value of $\int_0^{\frac{\pi}{3}} \mathrm{e}^x \sin x \, \mathrm{d}x$. [4]

4 Given that the first three terms, in ascending powers of y, of the expansion of $(a+y)^n$, where a and n are positive real constants, are $64 + 192y + 240y^2$.

(i) By considering the ratio of the coefficients of the first two terms, show that $a = \frac{1}{3}n$. [3]

(ii) Find the value of a and of n. [4]

5 **(a)** Using the substitution $u = 2^x$, solve the equation $4^{x+1} = 2^x + 3$. [4]

(b) The quantity, N, of a radioactive substance, at time t years, is given by $N = N_0\mathrm{e}^{-kt}$, where N_0 and k are positive constants.

(i) Sketch the graph of N against t, labelling any axes intercepts. [2]

(ii) State the significance of N_0. [1]

(iii) The quantity halves every 5 years. Calculate the value of k. [3]

6 **Solutions to this question by accurate drawing will not be accepted.**

The coordinates of the points P and Q are $(-5,2)$ and $(7,6)$ respectively. Find

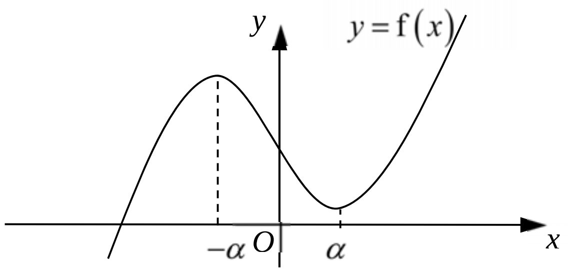
(i) the equation of the line parallel to PQ and passing through the point $(-2,3)$, [3]

(ii) the equation of the perpendicular bisector of PQ. [3]

A point R is such that the shortest distance of R from the line passing through P and Q is $\sqrt{10}$ units.

(iii) Find the area of triangle PQR. [3]

7 The diagram shows a sketch of the curve $y=\mathrm{f}(x)$. The x-coordinates of the maximum and minimum points are $-\alpha$ and α, where $k>0$.



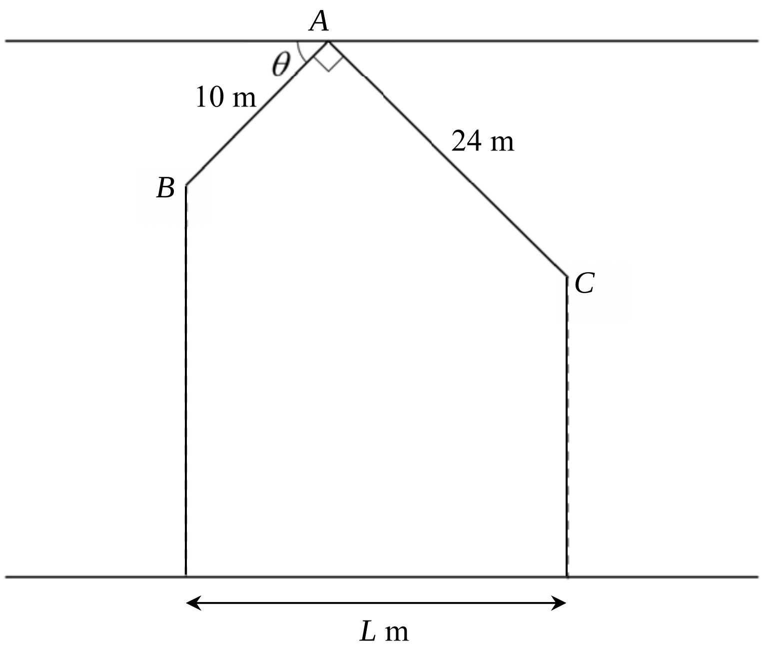
It is given that $\mathrm{f}'(x)=ax^2+bx+c$, where a, b and c are real constants. For each of the following, state, with reasons, whether they are positive, negative or zero.

(i) b^2-4ac, [2]

(ii) $\dfrac{b}{a}$, [2]

(iii) $\dfrac{c}{a}$. [2]

8 The diagram shows the cross-section of a house with a rooftop *BAC*. The length of *AB* and *AC* are 10 m and 24 m respectively. The angle between *AB* and the horizontal through *A* is θ degrees and $\angle BAC = 90°$.



The base of the house is of length *L* m.

(i) Show that $L = 10\cos\theta + 24\sin\theta$. [2]

(ii) Express *L* in the form $R\sin(\theta+\alpha)$, where $R > 0$ and α is an acute angle. [4]

(iii) Find the longest possible base of the house and the corresponding value of θ. [3]

9 **(a)** The equation of a curve is $y = \dfrac{2x}{1+x}$.

(i) Find the equation of the tangent to the curve at point $P(1,1)$. [4]

(ii) The tangent cuts the axes at *Q* and *R* respectively. Find the area of triangle *OPQ*. [2]

(b) A curve has equation $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = \dfrac{1}{3}x^3 - 2x^2 + 13x + 5$.

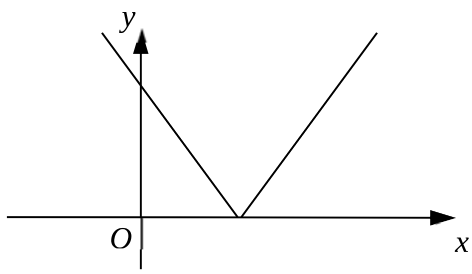
Determine, with explanation, whether f is an increasing or decreasing function. [4]

10 **(a)** **(i)** Solve the equation $\left|x^2-3x+2\right|+x=1$. [3]

(ii) What can be deduced about the number of points of intersections of the graphs of $y=\left|x^2-3x+2\right|$ and $y=-x+1$? [1]

(iii) Hence, on a single diagram, sketch the graphs of $y=\left|x^2-3x+2\right|$ and $y=-x+1$, indicating the coordinates of any axial intercepts and turning point. [4]

(b) The diagram shows part of the graph of $y=|k-x|$, where k is a constant.



A line $y=mx+c$ is drawn to determine the number of solutions to the equation $|k-x|=mx+c$.

(i) If $m=1$, state the range of values of c, in terms of k, such that the equation has one solution. [1]

(ii) If $c=0$, state the range of values of m such that the equation has no solutions. [2]

11 **(a)** State the principal range of $\sin^{-1}x$, leaving your answers in terms of π. [1]

(b) **(i)** Prove that $\dfrac{1+\tan x}{1-\tan x}=\sec 2x+\tan 2x$. [5]

(ii) Hence find the reflex angle x such that $3\sec 2x+3\tan 2x=1$. [3]

(c) A buoy floats and its height above the seabed, h m, is given by $h=a\cos bt+c$, where t is time measured in hours from 0000 hours and a, b and c are constants. The least height of the buoy above seabed is 180 metres and is recorded at 0000 hours. The greatest height of the buoy above seabed is 196 metres and is first recorded at 0600 hours.

(i) Find the values of a, b and c. [3]

(ii) Using values found in **(i)**, sketch the graph of $h=a\cos bt+c$ for $0\le t\le 24$. [2]

(ii) The buoy floats above the top of a huge rock first at 0500 hours. State the number of hours in each day that the buoy is above the rock. [1]

END OF PAPER

Question 1

(i)	0.25 units/s
(ii)	$x = \pm 1$

Question 2

(i)	$x = \frac{23}{2}$
(ii)	$y = e^{-2}$ or $y = e$

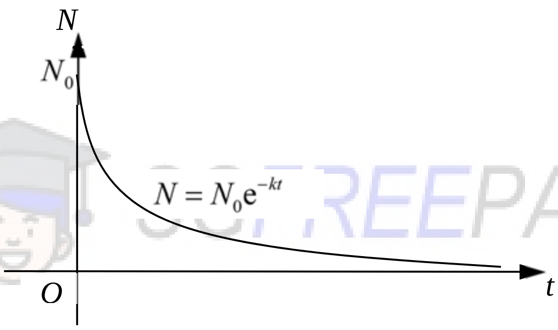
Question 3

(ii)	$1.02 (3\text{s.f.})$

Question 4

(ii)	$n = 6, a = 2$

Question 5

(a)	$x = 0$
(a)(i)	
(ii)	It represents the initial amount of radioactive substance.
(iii)	0.139

Question 6

(i)	$y = \frac{1}{3}x + 3\frac{2}{3}$
(ii)	$y = -3x + 7$
(iii)	20 units^2

Question 7

(i)	$b^2 - 4ac > 0$.
(ii)	$\frac{b}{a} > 0$
(iii)	$\frac{c}{a} < 0$

Question 8

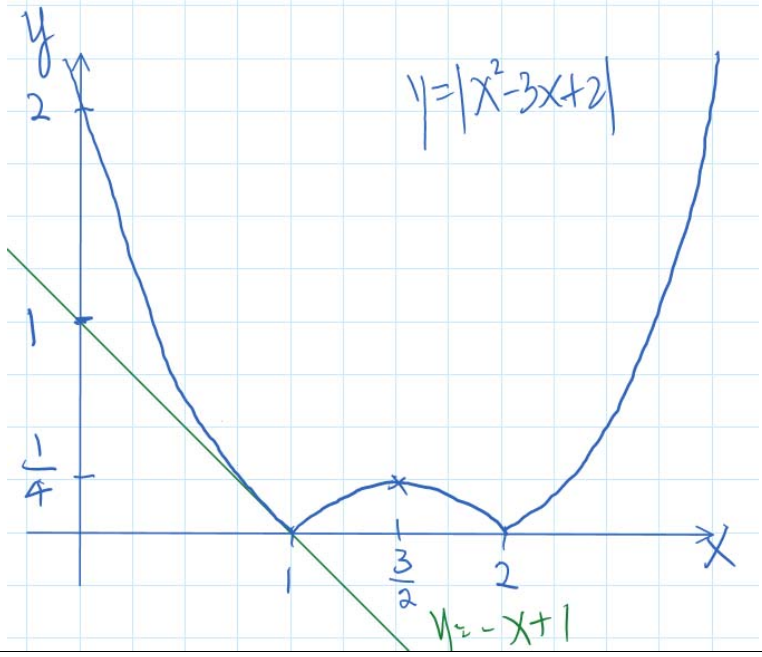
(ii)	$L = 26\sin(\theta + 22.6°)$
(iii)	Longest possible base is 26 m.

	$\theta = 67.4°(1 \text{ d.p.})$

Question 9

(a)(i)	$y = \frac{1}{2}x + \frac{1}{2}$
(ii)	$\frac{1}{4}$ units^2

Question 10

(a)(i)	$x = 1$
(ii)	The line $y = -x + 1$ is tangential to $y = \lvert x^2 - 3x + 2 \rvert$.
(ii)	
(b)(i)	$c > -k$
(ii)	$-1 \le m < 0$

Question 11

(a)	$-\frac{\pi}{2} \le \sin^{-1} x \le \frac{\pi}{2}$
(b)(ii)	$x = 333.3°(1 \text{ d.p.})$
(c)(i)	$a = -8, b = \frac{\pi}{6}, c = 188$
(iii)	4 hours

1 **(i)** A particle moves along the curve $y = \ln\left(x^2+1\right)$ in such a way that the y-coordinate of the particle is decreasing at a constant rate of 0.2 units per second. Find the rate at which the x-coordinate of the particle is changing at the instant when $x = -0.5$. [3]

(ii) Find the x-coordinates of the point on the curve where the gradient is **stationary**. [3]

(i)	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2x}{x^2+1}$ $\frac{\mathrm{d}y}{\mathrm{d}t} = \frac{\mathrm{d}y}{\mathrm{d}x} \times \frac{\mathrm{d}x}{\mathrm{d}t}$ $-0.2 = \frac{2(-0.5)}{(-0.5)^2+1} \times \frac{\mathrm{d}x}{\mathrm{d}t}$ $\Rightarrow \frac{\mathrm{d}x}{\mathrm{d}t} = 0.25$ units/s	**B1** **M1** **A1**
(ii)	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \frac{\left(x^2+1\right)(2) - 2x(2x)}{\left(x^2+1\right)^2}$ $= \frac{2-2x^2}{\left(x^2+1\right)^2}$ $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 0$ $2-2x^2 = 0$ $x = \pm 1$	**√M1** **M1** **A1**

2 **(i)** Solve the equation $\log_3(2x+1)-\log_3(2x-3)=1+\log_3\frac{2}{5}$. [4]

(ii) Solve the equation $\ln y+1=2\log_y \mathrm{e}$, giving your answer(s) in terms of e. [5]

(i)	$\log_3(2x+1)-\log_3(2x-3)=1+\log_3\frac{2}{5}$	
	$\log_3\frac{2x+1}{2x-3}=\log_3\left(3\times\frac{2}{5}\right)$	**B1**, **B1**
	$\frac{2x+1}{2x-3}=\frac{6}{5}$	**M1** – remove log
	$10x+5=12x-18$	
	$2x=23$	
	$x=\frac{23}{2}$	**A1**
(ii)	$\ln y+1=2\log_y \mathrm{e}$	
	$\ln y+1=\frac{2}{\ln y}$	**B1** – change base
	$(\ln y)^2+\ln y-2=0$	**B1**
	$(\ln y+2)(\ln y-1)=0$	**M1** – attempt to solve
	$\ln y=-2$ or 1	
	$y=\mathrm{e}^{-2}$ or $y=\mathrm{e}$	**A2**

3 Given that $y = e^x \sin x$,

(i) show that $2\frac{dy}{dx} - \frac{d^2y}{dx^2} = 2y$. [4]

(ii) Hence, or otherwise, find the value of $\int_0^{\frac{\pi}{3}} e^x \sin x \, dx$. [4]

(i)	$y = e^x \sin x$ $\frac{dy}{dx} = e^x \sin x + e^x \cos x$ $\frac{d^2y}{dx^2} = e^x \sin x + e^x \cos x - e^x \sin x + e^x \cos x$ $= 2e^x \cos x$ $-\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = -2e^x \cos x + 2\left(e^x \sin x + e^x \cos x\right)$ $= 2e^x \sin x$ $= 2y$ $2\frac{dy}{dx} - \frac{d^2y}{dx^2} = 2y$	**M1** – product rule **B1** **M1** – product rule **M1** **a.g.**
(ii)	$-\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = 2y$ $\therefore -\frac{dy}{dx} + 2y = 2\int e^x \sin x \, dx$ $\Rightarrow -e^x \sin x - e^x \cos x + 2e^x \sin x = 2\int e^x \sin x \, dx$ $\therefore \int e^x \sin x \, dx = \frac{1}{2}\left(e^x \sin x - e^x \cos x\right) + c$ $\int_0^{\frac{\pi}{3}} e^x \sin x \, dx = \left[\frac{1}{2}\left(e^x \sin x - e^x \cos x\right)\right]_0^{\frac{\pi}{3}}$ $= 1.02\,(3\text{s.f.})$	**M1** – integration **B1** – making integral the subject **M1** – substitution of limits **A1**

4 Given that the first three terms, in ascending powers of *y*, of the expansion of $(a+y)^n$, where *a* and *n* are positive real constants, are $64+192y+240y^2$.

(i) By considering the ratio of the coefficients of the first two terms, show that $a=\frac{1}{3}n$. [3]

(ii) Find the value of *a* and of *n*. [4]

(i)	$(a+y)^n = a^n + na^{n-1}y + \frac{n(n-1)}{2}a^{n-2}y^2 + \ldots$ By comparing coefficents, $a^n = 64 \quad ---(1)$ $na^{n-1} = 192 \quad ---(2)$ $\frac{n(n-1)}{2}a^{n-2} = 240 \quad ---(3)$ $\frac{(1)}{(2)}: \frac{a}{n} = \frac{64}{192} = \frac{1}{3} \Rightarrow a = \frac{1}{3}n ---(4)$	**B1** – award for first two terms **M1, A1**
(ii)	$\frac{(2)}{(3)}: \frac{2a}{n-1} = \frac{192}{240} = \frac{4}{5} \Rightarrow a = \frac{2}{5}(n-1) ---(5)$ $(4)=(5)$: $\frac{1}{3}n = \frac{2}{5}(n-1)$ $5n = 6n - 6$ $n = 6$ $\Rightarrow a = 2$	√**M1** √**M1** – simultaneous eqn **A1** **A1**

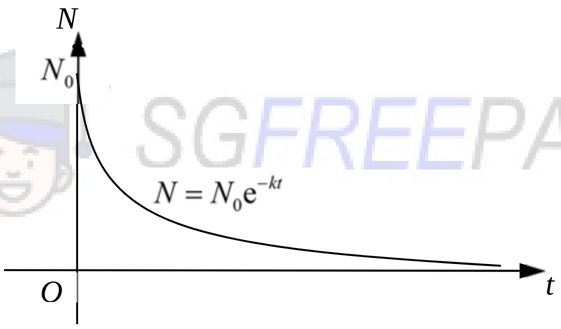
5 **(a)** Using the substitution $u = 2^x$, solve the equation $4^{x+1} = 2^x + 3$. [4]

(b) The quantity, N, of a radioactive substance, at time t years, is given by $N = N_0 e^{-kt}$, where N_0 and k are positive constants.

(i) Sketch the graph of N against t, labelling any axes intercepts. [2]

(ii) State the significance of N_0. [1]

(iii) The quantity halves every 5 years. Calculate the value of k. [3]

(a)	$4u^2 = u + 3$ $4u^2 - u - 3 = 0$ $(4u+3)(u-1) = 0$ $u = 1$ or $-\frac{3}{4}$ $x = 0$ or $2^x = -\frac{3}{4}$ (no solutions)	**B1** **M1** **A1, A1**
(a)(i)		**B1** – shape **B1** – $t > 0$ and label N_0
(ii)	It represents the initial amount of radioactive substance.	**B1**
(iii)	$\frac{1}{2} N_0 = N_0 e^{-k(5)}$ $\frac{1}{2} = e^{-5k}$ $-5k = \ln\frac{1}{2} = -\ln 2$ $t = \frac{\ln 2}{5} \approx 0.139$	**M1** **M1** **A1**

6 Solutions to this question by accurate drawing will not be accepted.

The coordinates of the points *P* and *Q* are $(-5,2)$ and $(7,6)$ respectively. Find

(i) the equation of the line parallel to *PQ* and passing through the point $(-2,3)$. [3]

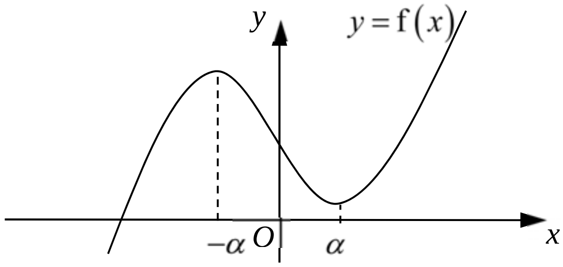
(ii) the equation of the perpendicular bisector of *PQ*. [3]

A point *R* is such that the shortest distance of *R* from the line passing through *P* and *Q* is $\sqrt{10}$ units.

(iii) Find the area of triangle *OQR*. [3]

(i)	$m_{PQ}=\dfrac{6-2}{7-(-5)}=\dfrac{1}{3}$	**B1**
	$y-3=\dfrac{1}{3}\left[x-(-2)\right]$	**M1**
	$y=\dfrac{1}{3}x+3\dfrac{2}{3}$	**A1**
(ii)	Midpoint of $PQ=\left(\dfrac{-5+7}{2},\dfrac{2+6}{2}\right)=(1,4)$	**B1**
	Gradient of perpendicular bisector $=-3$	
	$y-4=-3(x-1)$	√**M1**
	$y=-3x+7$	**A1**
(iii)	$PQ=\sqrt{(7-(-5))^2+(6-2)^2}=4\sqrt{10}$ units	**M1**
	Area $=\dfrac{1}{2}\left(4\sqrt{10}\right)\sqrt{10}$	√**M1**
	$=20$ units^2	**A1**

7 The diagram shows a sketch of the curve $y = \mathrm{f}(x)$. The x-coordinates of the minimum and maximum points are α and $-\alpha$, where $\alpha > 0$.



It is given that $\mathrm{f}'(x) = ax^2 + bx + c$, where a, b and c are real constants. For each of the following, state, with reasons, whether they are positive, negative or zero.

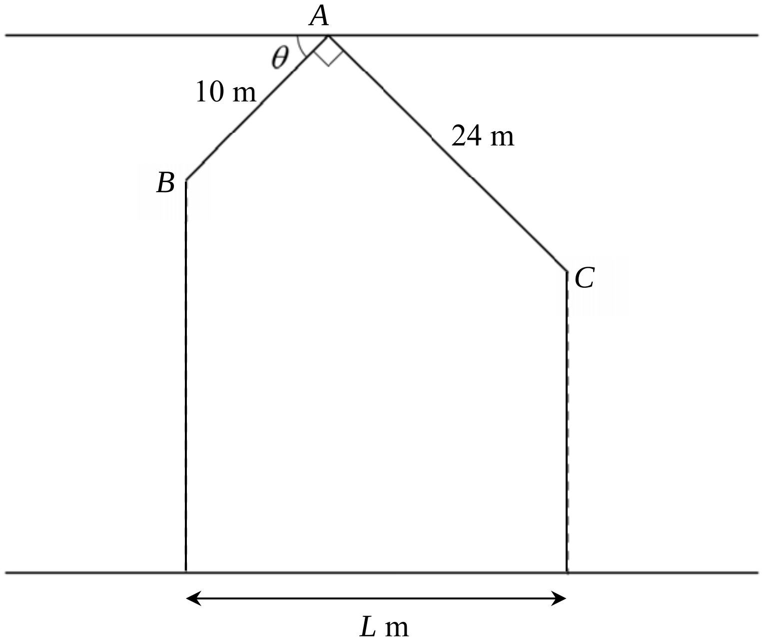
(i) $b^2 - 4ac$, [2]

(ii) $\dfrac{b}{a}$, [2]

(iii) $\dfrac{c}{a}$. [2]

(i)	Since there are two stationary points, $\mathrm{f}'(x) = 0$ has two real roots, therefore $b^2 - 4ac > 0$.	**M1** **A1**
(ii)	Since $\lvert\alpha\rvert > \lvert\beta\rvert$ and $\alpha < 0$, $\alpha + \beta < 0$, $\therefore \dfrac{b}{a} = -(\alpha + \beta) > 0$	**M1** **A1**
(iii)	Since $\alpha < 0$ and $\beta > 0$, $\alpha\beta < 0$, $\therefore \dfrac{c}{a} = \alpha\beta < 0$	**M1** **A1**

8 The diagram shows the cross-section of a house with a rooftop BAC. The length of AB and AC are 10 m and 24 m respectively. The angle between AB and the horizontal through A is θ degrees and $\angle BAC = 90°$.



The base of the house is of length L m.

(i) Show that $L = 10\cos\theta + 24\sin\theta$. [2]

(ii) Express L in the form $R\sin(\theta+\alpha)$, where $R > 0$ and α is an acute angle. [4]

(iii) Find the longest possible base of the house and the corresponding value of θ. [3]

(i)	Let the point vertically above B and C be M and N respectively. $\angle ACN = 90°$ $AM = 10\cos\theta$ and $AN = 24\sin\theta$ $L = MN = 10\cos\theta + 24\sin\theta$	**B1, B1**
(ii)	$R = \sqrt{10^2 + 24^2}$ $= 26$ $\alpha = \tan^{-1}\left(\frac{10}{24}\right)$ $= 22.620° \text{ (3 d.p.)}$ $L = 26\sin(\theta + 22.6°)$	**M1** **A1** **M1** **A1**
(iii)	Longest possible base is 26 m. $\theta + 22.620° = 90°$ $\theta = 67.4° \text{ (1 d.p.)}$	**B1** **√M1** **A1**

9 **(a)** The equation of a curve is $y = \dfrac{2x}{1+x}$.

(i) Find the equation of the tangent to the curve at point $P(1,1)$. [4]

(ii) The tangent cuts the axes at Q and R respectively. Find the area of triangle PQR. [2]

(b) A curve has equation $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = \dfrac{1}{3}x^3 - 2x^2 + 13x + 5$.

Determine, with explanation, whether f is an increasing or decreasing function. [4]

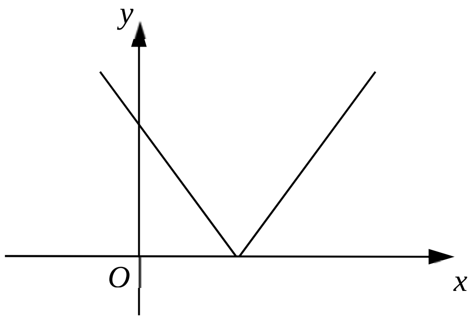
(a) (i)	$\dfrac{dy}{dx} = \dfrac{(1+x)(2)-(2x)(1)}{(1+x)^2}$ $= \dfrac{2}{(1+x)^2}$ $\left.\dfrac{dy}{dx}\right\|_{x=1} = \dfrac{1}{2}$ Equation of Tangent: $y - 1 = \dfrac{1}{2}(x-1) \Rightarrow y = \dfrac{1}{2}x + \dfrac{1}{2}$	**M1** **B1** **M1** – substitution of point **A1**
(ii)	$Q(-1,0)$ and $R\left(0,\dfrac{1}{2}\right)$ Area of Triangle $= \dfrac{1}{2}(1)\left(\dfrac{1}{2}\right) = \dfrac{1}{4}$ units^2	**√B1** **√B1**
(b)	$\mathrm{f}'(x) = x^2 - 4x + 13$ $= (x-2)^2 - 2^2 + 13$ $= (x-2)^2 + 9$ $(x-2)^2 \geq 0 \Rightarrow (x-2)^2 + 9 > 0$ $\therefore \mathrm{f}'(x) > 0$, f is an increasing function.	**B1** **M1** - complete the square **M1** **A1**

10 (a) (i) Solve the equation $\left|x^2 - 3x + 2\right| + x = 1$. [3]

(ii) What can be deduced about the number of points of intersections of the graphs of $y = \left|x^2 - 3x + 2\right|$ and $y = -x + 1$? [1]

(iii) Hence, on a single diagram, sketch the graphs of $y = \left|x^2 - 3x + 2\right|$ and $y = -x + 1$, indicating any axial intercepts. [4]

(b) The diagram shows part of the graph of $y = |k - x|$, where k is a constant.

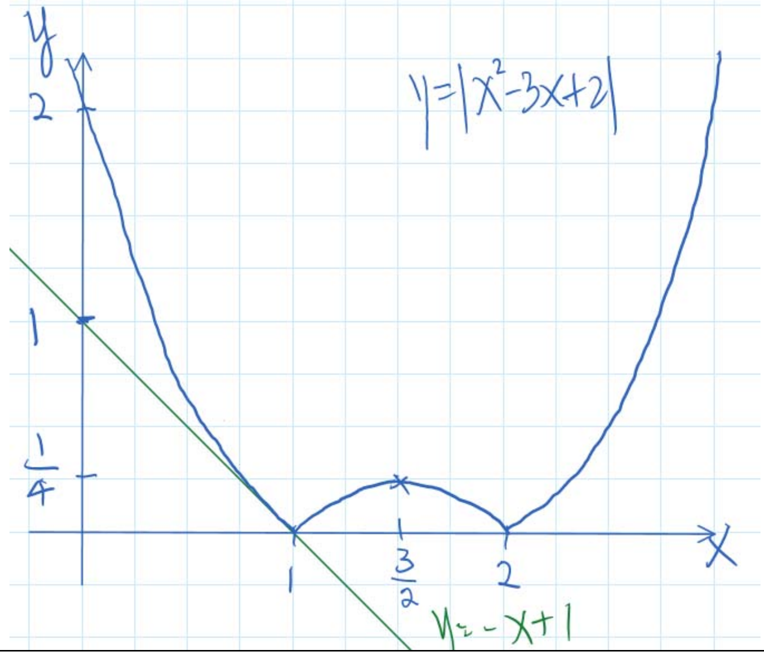


A line $y = mx + c$ is drawn to determine the number of solutions to the equation $|k - x| = mx + c$.

(i) If $m = 1$, state the range of values of c, in terms of k, such that the equation has one solution. [1]

(ii) If $c = 0$, state the range of values of m such that the equation has no solutions. [2]

(a) (i)	$x^2 - 3x + 2 = -x + 1$ or $x^2 - 3x + 2 = -(-x + 1)$ $x^2 - 2x + 1 = 0$ $x^2 - 4x + 3 = 0$ $(x-1)^2 = 0$ $(x-3)(x-1) = 0$ $x = 1$ $x = 1$ or $x = 3$ (rejected)	**M1** **A1, A1**
(ii)	The line $y = -x + 1$ is tangential to $y = \left|x^2 - 3x + 2\right|$.	**B1**

(ii)		**B1** - modulus graph **B1** – axial intercepts & turning point **B1** – line with intercepts **B1** – line tangent to modulus
(b)(i)	$c > -k$	**B1**
(ii)	$-1 \le m < 0$	**A1**, **A1**

11 **(a)** State the principal range of $\sin^{-1} x$, leaving your answers in terms of π. [1]

(b) **(i)** Prove that $\dfrac{1+\tan x}{1-\tan x} = \sec 2x + \tan 2x$. [5]

(ii) Hence find the reflex angle x such that $\sec 2x + \tan 2x = \dfrac{1}{3}$. [3]

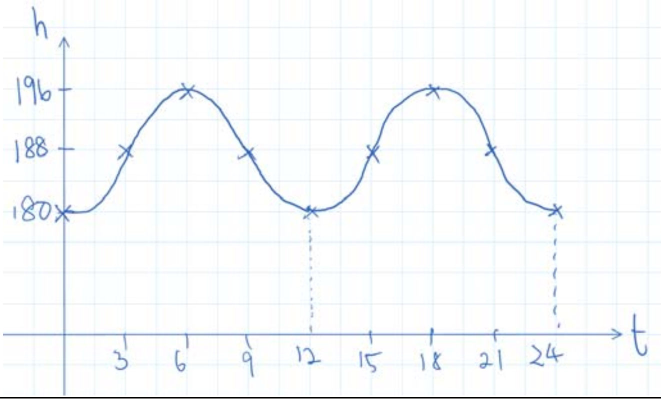
(c) A buoy floats and its height above the seabed, h m, is given by $h = a\cos bt + c$, where t is time measured in hours from 0000 hours and a, b and c are constants. The least height of the buoy above seabed is 180 metres and is recorded at 0000 hours. The greatest height of the buoy above seabed is 196 metres and is first recorded at 0600 hours.

(i) Find the values of a, b and c. [3]

(ii) Using values found in **(i)**, sketch the graph of $h = a\cos bt + c$ for $0 \le t \le 24$. [2]

(ii) The buoy floats above the top of a huge rock first at 0500 hours. State the number of hours in each day that the buoy is above the rock. [1]

(a)	$-\dfrac{\pi}{2} \le \sin^{-1} x \le \dfrac{\pi}{2}$	**B1**
(b) (i)	$\dfrac{1+\tan x}{1-\tan x} = \dfrac{1+\dfrac{\sin x}{\cos x}}{1-\dfrac{\sin x}{\cos x}}$ $= \dfrac{\cos x + \sin x}{\cos x - \sin x}$ $= \dfrac{(\cos x + \sin x)^2}{\cos^2 x - \sin^2 x}$ $= \dfrac{1 + 2\sin x \cos x}{\cos 2x}$ $= \dfrac{1+\sin 2x}{\cos 2x}$ $= \sec 2x + \tan 2x$	 **M1** **M1** **M1** – double angle **M1** – double angle **A1**
(ii)	$\dfrac{1+\tan x}{1-\tan x} = \dfrac{1}{3}$ $3 + 3\tan x = 1 - \tan x$ $4\tan x = -2$ $\tan x = -\dfrac{1}{2}$ $\alpha = 26.565°(3 \text{ d.p.})$ $x = 333.3°(1 \text{ d.p.})$	**M1** **B1** **A1**

(c) (i)	$\dfrac{196-180}{2}=8 \Rightarrow a=-8$ $c=\dfrac{196+180}{2}=188$ $b=\dfrac{2\pi}{12}=\dfrac{\pi}{6}$	**B1** **B1** **B1**
(ii)		**B1** – shape **B1** - points
(iii)	4 hours	**B1**

Class | Index Number

Name : ________________________________

METHODIST GIRLS' SCHOOL

Founded in 1887

PRELIMINARY EXAMINATION 2018
Secondary 4

Thursday **ADDITIONAL MATHEMATICS** **4047/1**

2 August 2018 **Paper 1** 2 h

INSTRUCTIONS TO CANDIDATES
Write your class, index number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all questions.

Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the quadratic equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion $$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)...(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\text{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$$

$$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$$

$$\sin 2A = 2\sin A\cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2}ab \sin C$$

1 The function f is defined, for all values of x, by

$$\mathrm{f}(x) = x^2 e^{2x}.$$

Find the values of x for which f is a decreasing function. [4]

2 A man buys an antique porcelain at the beginning of 2015. After t years, its value, \$$V$, is given by $V = 15\ 000 + 3000e^{0.2t}$.

(i) Find the value of the porcelain when the man first bought it. [1]

(ii) Find the year in which the value of the porcelain first reached \$50 000. [3]

3 Given the identity $\cos 3x = 4\cos^3 x - 3\cos x$, find the value of $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cos^3 x \ \mathrm{d}x$. [3]

4 **(i)** Sketch the graph of $y = 4x^{\frac{1}{3}}$ for $x \geq 0$. [2]

The line $y = x$ intersects the curve $y = 4x^{\frac{1}{3}}$ at the points A and B.

(ii) Show that the perpendicular bisector of AB passes through the point (5, 3). [4]

5 Solve the following equations:

(i) $\log_8 y + \log_2 y = 4$ [2]

(ii) $10^{2x+1} = 7(10^x) + 26$ [4]

6 **(i)** Show that $(\operatorname{cosec} x - 1)(\operatorname{cosec} x + 1)(\sec x - 1)(\sec x + 1) \equiv 1$. [2]

(ii) Hence solve $(\operatorname{cosec} x - 1)(\operatorname{cosec} x + 1)(\sec x - 1)(\sec x + 1) = 2\tan^2 2x - 5\sec 2x$ for $0 \leq x \leq 360°$. [4]

7 The function $f(x) = \sin^2 x + 2 - 3\cos^2 x$ is defined for $0 \le x \le 2\pi$.

(i) Express f(x) in the form $a + b\cos 2x$, stating the values of a and b. [2]

(ii) State the period and amplitude of f(x). [2]

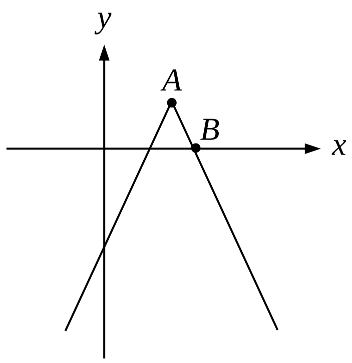
(iii) Sketch the graph of y = f(x) and hence state the number of solutions of the equation $\frac{1}{2} - \frac{x}{2\pi} + \cos 2x = 0$. [4]

8 A particle moves in a straight line passes through a fixed point X with velocity 5 m/s. Its acceleration is given by $a = 4 - 2t$, where t is the time in seconds after passing X. Calculate

(i) the value of t when the particle is instantaneously at rest, [4]

(ii) the total distance travelled by the particle in the first 6 seconds. [4]

9 **(i)** The diagram shows part of the graph of $y = 1 - |2x - 6|$. Find the coordinates of A and B. [3]

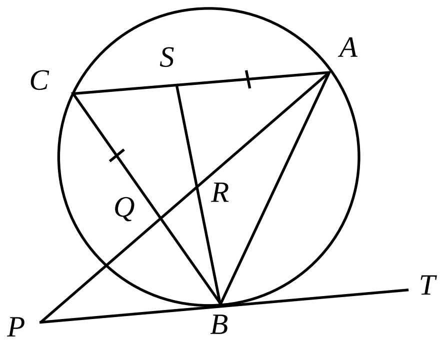


A line of gradient m passes through the point (4, 1).

(ii) In the case where m = 2, find the coordinates of the points of intersection of the line and the graph of $y = 1 - |2x - 6|$. [4]

(iii) Determine the sets of values of m for which the line intersects the graph of $y = 1 - |2x - 6|$ in two points. [1]

10 An equilateral triangle *ABC* is inscribed in a circle. *PT* is a tangent to the circle at *B*. It is given that *AS* = *QC*. *PQA* is a straight line and *BS* meets *AQ* at *R*.

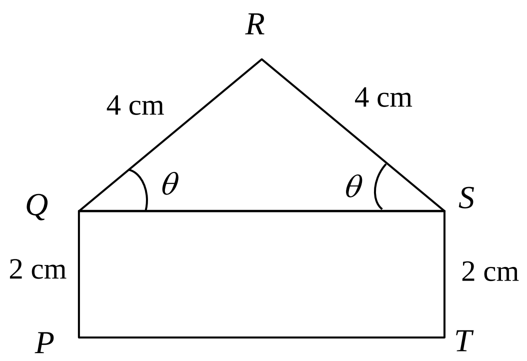


(i) Show that *AC* is parallel to *PB*. [2]

(ii) Prove that $\triangle ABS$ is congruent to $\triangle CAQ$. [2]

(iii) Prove that $\angle PBQ = \angle BRQ$. [3]

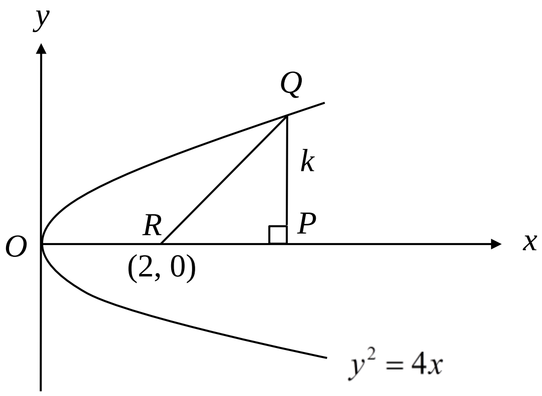
11 In the diagram, *PQRST* is a piece of cardboard. *PQST* is a rectangle with $PQ = 2$ cm and *QRS* is an isosceles triangle with $QR = RS = 4$ cm. $\angle RSQ = \angle RQS = \theta$ radians.



(i) Show that the area, *A* cm², of the cardboard is given by $A = 8\sin 2\theta + 16\cos\theta$. [3]

(ii) Given that θ can vary, find the stationary value of *A* and determine whether it it is a maximum or a minimum. [6]

12 **(a)**

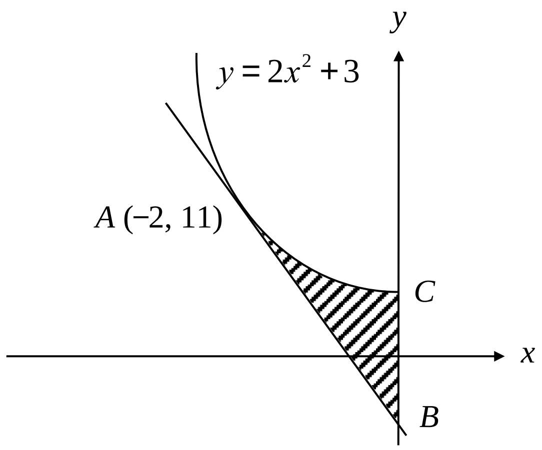


The diagram shows part of a curve $y^2 = 4x$. The point *P* is on the *x*-axis and the point *Q* is on the curve. *PQ* is parallel to the *y*-axis and *k* is units in length. Given *R* is (2, 0), express the area, *A*, of the ΔPQR in terms of *k* and hence show that $\frac{dA}{dk} = \frac{3k^2 - 8}{8}$.

The point *P* moves along the *x*-axis and the point *Q* moves along the curve in such a way that *PQ* remains parallel to the *y*-axis. *k* increases at the rate of 0.2 units per second.

Find the rate of increase of *A* when *k* = 6 units. [5]

(b)



The diagram shows part of the curve $y = 2x^2 + 3$.

The tangent to the curve at the point $A(-2, 11)$ intersects the *y*-axis at *B*. Find the area of the shaded region *ABC*. [6]

~ **End of Paper** ~

Class	Index Number

Name : ___________________________________

METHODIST GIRLS' SCHOOL

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PRELIMINARY EXAMINATION 2018
Secondary 4

Thursday	**ADDITIONAL MATHEMATICS**	**4047/1**
2 August 2018	**Paper 1**	2 h

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$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\ \cos A$$

$$\Delta = \frac{1}{2}ab\ \sin C$$

1 The function f is defined, for all values of x, by

$$\mathrm{f}(x) = x^2 e^{2x}.$$

Find the values of x for which f is a decreasing function. [4]

$\mathrm{f}(x) = x^2 e^{2x}$

$\mathrm{f}'(x) = e^{2x}(2x) + x^2(2e^{2x})$

$\mathrm{f}'(x) = 2xe^{2x}(1+x)$

For increasing function,

$\mathrm{f}'(x) < 0$

$2xe^{2x}(1+x) < 0$

Since $e^{2x} > 0$

$x(1+x) < 0$

Ans : $-1 < x < 0$

2 A man buys an antique porcelain at the beginning of 2015. After t years, its value, \$$V$, is given by $V = 15\ 000 + 3000e^{0.2t}$.

(i) Find the value of the porcelain when the man first bought it. [1]

(ii) Find the year in which the value of the porcelain first reached \$50 000. [3]

(i) at t = 0,

$V = 15\ 000 + 3000e^{0} = 18\ 000$

(ii) $50\ 000 = 15\ 000 + 3000e^{0.2t}$

$35\ 000 = 3000e^{0.2t}$

$\frac{35}{3} = e^{0.2t}$

$0.2t = \ln\left(\frac{35}{3}\right)$

$t = 12.283...$

Ans : 2027

3 Given the identity $\cos 3x = 4\cos^3 x - 3\cos x$, find the value of $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cos^3 x \ \mathrm{d}x$. [3]

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cos^3 x \ \mathrm{d}x$$

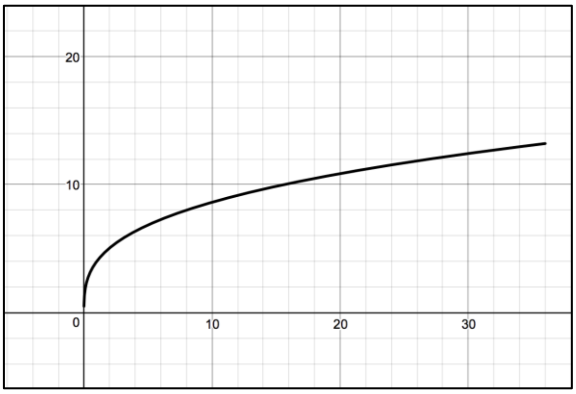
$$= \frac{1}{4}\left[\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \left(\cos 3x + 3\cos x\right) \ \mathrm{d}x\right]$$

$$= \frac{1}{4}\left[\frac{\sin 3x}{3} + 3\sin x\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$$

$$= \frac{1}{4}\left[\left(\frac{-1}{3} + 3\right) - \left(\frac{1}{3} + \frac{3}{2}\right)\right]$$

$$= \frac{5}{24}$$

4 **(i)** Sketch the graph of $y = 4x^{\frac{1}{3}}$ for $x \geq 0$. [2]



The line $y = x$ intersects the curve $y = 4x^{\frac{1}{3}}$ at the points *A* and *B*.

(ii) Show that the perpendicular bisector of *AB* passes through the point (5, 3). [4]

$x = 4x^{\frac{1}{3}}$

$x - 4x^{\frac{1}{3}} = 0$

$x^{\frac{1}{3}}\left(x^{\frac{2}{3}} - 4\right) = 0$

$x^{\frac{1}{3}} = 0$ or $x^{\frac{2}{3}} = 4$

$x = 0$ or $x = 4^{\frac{3}{2}}$

$x = 0$ or $x = 8$ $(x \geq 0)$

$A(0,0)$, $B(8,8)$

mid-point of *AB* = (4, 4)

gradient *AB* = 1

eqn of perpendicular bisector ,

$y - 4 = -1(x - 4)$

$y = -x + 8$

when $x = 5$, $y = 3$.

Therefore the perpendicular bisector passes through (5, 3).

5 Solve the following equations:

(i) $\log_8 y + \log_2 y = 4$ [2]

(ii) $10^{2x+1} = 7(10^x) + 26$ [4]

(i) $\log_8 y + \log_2 y = 4$

$$\frac{\log_2 y}{\log_2 8} + \log_2 y = 4$$

$$\frac{\log_2 y}{3} + \log_2 y = 4$$

$$\frac{4}{3}\log_2 y = 4$$

$$\log_2 y = 3$$

$$y = 8$$

(ii) $10^{2x+1} = 7(10^x) + 26$

$$10^{2x}(10^1) = 7(10^x) + 26$$

let $p = 10^x$,

$$10p^2 - 7p - 26 = 0$$

$$(10p + 13)(p - 2) = 0$$

$$p = -\frac{13}{10} \quad \text{or} \quad p = 2$$

$$10^x = -\frac{13}{10} \quad \text{or} \quad 10^x = 2$$

$$\text{(NA)} \quad \text{or} \quad x = \lg 2 = 0.301$$

6 **(i)** Show that $(\operatorname{cosec} x - 1)(\operatorname{cosec} x + 1)(\sec x - 1)(\sec x + 1) \equiv 1$. [2]

(ii) Hence solve $(\operatorname{cosec} x - 1)(\operatorname{cosec} x + 1)(\sec x - 1)(\sec x + 1) = 2\tan^2 2x - 5\sec 2x$ for $0 \le x \le 360°$. [4]

(i) LHS,

$(\operatorname{cosec} x - 1)(\operatorname{cosec} x + 1)(\sec x - 1)(\sec x + 1)$

$= (\operatorname{cosec}^2 x - 1)(\sec^2 x - 1)$

$= (\cot^2 x)(\tan^2 x)$

$= 1$

(ii) $(\operatorname{cosec} x - 1)(\operatorname{cosec} x + 1)(\sec x - 1)(\sec x + 1) = 2\tan^2 2x - 5\sec 2x$

$1 = 2\tan^2 2x - 5\sec 2x$

$2(\sec^2 2x - 1) - 5\sec 2x - 1 = 0$

$2\sec^2 2x - 5\sec 2x - 3 = 0$

$(\sec 2x - 3)(2\sec 2x + 1) = 0$

$\sec 2x = 3$ or $\sec 2x = -\frac{1}{2}$

$\cos 2x = \frac{1}{3}$ or $\cos 2x = -2$

basic angle, $\alpha = 70.529...$ or NA

$2x = \alpha, 360° - \alpha, 360° - \alpha, 720° - \alpha$

$x = 35.3°,\ 144.7°,\ 215.3°,\ 324.7°$

7 The function $\mathrm{f}(x)=\sin^2 x+2-3\cos^2 x$ is defined for $0\le x\le 2\pi$.

(i) Express f(x) in the form $a+b\cos 2x$, stating the values of a and b. [2]

(ii) State the period and amplitude of f(x). [2]

(iii) Sketch the graph of y = f(x) and hence state the number of solutions of the equation $\frac{1}{2}-\frac{x}{2\pi}+\cos 2x=0$. [4]

(i) $\mathrm{f}(x)=\sin^2 x+2-3\cos^2 x$

$\mathrm{f}(x)=\sin^2 x+\cos^2 x+2-4\cos^2 x$

$\mathrm{f}(x)=3-2(2\cos^2 x)$

$\mathrm{f}(x)=1-2(2\cos^2 x-1)$

$\mathrm{f}(x)=1-2\cos 2x$

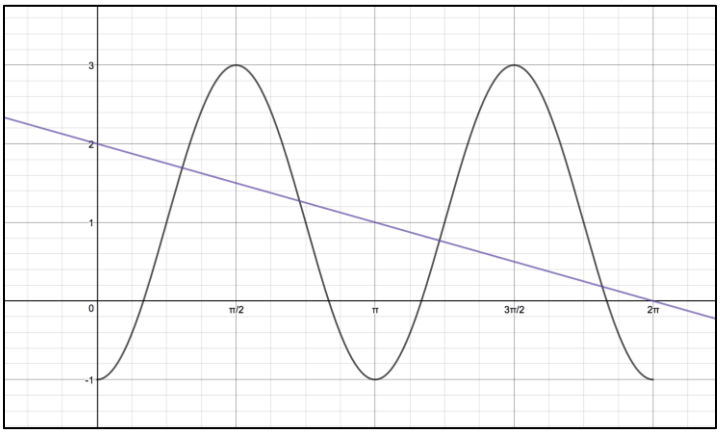
(ii) Amplitude = 2

Period = $\frac{2\pi}{2}=\pi$

(iii) $\frac{1}{2}-\frac{x}{2\pi}+\cos 2x=0$

$1-\frac{x}{\pi}=-2\cos 2x$

$2-\frac{x}{\pi}=1-2\cos 2x$



No. of solutions = 4

8 A particle moves in a straight line passes through a fixed point *X* with velocity 5 m/s. Its acceleration is given by $a = 4 - 2t$, where *t* is the time in seconds after passing *X*. Calculate

(i) the value of *t* when the particle is instantaneously at rest, [4]

(ii) the total distance travelled by the particle in the first 6 seconds. [4]

(i)

$$a = 4 - 2t$$
$$v = \int (4 - 2t)\ \mathrm{d}t$$
$$v = 4t - t^2 + c$$

at $t = 0$, $v = 5$,
$5 = c$

$$v = 4t - t^2 + 5$$

at $v = 0$,
$$0 = 4t - t^2 + 5$$
$$t^2 - 4t - 5 = 0$$
$$(t - 5)(t + 1) = 0$$
$$t = 5 \text{ or } t = -1$$
(NA)

(ii)

$$s = \int (4t - t^2 + 5)\ \mathrm{d}t$$
$$s = 2t^2 - \frac{t^3}{3} + 5t + c_1$$

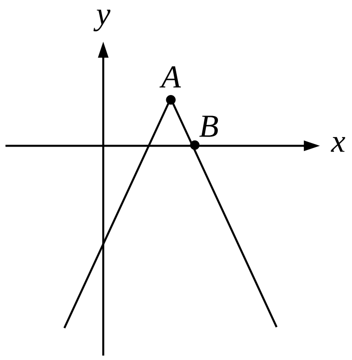
at $t = 0$, $s = 0$,
$c_1 = 0$

$$s = 2t^2 - \frac{t^3}{3} + 5t$$

at $t = 0$, $s = 0$
at $t = 5$, $s = \frac{100}{3}$
at $t = 6$, $s = 30$

Total Distance = $\left(2 \times \frac{100}{3}\right) - 30 = 36\frac{2}{3}$

9 **(i)** The diagram shows part of the graph of $y=1-|2x-6|$. Find the coordinates of A and B. [3]



$2x-6=0$ | $|2x-6|=1$

$x=3$ | $2x-6=1$ or $2x-6=-1$

$A(3, 1)$ | $x=3.5$ or $x=2.5$

$B(3.5, 0)$

A line of gradient m passes through the point (4, 1).

(ii) In the case where $m = 2$, find the coordinates of the points of intersection of the line and the graph of $y=1-|2x-6|$. [4]

(iii) Determine the sets of values of m for which the line intersects the graph of $y=1-|2x-6|$ in two points. [1]

(ii) $y=2x+c$

at (4, 1),

$1=8+c$

$c=-7$

$y=2x-7$

$y=1-|2x-6|$

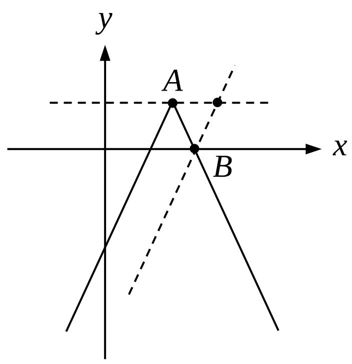
$2x-7=1-|2x-6|$

$|2x-6|=8-2x$

$2x-6=8-2x$ or $2x-6=-(8-2x)$

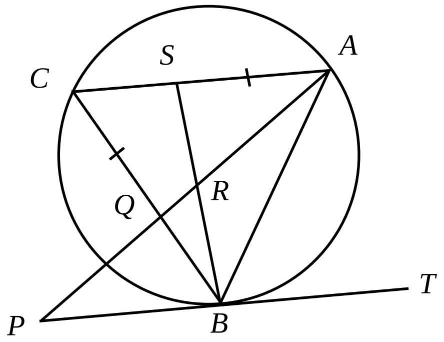
$4x=14$ or $2x-6=-8+2x$

$x=3.5$ or NA



(iii) $0<m<2$

10 An equilateral triangle *ABC* is inscribed in a circle. *PT* is a tangent to the circle at *B*. It is given that *AS* = *QC*. *PQA* is a straight line and *BS* meets *AQ* at *R*.



(i) Show that *AC* is parallel to *PB*. [2]

(ii) Prove that $\triangle ABS$ is congruent to $\triangle CAQ$. [2]

(iii) Prove that $\angle PBQ = \angle BRQ$. [3]

(i) $\angle ACB = \angle BAC = 60°$ (equilateral triangle)
$\angle PBC = \angle BAC$ (Alternate Segment Theorem)
Since $\angle PBC = \angle ACB$,
AC is parallel to *PB* (alternate angle)

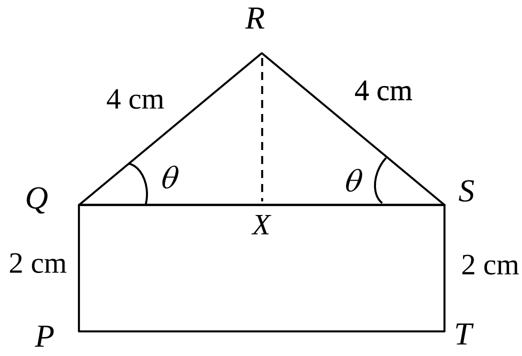
(ii) *AS* = *CQ* (given)
$\angle BAS = \angle ACQ = 60°$ (equilateral triangle)
AB = *AC* (sides of a equilateral triangle)
$\therefore$ $\triangle ABS \equiv \triangle CAQ$ (SAS)

(iii) let $\angle RBQ = x$,
$\angle RBA = 60° - x$ (equilateral triangle)
$\angle ASB = 180° - (60° - x) - 60° = 60° + x$ (angle sum of triangle)

$\angle RBA = \angle RAS = 60° - x$ ($\triangle ABS \equiv \triangle CAQ$)

$\angle ARS = 180° - (60° + x) - (60° - x) = 60°$ (angle sum of triangle)
$\angle BRQ = 60°$ (vertically opposite angle)
so, $\angle PBQ = \angle BRQ$

11 In the diagram, $PQRST$ is a piece of cardboard. $PQST$ is a rectangle with $PQ = 2$ cm and QRS is an isosceles triangle with $QR = RS = 4$ cm. $\angle RSQ = \angle RQS = \theta$ radians.



(i) Show that the area, A cm^2, of the cardboard is given by $A = 8\sin 2\theta + 16\cos\theta$. [3]

(ii) Given that θ can vary, find the stationary value of A and determine whether it it is a maximum or a minimum. [6]

(i) $QS = 2(4\cos\theta) = 8\cos\theta$

$RX = 4\sin\theta$

Area, $A = \frac{1}{2}(4\sin\theta)(8\cos\theta) + 2(8\cos\theta)$

$= 16\sin\theta\cos\theta + 16\cos\theta$

$= 8\sin 2\theta + 16\cos\theta$

(ii) $\frac{dA}{d\theta} = (8\cos 2\theta)(2) + 16(-\sin\theta)$

$\frac{dA}{d\theta} = 16(\cos 2\theta - \sin\theta)$

For $\frac{dA}{d\theta} = 0$,

$\cos 2\theta - \sin\theta = 0$

$1 - 2\sin^2\theta - \sin\theta = 0$

$2\sin^2\theta + \sin\theta - 1 = 0$

$(2\sin\theta - 1)(\sin\theta + 1) = 0$

$\sin\theta = 0.5$ or $\sin\theta = -1$

$\theta = \frac{\pi}{6} \approx 0.524$ or NA

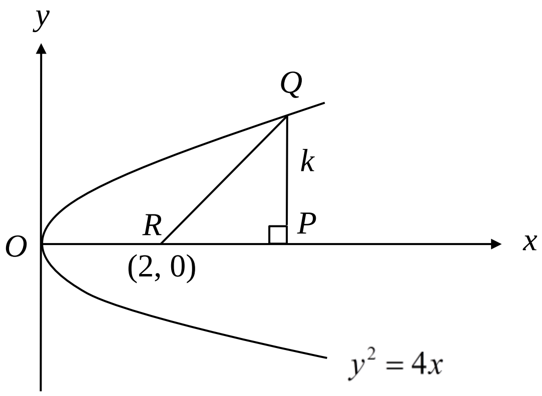
$A = 12\sqrt{3} = 20.8$

$\frac{d^2A}{d\theta^2} = 16(-2\sin 2\theta - \cos\theta)$

For $\theta = \frac{\pi}{6} \approx 0.524$

$\frac{d^2A}{d\theta^2} < 0$ => max. area

12 **(a)**



The diagram shows part of a curve $y^2 = 4x$. The point P is on the x-axis and the point Q is on the curve. PQ is parallel to the y-axis and k is units in length. Given R is (2, 0), express the area, A, of the ΔPQR in terms of k and hence show that $\frac{dA}{dk} = \frac{3k^2 - 8}{8}$.

The point P moves along the x-axis and the point Q moves along the curve in such a way that PQ remains parallel to the y-axis. k increases at the rate of 0.2 units per second.

Find the rate of increase of A when k = 6 units. [5]

$y^2 = 4x$

at Q, $k^2 = 4x$

$$x = \frac{k^2}{4}$$

$$A = \frac{1}{2}(k)\left(\frac{k^2}{4} - 2\right)$$

$$A = \frac{k^3}{8} - k$$

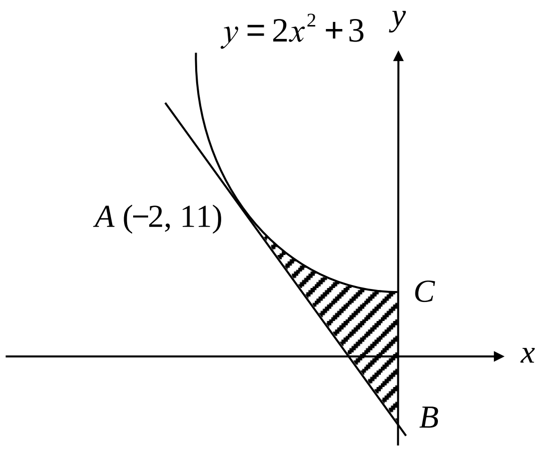
$$\frac{dA}{dk} = \frac{3k^2}{8} - 1$$

$$\frac{dA}{dk} = \frac{3k^2 - 8}{8}$$

$$\frac{dA}{dt} = \frac{dA}{dk} \times \frac{dk}{dt}$$

at p = 6, $\frac{dA}{dt} = \left(\frac{3(6)^2 - 8}{8}\right) \times 0.2 = 2.5$

(b)



The diagram shows part of the curve $y = 2x^2 + 3$.

The tangent to the curve at the point $A(-2, 11)$ intersects the *y*-axis at *B*. Find the area of the shaded region *ABC*. [6]

$\frac{dy}{dx} = 4x$

at *A*, $m = -8$

let *B* (0, *y*)

$m_{AB} = \frac{11 - y}{-2 - 0}$

$-8 = \frac{11 - y}{-2}$

$y = -5$

B (0, –5)

at *C*,

$y = 2(0)^2 + 3 = 3$

C(3, 0)

eqn *AB*

$y = -8x - 5$

Area = $\int_{-2}^{0} \left[\left(2x^2 + 3\right) - \left(-8x - 5\right) \right]$

$= \int_{-2}^{0} \left[2x^2 + 8x + 8 \right]$

$= \left[\frac{2x^3}{3} + 4x^2 + 8x \right]_{-2}^{0}$

$= 0 - \left[\frac{-16}{3} + 16 - 16 \right] = \frac{16}{3}$

~ End of Paper ~

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the quadratic equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion $(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n$,

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)...(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2}ab \sin C$$

1. The equation $2x^2 + px + 3 = 0$, where $p > 0$, has roots α and β.

(i) Given that $\alpha^2 + \beta^2 = 1$, show that $p = 4$. **[3]**

(ii) Find the value of $\alpha^3 + \beta^3$. **[2]**

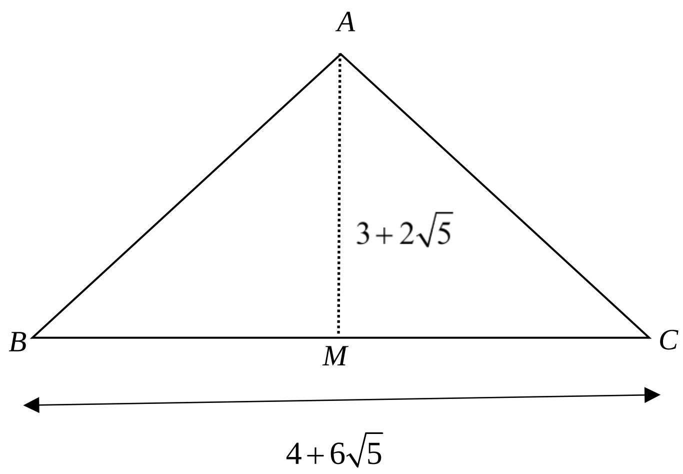
(iii) Find a quadratic equation with roots $\dfrac{2\alpha}{\beta^2}$ and $\dfrac{2\beta}{\alpha^2}$. **[3]**

2. **(a)** Find the term independent of x in the expansion of $2x\left(2x - \dfrac{1}{x^2}\right)^8$. **[4]**

(b) The first 3 terms in the binomial expansion $(1+kx)^n$ are $1 + 5x + \dfrac{45}{4}x^2 + ...$

Find the value of n and of k. **[4]**

3.



The diagram shows an isosceles triangle ABC, where $AB = AC$. The point M is the mid-point of BC. Given that $AM = (3 + 2\sqrt{5})cm$ and $BC = (4 + 6\sqrt{5})cm$.

Without the use of a calculator, find

(i) the area of triangle ABC, **[2]**

(ii) AB^2, **[3]**

(iii) $\sin \angle BAC$, giving your answer in the form $\dfrac{p + q\sqrt{5}}{r}$ where p, q and r are positive integers. **[3]**

4. **(i)** Given that $\frac{6x^3-15x^2+6x-5}{2x^2-x}=ax+b+\frac{c}{2x^2-x}$, where *a*, *b* and *c* are integers,

express $\frac{6x^3-15x^2+6x-5}{2x^2-x}$ in partial fractions. **[5]**

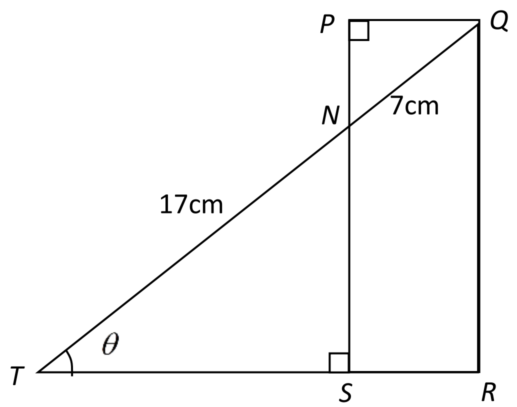
(ii) **Hence** find $\int \frac{6x^3-15x^2+6x-5}{2x^2-x}\,dx$. **[3]**

5. The term containing the highest power of *x* and the term independent of *x* in the polynomial *f*(*x*) are $2x^4$ and -3 respectively. It is given that $(2x^2+x-1)$ is a quadratic factor of *f*(*x*) and the remainder when *f*(*x*) is divided by (*x* – 1) is 4.

(i) Find an expression for f(x) in descending powers of x, **[5]**

(ii) Explain why the equation *f*(*x*) = 0 has only 2 real roots and state the values. **[4]**

6. *PQRS* is a rectangle. A line through *Q*, intersects *PS* at *N* and *RS* produced at *T*, where *QN*= 7cm, *NT* = 17cm, $\angle NTS = \theta$, and θ varies.



(i) Show that the perimeter of *PQRS*, *P* cm, is given by $P = 14\cos\theta + 48\sin\theta$. **[2]**

(ii) Express *P* in the form of $R\cos(\theta-\alpha)$ and state the value of *R* and α in degree. **[3]**

(iii) Without evaluating θ, justify with reasons if *P* can have a value of 48 cm. **[1]**

(iv) Find the value of *P* for which *QR* =12 cm. **[2]**

7. Variables x and y are related by the equation $\frac{x+sy}{t} = xy$, where s and t are constants.

The table below shows the measured values of x and y during an experiment.

x	1.00	1.50	2.00	2.50	3.00
y	0.48	0.65	0.85	1.00	1.13

(i) On graph paper, draw a straight line graph of $\frac{x}{y}$ against x, using a scale of 4 cm to represent 1 unit on the x – axis. The vertical $\frac{x}{y}$ – axis should start at 1.5 and have a scale of 1 cm to 0.1 units. **[3]**

(ii) Determine which value of y is inaccurate and estimate its correct value. **[1]**

(iii) Use your graph to estimate the value of s and of t. **[2]**

(iv) By adding a suitable straight line on the **same axes**, find the value of x and y which satisfy the following pair of simultaneous equations.

$$\frac{x+sy}{t} = xy$$

$$5y - 2x = 2xy.$$ **[3]**

8. The equation of a circle C_1, is $x^2 + y^2 - 2x - y - 10 = 0$.

(i) Find the centre and the radius of the circle. **[3]**

(ii) The equation of a tangent to the circle C_1 at the point A is $y + 2x = k$, where $k > 0$.

Find the value of the constant k. **[4]**

A second circle C_2 has its centre at point A and its lowest point B lies on the x-axis.

(iii) Find the equation of the circle C_2. **[2]**

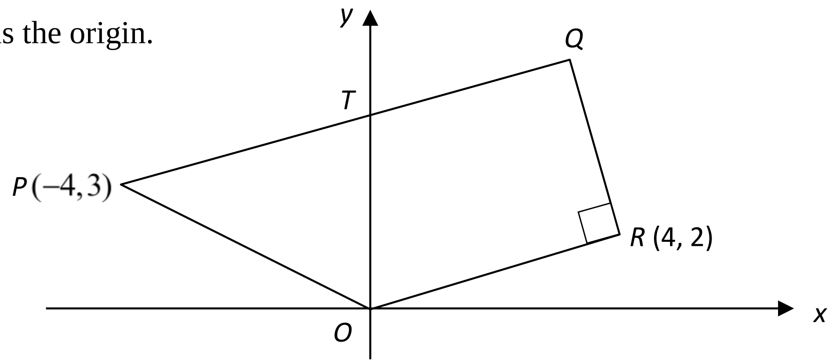
9. **(a)** The curve $y = \frac{2x-5}{1-2x}$ passes through the point A where $x = 1$.

(i) Find the equation of the normal to the curve at the point A. **[4]**

(ii) Find the acute angle the tangent makes with the positive x-axis. **[2]**

9. **(b)** The curve $y = f(x)$ is such that $f''(x) = 3(e^{x} - e^{-3x})$ and the point $P(0, 2)$ lies on the curve. Given that the gradient of the curve at P is 5, find the equation of the curve. **[6]**

10. The diagram (not drawn to scale) shows a trapezium *OPQR* in which *PQ* is parallel to *OR* and $\angle ORQ = 90°$. The coordinates of *P* and *R* are $(-4, 3)$ and $(4, 2)$ respectively and *O* is the origin.



(i) Find the coordinates of *Q*. **[3]**

(ii) *PQ* meets the *y*-axis at *T*. Show that triangle *ORT* is isosceles. **[2]**

(iii) Find the area of the trapezium *OPQR*. **[2]**

(iv) *S* is a point such that *ORPS* forms a parallelogram, find the coordinates of *S*. **[2]**

11. **(a)** Given that $y = x^2\sqrt{2x+1}$, show that $\dfrac{dy}{dx} = \dfrac{x(5x+2)}{\sqrt{2x+1}}$. **[3]**

(b) Hence

(i) find the coordinates of the stationary points on the curve $y = x^2\sqrt{2x+1}$ and determine the nature of these stationary points. **[5]**

(ii) evaluate $\displaystyle\int_1^5 \frac{5x^2 + 2x - 3}{\sqrt{2x+1}}\, dx$. **[4]**

~~ ***End of Paper*** ~~

METHODIST GIRLS' SCHOOL

Founded in 1887

PRELIMINARY EXAMINATION 2018
Secondary 4

Thursday	**ADDITIONAL MATHEMATICS**	**4047/02**
3 Aug 2018	**Paper 2**	2 h 30 min

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions**.**
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the quadratic equation $ax^2 + bx + c = 0$, $\quad x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Binomial Expansion $\quad (a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n$,

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$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

$$\sin A + \sin B = 2\sin\frac{1}{2}(A+B)\cos\frac{1}{2}(A-B)$$

$$\sin A - \sin B = 2\cos\frac{1}{2}(A+B)\sin\frac{1}{2}(A-B)$$

$$\cos A + \cos B = 2\cos\frac{1}{2}(A+B)\cos\frac{1}{2}(A-B)$$

$$\cos A - \cos B = -2\sin\frac{1}{2}(A+B)\sin\frac{1}{2}(A-B)$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

1. The equation $2x^2 + px + 3 = 0$, where $p > 0$, has roots α and β.

(i) Given that $\alpha^2 + \beta^2 = 1$, show that $p = 4$. **[3]**

(ii) Find the value of $\alpha^3 + \beta^3$. **[2]**

(iii) Find a quadratic equation with roots $\dfrac{2\alpha}{\beta^2}$ and $\dfrac{2\beta}{\alpha^2}$. **[3]**

(i) $\alpha + \beta = -\dfrac{p}{2} \quad and \quad \alpha\beta = \dfrac{3}{2}$

$$\alpha^2 + \beta^2 = 1$$
$$(\alpha + \beta)^2 - 2\alpha\beta = 1$$
$$\frac{p^2}{4} - 3 = 1$$
$$p^2 = 16$$
$$p = 4 \quad or \quad p = -4$$

Since $p > 0$, $p = 4$ (Shown)

(ii)
$$\alpha^3 + \beta^3 = (\alpha + \beta)(\alpha^2 - \alpha\beta + \beta^2)$$
$$= -2(1 - \frac{3}{2})$$
$$= 1$$

(iii)
$$\frac{2\alpha}{\beta^2} + \frac{2\beta}{\alpha^2} = \frac{2\left(\alpha^3 + \beta^3\right)}{\alpha^2\beta^2}$$
$$= \frac{8}{9}$$
$$\frac{2\alpha}{\beta^2} \times \frac{2\beta}{\alpha^2} = \frac{4}{\alpha\beta}$$
$$= \frac{8}{3}$$

Required quadratic equation : $x^2 - \dfrac{8}{9}x + \dfrac{8}{3} = 0$ or $9x^2 - 8x + 24 = 0$

2. **(a)** Find the term independent of x in the expansion of $2x\left(2x-\frac{1}{x^2}\right)^8$. **[4]**

(b) The first 3 terms in the binomial expansion $(1+kx)^n$ are $1+5x+\frac{45}{4}x^2+...$

Find the value of n and of k. **[4]**

(a) For $\left(2x-\frac{1}{x^2}\right)^8$, $T_{r+1}=\binom{8}{r}(2x)^{8-r}\left(-\frac{1}{x^2}\right)^r$

For x^{-1}, $8-r-2r=-1$

$r=3$

Coefficient of $x^{-1}=\binom{8}{3}(2)^5(-1)^3$ = -1792

Term independent of x in $2x\left(2x-\frac{1}{x^2}\right)^8=-3584$.

(b) $(1+kx)^n=1+\binom{n}{1}kx+\binom{n}{2}k^2x^2+....$

$=1+nkx+\frac{n(n-1)k^2}{2}x^2+...$

Comparing coefficients : $nk=5$ ……….. (1)

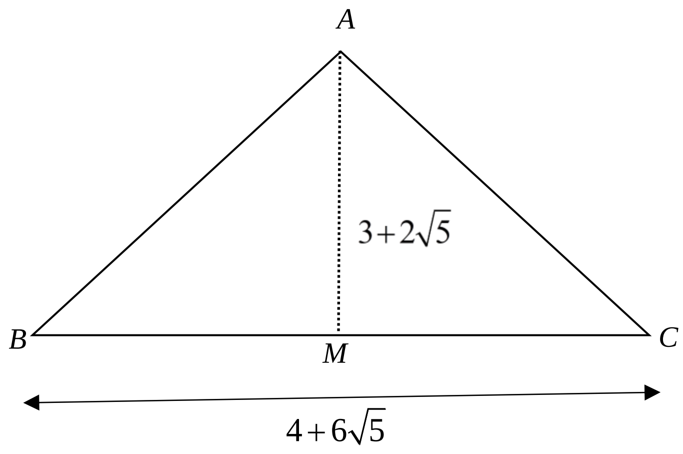
$\frac{n(n-1)k^2}{2}=\frac{45}{4}$

$2n^2k^2-2nk^2=45$……….. (2)

Subst (1) in (2) : $50-10k=45$

$\therefore k=\frac{1}{2}$ and $n=10$

3.



The diagram shows an isosceles triangle *ABC*, where *AB* = *AC*. The point *M* is the mid-point of *BC*. Given that *AM* = $(3+2\sqrt{5})cm$ and $BC=(4+6\sqrt{5})cm$.

Without the use of a calculator, find

(i) the area of triangle *ABC*, **[2]**

(ii) AB^2, **[3]**

(iii) $\sin\angle BAC$, giving your answer in the form $\dfrac{p+q\sqrt{5}}{r}$ where *p*, *q* and *r* are positive integers. **[3]**

(i) Area of triangle *ABC* = $\frac{1}{2}(4+6\sqrt{5})(3+2\sqrt{5})$

$$=(2+3\sqrt{5})(3+2\sqrt{5})$$

$$=(36+13\sqrt{5})\ cm^2$$

(ii) $AB^2=(3+2\sqrt{5})^2+(2+3\sqrt{5})^2$

$$=9+12\sqrt{5}+20+4+12\sqrt{5}+45$$

$$=(78+24\sqrt{5})\ cm^2$$

(iii) $\frac{1}{2}(78+24\sqrt{5})\ \sin\angle BAC=36+13\sqrt{5}$

$$\sin\angle BAC=\frac{36+13\sqrt{5}}{39+12\sqrt{5}}$$

$$=\frac{36+13\sqrt{5}}{39+12\sqrt{5}}\times\frac{39-12\sqrt{5}}{39-12\sqrt{5}}$$

$$=\frac{1404-432\sqrt{5}+507\sqrt{5}-780}{801}$$

$$=\frac{624+75\sqrt{5}}{801}$$

$$=\frac{208+25\sqrt{5}}{267}$$

4. **(i)** Given that $\dfrac{6x^3-15x^2+6x-5}{2x^2-x}=ax+b+\dfrac{c}{2x^2-x}$, where a, b and c are integers,

express $\dfrac{6x^3-15x^2+6x-5}{2x^2-x}$ in partial fractions. **[5]**

(ii) **Hence** find $\displaystyle\int\frac{6x^3-15x^2+6x-5}{2x^2-x}\,dx$. **[3]**

(i) Using long division, $\dfrac{6x^3-15x^2+6x-5}{2x^2-x}=3x-6-\dfrac{5}{2x^2-x}$

Let $\dfrac{-5}{x(2x-1)}=\dfrac{A}{x}+\dfrac{B}{2x-1}$

$-5=A(2x-1)+Bx$

Put $x = 0$: $A = 5$

Put $x=\dfrac{1}{2}$: $\dfrac{1}{2}B=-5$

$B = -10$

$\therefore \dfrac{6x^3-15x^2+6x-5}{2x^2-x}=3x-6+\dfrac{5}{x}-\dfrac{10}{2x-1}$

(ii) $\displaystyle\int\frac{6x^3-15x^2+6x-5}{2x^2-x}dx=\int(3x-6+\frac{5}{x}-\frac{10}{2x-1})dx$

$=\dfrac{3x^2}{2}-6x+5\ln x-5\ln(2x-1)+C$

5. The term containing the highest power of x and the term independent of x in the polynomial $f(x)$ are $2x^4$ and -3 respectively. It is given that $(2x^2+x-1)$ is a quadratic factor of $f(x)$ and the remainder when $f(x)$ is divided by $(x-1)$ is 4.

(i) Find an expression for $f(x)$ in descending powers of x, **[5]**

(ii) Explain why the equation $f(x) = 0$ has only 2 real roots and state the values. **[4]**

(i) $f(x)=(2x^2+x-1)(x^2+bx+3)$

$f(1) = 4$

$2(4 + b) = 4$
$b = -2$

$f(x)=(2x^2+x-1)(x^2-2x+3)$

$=2x^4-4x^3+6x^2+x^3-2x^2+3x-x^2+2x-3$
$=2x^4-3x^3+3x^2+5x-3$

(ii) $f(x)=(2x^2+x-1)(x^2-2x+3)$

$=(2x-1)(x+1)(x^2-2x+3)$
$(2x-1)(x+1)(x^2-2x+3)=0$

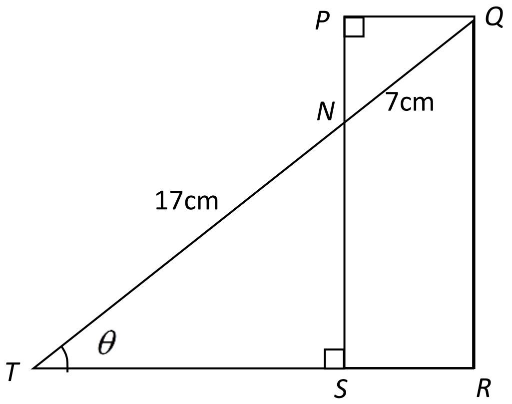
$x=\frac{1}{2}$ *or* $x=-1$

$x^2-2x+3=0$

$D=(-2)^2-4(1)(3)=-8<0$

$\therefore f(x) = 0$ has only 2 real roots (Shown)

6. *PQRS* is a rectangle. A line through *Q*, intersects *PS* at *N* and *RS* produced at *T*, where *QN*= 7cm, *NT* = 17cm, $\angle NTS = \theta$, and θ varies.



(i) Show that the perimeter of *PQRS*, *P* cm, is given by $P = 14\cos\theta + 48\sin\theta$. **[2]**

(ii) Express *P* in the form of $R\cos(\theta - \alpha)$ and state the value of *R* and α in degrees. **[3]**

(iii) Without evaluating θ, justify with reasons if *P* can have a value of 48 cm **[1]**

(iv) Find the value of *P* for which *QR* =12 cm. **[3]**

(i) $P = 2(7\cos\theta) + 2(24\sin\theta)$

$= 14\cos\theta + 48\sin\theta$

$14\cos\theta + 48\sin\theta = R\cos(\theta - \alpha) = R\cos\theta\cos\alpha + R\sin\theta\sin\alpha$

$R\cos\alpha = 14$ and $R\sin\alpha = 48$

$R = \sqrt{14^2 + 48^2} = \sqrt{2500} = 50$

$\tan\alpha = \frac{48}{14}$

$\alpha = 73.74°$

$= 73.7°$

$14\cos\theta + 48\sin\theta = 50\cos(\theta - 73.74°)$

(ii) Since maximum value of *P* = 50, *P* can have a value of 48 cm.

Or $\cos(\theta - 73.74°) = \frac{48}{50} < 1$, *P* can have a value of 48 cm.

When *QR*=12cm, $\sin\theta = \frac{12}{24} = \frac{1}{2}$

$\theta = 30°, 150°$ (NA $\because \theta < 90°$)

$\therefore P = 50\cos(30° - 73.74°)$

= 36.1 cm (3sf)

7. Variables x and y are related by the equation $\frac{x+sy}{t}=xy$, where s and t are constants.
The table below shows the measured values of x and y during an experiment.

x	1.00	1.50	2.00	2.50	3.00
y	0.48	0.65	0.85	1.00	1.13

(i) On graph paper, draw a straight line graph of $\frac{x}{y}$ against x, using a scale of 4 cm to represent 1 unit on the x – axis. The vertical $\frac{x}{y}$ – axis should start at 1.5 and have a scale of 1 cm to 0.1 units. **[3]**

(ii) Determine which value of y is inaccurate and estimate its correct value. **[1]**

(iii) Use your graph to estimate the value of s and of t. **[2]**

(iv) By adding a suitable straight line on the **same axes**, find the value of x and y which satisfy the following pair of simultaneous equations.

$$\frac{x+sy}{t}=xy$$

$$5y-2x=2xy\,.$$ **[3]**

(i) $x+sy=xyt$

$$\frac{x}{y}=tx-s$$

Gradient = t and $\frac{x}{y}-\text{int}\,ercept=-s$

(ii) Incorrect value of y = 0.65.

From graph, correct value of $\frac{x}{y}=2.2$

Estimated correct value of y = 0.68

(iii) From the graph,
s = -1.75 (-1.82 ~ -1.72)
t = 0.3 (0.28 ~ 0.32)

(iv) Draw the line : $\frac{x}{y}=-x+\frac{5}{2}$

From graph, x =0.575 (0.55 ~ 0.60)

and $\frac{x}{y}=1.93(1.92\ \sim\ 1.95)\Rightarrow y=0.30$

8. The equation of a circle C_1, is $x^2 + y^2 - 2x - y - 10 = 0$.

(i) Find the centre and the radius of the circle. **[3]**

(ii) The equation of a tangent to the circle C_1 at the point A is $y + 2x = k$, where $k > 0$.

Find the value of the constant k. **[4]**

A second circle C_2 has its centre at point A and its lowest point B lies on the x-axis.

Find the equation of the circle C_2. **[2]**

(i) $x^2 + y^2 - 2x - y - 10 = 0$

$$(x-1)^2 - 1 + \left(y - \frac{1}{2}\right)^2 - \frac{1}{4} - 10 = 0$$

$$(x-1)^2 + \left(y - \frac{1}{2}\right)^2 = 11\frac{1}{4}$$

$\therefore$ centre of circle = $\left(1, \frac{1}{2}\right)$ and radius = $\frac{\sqrt{45}}{2} = \frac{3\sqrt{5}}{2}$ *units*

(ii) $x^2 + (k-2x)^2 - 2x - (k-2x) - 10 = 0$

$5x^2 - 4kx + k^2 - 2x - k + 2x - 10 = 0$

$5x^2 - 4kx + k^2 - k - 10 = 0$

Since line is a tangent to the circle, Discriminant = 0

$(-4k)^2 - 4(5)(k^2 - k - 10) = 0$

$-4k^2 + 20k + 200 = 0$

$k^2 - 5k - 50 = 0$

$k = 10 \quad or \quad k = -5 \ (NA \because k > 0)$

(iii) When k = 10, $5x^2 - 40x + 80 = 0$

$x^2 - 8x + 16 = 0$

$\therefore x = 4 \quad and \quad y = 2$

$A(4, \ 2)$

Since lowest point lies on x-axis, radius of circle C_2 = 2 units

Equation of circle C_2 : $(x-4)^2 + (y-2)^2 = 4$.

9. **(a)** The curve $y = \frac{2x-5}{1-2x}$ passes through the point A where $x = 1$.

(i) Find the equation of the normal to the curve at the point A. **[4]**

(ii) Find the acute angle the tangent makes with the positive x-axis. **[2]**

(a)(i)

$$y = \frac{2x-5}{1-2x}$$

$$\frac{dy}{dx} = \frac{(1-2x)(2)-(2x-5)(-2)}{(1-2x)^2}$$

$$= \frac{2-4x+4x-10}{(1-2x)^2}$$

$$= \frac{-8}{(1-2x)^2}$$

$$m_{\tan gent} = -8$$

$$m_{normal} = \frac{1}{8}$$

$$y = 3$$

$$y - 3 = \frac{1}{8}(x-1)$$

$$y = \frac{1}{8}x + \frac{23}{8} \quad or\ 8y = x + 23$$

(ii)

$$\tan\theta = 8$$

$$\theta = 82.9^0 \quad \text{or} \quad 1.45 rad$$

9. **(b)** The curve $y = f(x)$ is such that $f''(x) = 3(e^x - e^{-3x})$ and the point $P(0, 2)$ lies on the curve. Given that the gradient of the curve at P is 5, find the equation of the curve. **[6]**

$f'(x) = 3e^x + e^{-3x} + C$, where C is an arbitrary constant.

$f'(0) = 5$

$3e^0 + e^0 + C = 5$
$C = 1$

$\therefore f'(x) = 3e^x + e^{-3x} + 1$

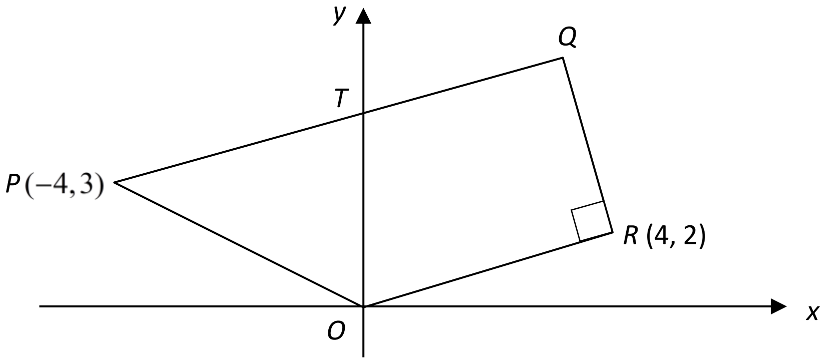
$f(x) = \int (3e^x + e^{-3x} + 1)dx$

$= 3e^x - \dfrac{e^{-3x}}{3} + x + D$, where D is an arbitrary constant.

$f(0) = 2$
$3 - \dfrac{1}{3} + 0 + D = 2$
$D = -\dfrac{2}{3}$

Equation of curve : $y = 3e^x - \dfrac{1}{3e^{3x}} + x - \dfrac{2}{3}$.

10. The diagram (not drawn to scale) shows a trapezium *OPQR* in which *PQ* is parallel to *OR* and $\angle ORQ = 90°$. The coordinates of *P* and *R* are $(-4, 3)$ and (4, 2) respectively and *O* is the origin.



(i) Find the coordinates of *Q*. **[3]**

(ii) *PQ* meets the *y*-axis at *T*. Show that triangle *ORT* is isosceles. **[2]**

(iii) Find the area of the trapezium *OPQR*. **[2]**

(iv) *S* is a point such that *ORPS* forms a parallelogram, find the coordinates of *S*.

[2]

(i) Gradient of *PQ* = gradient of *OR*= 0.5

Eqn of PQ: $y - 3 = \frac{1}{2}(x+4)$

$y = \frac{1}{2}x + 5$ --------(1)

Gradient of QR = -2

Eqn of QR: $y - 2 = -2(x-4)$

$y = -2x + 10$ ------(2)

(1)=(2)

$-2x + 10 = \frac{1}{2}x + 5$

$\frac{5}{2}x = 5$

$x = 2$

$y = -2(2) + 10 = 6$

$\therefore Q(2, 6)$

(ii) In eqn (1), let $x = 0$, $y = 5$, $\therefore OT = 5 units$

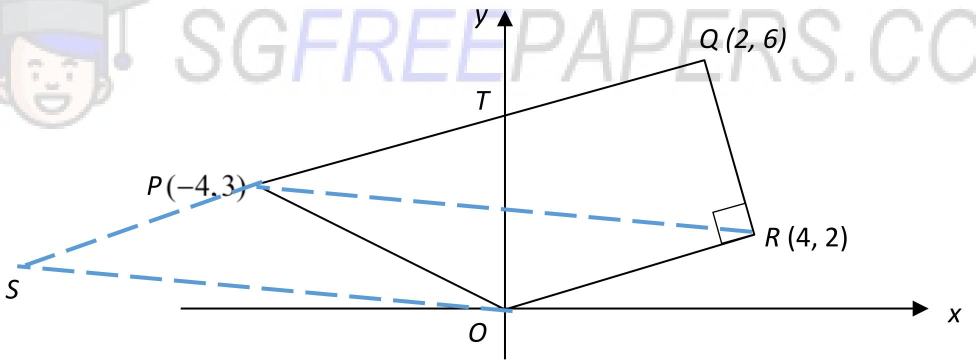
$$RT = \sqrt{(4-0)^2 + (2-5)^2}$$
$$RT = \sqrt{25} = 5$$

Since $OT = RT = 5$ units

$\therefore \Delta ORT$ is isosceles.

Area of trapezium *OPQR*

$$= \frac{1}{2}\begin{vmatrix} 0 & -4 & 2 & 4 & 0 \\ 0 & 3 & 6 & 2 & 0 \end{vmatrix}$$
$$= \frac{1}{2}\left|-24+4-24-6\right|$$
$$= \frac{1}{2}\left|-50\right|$$
$$= 25 units^2$$



(iii) Let $S\ (a, b)$

Midpoint of *RS* = Midpoint of *OP*

$$\left(\frac{a+4}{2}, \frac{b+2}{2}\right) = \left(-\frac{4}{2}, \frac{3}{2}\right)$$
$$a + 4 = -4 \quad \& \quad b + 2 = 3$$
$$a = -8 \qquad\qquad b = 1$$

Hence coordinates of $S\ (-8, 1)$

11. **(a)** Given that $y = x^2\sqrt{2x+1}$, show that $\dfrac{dy}{dx} = \dfrac{x(5x+2)}{\sqrt{2x+1}}$. **[3]**

(a) $y = x^2\sqrt{2x+1}$

$$\frac{dy}{dx} = x^2[\frac{1}{2}(2x+1)^{-\frac{1}{2}}(2)] + 2x(2x+1)^{\frac{1}{2}}$$

$$= x(2x+1)^{-\frac{1}{2}}(x+4x+2)$$

$$= x(5x+2)(2x+1)^{-\frac{1}{2}}$$

$\therefore \dfrac{dy}{dx} = \dfrac{x(5x+2)}{\sqrt{2x+1}}$ (shown)

(b) Hence

(i) find the coordinates of the stationary points on the curve $y = x^2\sqrt{2x+1}$ and determine the nature of these stationary points. **[5]**

(ii) evaluate $\displaystyle\int_0^4 \frac{5x^2+2x-3}{\sqrt{2x+1}}\,dx$. **[4]**

(b)(i) For stationary points, $\dfrac{dy}{dx} = \dfrac{x(5x+2)}{\sqrt{2x+1}} = 0$

$x = 0$ or $x = -\dfrac{2}{5}$

Stationary points are (0, 0) and $(-\dfrac{2}{5}, 0.0716)$

Using 1st derivative test :

x	-0.5	-0.4	-0.3	-0.1	0	0.1
$\dfrac{dy}{dx}$	>0	0	<0	< 0	0	>0
Sketch of tangent	/	—	\	\	—	/

$(-\dfrac{2}{5}, 0.0716)$ is a maximum point and (0, 0) is a minimum point.

(ii) $$\int_1^5 \frac{5x^2+2x-3}{\sqrt{2x+1}}\,dx = \int_1^5 \frac{x(5x+2)}{\sqrt{2x+1}}dx - 3\int_1^5 (2x+1)^{-\frac{1}{2}}dx$$

$$= [x^2\sqrt{2x+1}]_1^5 - 3\left[\sqrt{2x+1}\right]_1^5$$

$$= 76.4$$

Qn	Answer Key	Qn	Answer Key
1(ii)	$\alpha^3+\beta^3=1$	**7(iii)**	From the graph, s = -1.75 (-1.82 ~ -1.72) t = 0.3 (0.28 ~ 0.32)
(iii)	$9x^2-8x+24=0$	**7(iv)**	$\frac{x}{y}=1.93(1.92\ \sim\ 1.95)\Rightarrow y=0.30$
2(a)	Term independent of x in $2x\left(2x-\frac{1}{x^2}\right)^8=-3584$.	**8(i)**	centre of circle = $\left(1,\frac{1}{2}\right)$ radius = $\frac{\sqrt{45}}{2}=\frac{3\sqrt{5}}{2}\,units$
2(b)	$\therefore k=\frac{1}{2}$ and n = 10	**8(ii)**	k = 10
3(i)	$(36+13\sqrt{5})\ cm^2$	**8(iii)**	Equation of circle C_2 : $(x-4)^2+(y-2)^2=4$.
3(ii)	$(78+24\sqrt{5})\ cm^2$	**9(ai)**	$y=\frac{1}{8}x+\frac{23}{8}$ or $8y=x+23$
3(iii)	$\frac{208+25\sqrt{5}}{267}$	**9(aii)**	$\tan\theta=8$ $\theta=82.9^0$ or $1.45rad$
4(i)	$3x-6+\frac{5}{x}-\frac{10}{2x-1}$	**9(b)**	Equation of curve : $y=3e^x-\frac{1}{3e^{3x}}+x-\frac{2}{3}$.
4(ii)	$\frac{3x^2}{2}-6x+5\ln x-5\ln(2x-1)+C$	**10(i)**	$Q(2,6)$
5(i)	$f(x)=2x^4-3x^3+3x^2+5x-3$	**10(ii)**	Since OT = RT = 5 units $\therefore$ ΔORT is isosceles.
5(ii)	$x^2-2x+3=0$ $D=(-2)^2-4(1)(3)=-8<0$ $f(x)$ = 0 has only 2 real roots (Shown)	**10(iii)**	Area of trapezium $OPQR$ $=25units^2$
6(ii)	$14\cos\theta+48\sin\theta=50\cos(\theta-73.74°)$	**10(iv)**	$S(-8,1)$
6(iii)	Since maximum value of P = 50, P can have a value of 48 cm. **Or** $\cos(\theta-73.74°)=\frac{48}{50}<1$, P can have a value of 48 cm.	**11(bi)**	$(-\frac{2}{5},0.0716)$ is a maximum point and (0, 0) is a minimum point.
6(iv)	36.1 cm (3sf)	**11(bii)**	76.4
7 (ii)	Incorrect value of y = 0.65. Estimated correct value of y = 0.68		

Candidate Name ....

Subject 2018 A Math Prelim Exam ..... Paper 2

Question No. 7

Centre Number | Index Number

x	1.00	1.50	2.00	2.50	3.00
$\frac{x}{y}$	2.08	2.31	2.35	2.50	2.65

(i) $x + sy = xyt$

$\frac{x}{y} = tx - s$

Gradient $= t$, $\frac{x}{y}$-int $= -s$

(ii) Incorrect value of $y = 0.65$

Correct value of $\frac{x}{y} = 2.2$

$\therefore$ Correct value of $y = 0.68$

(iii) From the graph, $-s = 1.75$

$\therefore s = -1.75$

Gradient $= t = \frac{2.65 - 2.35}{3 - 2}$

$= 0.3$

(iv) $5y - 2x = 2xy$

$5 - \frac{2x}{y} = 2x$

$\frac{2x}{y} = -2x + 5$

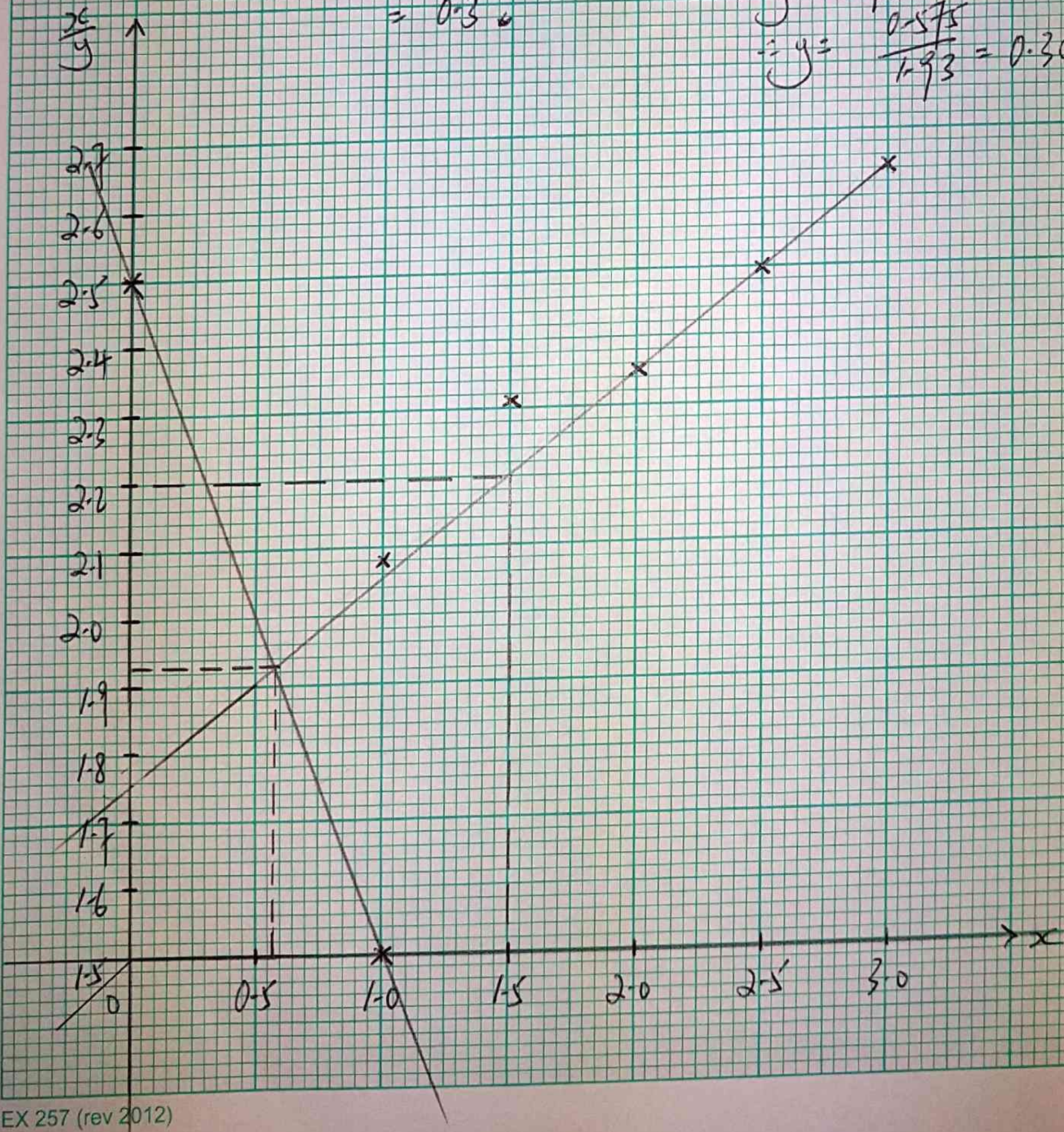
$\frac{x}{y} = -x + \frac{5}{2}$

x	0	1
$\frac{x}{y}$	2.5	1.5

$\therefore x = 0.575$

$\frac{x}{y} = 1.93$

$\therefore y = \frac{0.575}{1.93} = 0.30$



EX 257 (rev 2012)

Name: **Register Number:** **Class:**

南僑中學

NAN CHIAU HIGH SCHOOL

PRELIMINARY EXAMINATION (2) 2018
SECONDARY FOUR EXPRESS

ADDITIONAL MATHEMATICS **4047/01**
Paper 1 **11 September 2018, Tuesday**

Additional Materials : Writing Papers (7 sheets)
Graph Paper (1 sheet)
2 hours

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen on the separate writing papers provided.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
Calculators should be used where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 80.

Setter: Ms Renuka Ramakrishnan

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \cdots + \binom{n}{r} a^{n-r} b^r + \cdots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)...(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$cosec^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

Answer ALL Questions

1 **(i)** On the same axes, sketch the curves $y = \frac{2}{x^2}$ and $y^2 = 128x$. [2]

(ii) Find the coordinates of the point of intersection of the two curves. [2]

2 **(i)** Factorise completely the cubic polynomial $2x^3 - 11x^2 + 12x + 9$. [3]

(ii) Hence, express $\frac{6x^3-33x^2+35x+51}{2x^3-11x^2+12x+9}$ in partial fractions. [5]

3 A quadratic curve passes through $(0, -1)$ and $(2, 7)$. The gradient of the curve at $x = -2$ is -8. Find the equation of the curve. [5]

4 **(i)** Show that $\cos 3\theta - \cos\theta = -2\sin 2\theta \sin\theta$. [3]

(ii) Hence find the values of θ between $0°$ and $360°$ for which $\cos 3\theta - \cos\theta = \sin 2\theta$. [3]

5 The volume of a right square pyramid of length $(3 + \sqrt{2})$ cm is $\frac{1}{3}(29 - 2\sqrt{2})$ cm³. Without using a calculator, find the height of the pyramid in the form $(a + b\sqrt{2})$ cm, where a and b are integers. [5]

6 The roots of the quadratic equation $6x^2 - 5x + 2 = 0$ are $\frac{2}{\alpha}$ and $\frac{2}{\beta}$.

(i) Find the value of $\frac{\alpha}{\alpha+2} + \frac{\beta}{\beta+2}$. [5]

(ii) Find a quadratic equation whose roots are $\frac{\alpha}{\alpha+2}$ and $\frac{\beta}{\beta+2}$. [2]

7 A particle moves in a straight line such that, t seconds after leaving a fixed point O, its velocity, v ms^{-1}, is given by $v = t^2 - 5t + 4$.

(i) Find the acceleration of the particle when it first comes to an instantaneous rest. [3]

(ii) Find the average speed of the particle for the first 5 seconds. [4]

8 The following table shows the experimental values of two variables, x and y, which are related by the equation $y = ab^{x+1}$, where a and b are constants.

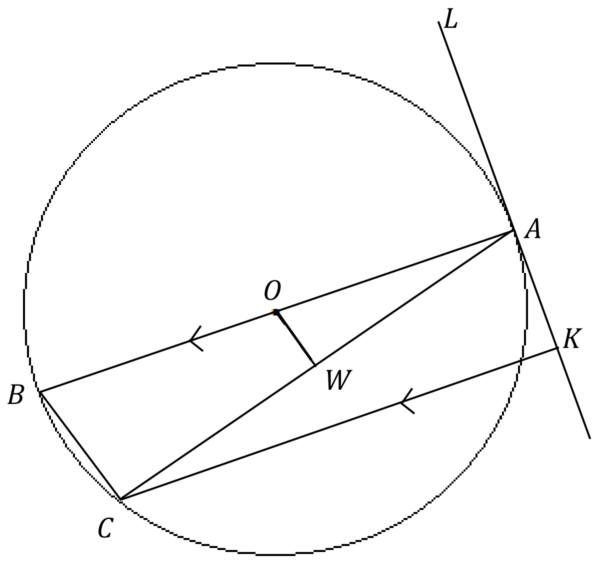
x	1	2	3	4
y	10.12	10.23	10.35	10.47

(i) On graph paper, plot $\lg y$ against x and draw a straight line graph. The vertical $\lg y$ – axis should start from 0.995 and have a scale of 4 cm to 0.005. [3]

(ii) Use your graph to estimate the value of a and of b. [4]

(iii) Explain how the value of a and of b will change if a graph of $\ln y$ against x was plotted instead. [1]

9



In the diagram, *A*, *B* and *C* are three points on the circle such that *AB* is the diameter of the circle and *W* is the midpoint of *AC*. *AB* and *CK* are parallel to each other and *KL* is a tangent to the circle at *A*.

(i) Prove that *OW* is parallel to *BC*. [2]

(ii) Prove that Angle AWO = Angle AKC. [3]

10 Diagram I shows a right angled ΔABC, with hypotenuse AB of length 4 m. This triangle is revolved around BC to generate a right circular cone as shown in Diagram II.

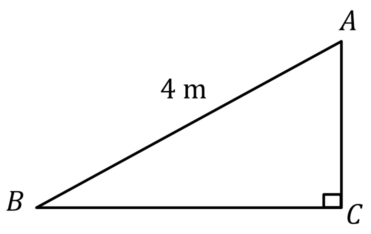


Diagram I

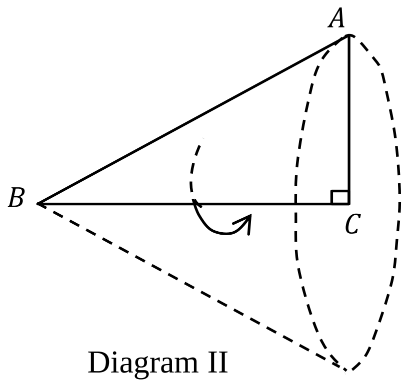


Diagram II

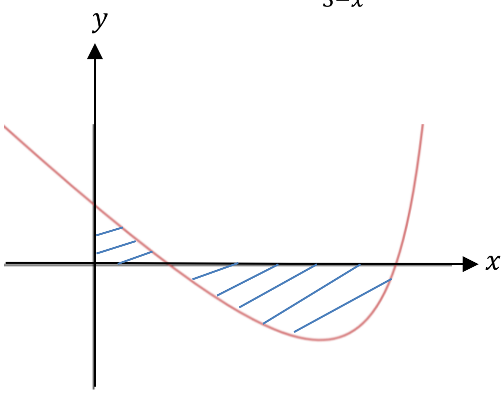
(i) Find the **exact** height that gives the maximum volume of the cone. [6]

(ii) Show that this maximum volume is obtained when $BC : CA = 1 : \sqrt{2}$. [2]

11 The equation of a curve is $y = \frac{4-5x+x^2}{5-x}$, $x \neq 5$.

(i) Find the set of values of x for which y is an increasing function of x. [3]

(ii) The diagram below shows part of the curve $y = \frac{4-5x+x^2}{5-x}$, $x \neq 5$.



By expressing $\frac{4-5x+x^2}{5-x}$ in the form $ax + \frac{b}{5-x}$, where a and b are constants, find the total area of the shaded regions. [5]

12 A circle C_1, with centre C, passes through four points A, B, F and G. The coordinates of A and B are (0, 4) and (8, 0) respectively. The equation of the normal to the circle at F is $y = -\frac{4}{3}x + 4$.

(i) Show that the coordinates of C is (3, 0). [5]

(ii) Hence find the equation of the circle. [2]

Another circle C_2 passes through the points C, F and G.

(iii) Given that GF is the diameter of the circle, calculate the radius of C_2. [2]

- End of Paper -

Answer Key

1ii) $\left(\frac{1}{2}, 8\right)$

2i) $(x-3)^2(2x+1)$

2ii) $3 + \frac{2}{2x+1} - \frac{1}{x-3} + \frac{3}{(x-3)^2}$

3) $y = 2x^2 - 1$

4ii) $\theta = 90°, 180°, 210°, 270°, 330°$

5) $(7 - 4\sqrt{2})cm$

6i) $\frac{17}{13}$

6ii) $x^2 - \frac{17}{13}x + \frac{6}{13} = 0$

7i) $-\frac{3m}{s^2}$

7ii) 1.63 m/s

8ii) $b = 1.01$, $a = 9.89$

8iii) remain unchanged

10i) $\frac{4\sqrt{3}}{3}$ cm

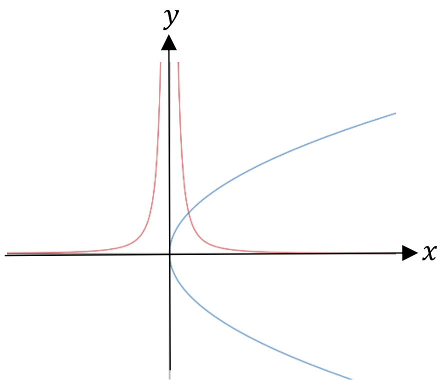
11i) $3 < x < 7, x \neq 5$

11ii) 2.35 $units^2$

12ii) $(x-3)^2 + y^2 = 25$

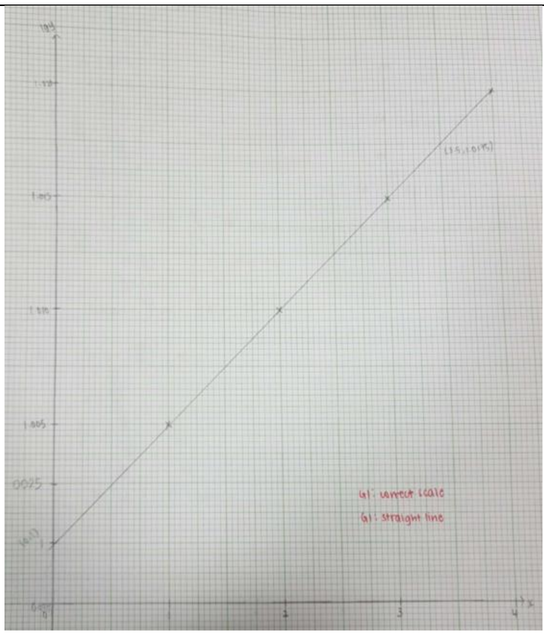
12iii) 3.54 $units$

NCHS 2018 Prelim 2 AM Paper 1 Solutions

1i		G1: graph of $y = \frac{2}{x^2}$ G1: graph of $y^2 = 128x$
1ii	$\left(\frac{2}{x^2}\right)^2 = 128x$ $x^5 = \frac{4}{128}$ $x = \frac{1}{2}$ $\rightarrow y = 8$ $\left(\frac{1}{2}, 8\right)$	M1: Equating both functions A1: award only if written as coordinates
2i	Let $f(x) = 2x^3 - 11x^2 + 12x + 9$ $f(3) = 0$ $\therefore (x-3)$ is a factor of $f(x)$. $f(x) = (x-3)(2x^2 - 5x - 3)$ $= (x-3)^2(2x+1)$	M1: show 1st factor using factor theorem M1: Find quadratic factor (by long division/synthetic method) A1
2ii	$\frac{6x^3 - 33x^2 + 35x + 51}{2x^3 - 11x^2 + 12x + 9} = 3 + \frac{-x+24}{(x-3)^2(2x+1)}$ $\frac{-x+24}{(x-3)^2(2x+1)} = \frac{A}{2x+1} + \frac{B}{x-3} + \frac{C}{(x-3)^2}$ $-x + 24 = A(x-3)^2 + B(x-3)(2x+1) + C(2x+1)$ ➔ Using substitution/comparing coefficient $A = 2, B = -1, C = 3$ $\therefore \frac{6x^3 - 33x^2 + 35x + 51}{2x^3 - 11x^2 + 12x + 9}$ $= 3 + \frac{2}{2x+1} - \frac{1}{x-3} + \frac{3}{(x-3)^2}$	M1: Change to proper fraction M1: Split correctly to respective partial fractions M2: Using substitution/comparing coefficient to find A, B and C A1

3	$y = ax^2 + bx + c$ When $x = 0, y = -1 \rightarrow c = -1$ $\frac{dy}{dx} = 2ax + b$ $When\ x = -2, \quad \frac{dy}{dx} = -8$ $-4a + b = -8$ $b = 4a - b$ --- (1) Sub $y = 7$ and $x = 2$ into $y = ax^2 + bx - 1$ $7 = 4a + 2b - 1$ $b = 4 - 2a$ --- (2) From (1) and (2), $4a - 8 = 4 - 2a$ $a = 2$ $\rightarrow b = 0$ $\therefore$ equation of curve:$y = 2x^2 - 1$	B1: writing quad eqn in a general form A1: Solving for c M1: differentiate quad function M1: forming 2 simultaneous equations and solving it A1
4i	$\cos 3\theta - \cos\theta$ $= \cos(2\theta + \theta) - \cos\theta$ $= \cos 2\theta \cos\theta - \sin 2\theta \sin\theta - \cos\theta$ $= \cos\theta\,(\cos 2\theta - 1) - \sin 2\theta \sin\theta$ $= \cos 2\theta\,(-2\sin^2\theta) - \sin 2\theta \sin\theta$ $= -2\sin^2\theta \cos 2\theta - \sin 2\theta \sin\theta$ $= (-2\sin\theta\cos\theta)\sin\theta - \sin 2\theta \sin\theta$ $= -\sin 2\theta \sin\theta - \sin 2\theta \sin\theta$ $= \ -2\sin 2\theta \sin\theta$	M1: applying addition formula to $\cos(2\theta + \theta)$ M1: changing $\cos 2\theta - 1 = -2\sin^2\theta$ A1: changing $-2\sin^2\theta\cos 2\theta = -\sin 2\theta \sin\theta$
4ii	$\cos 3\theta - \cos\theta \ = \sin 2\theta$ $\sin 2\theta + 2\sin 2\theta \sin\theta = 0$ $\sin 2\theta(1 + 2\sin\theta) = 0$ $\sin 2\theta = 0 \quad or \quad \sin\theta = -0.5$ $2\theta = 180, 360, 540 \quad or \quad \theta = 210, 330$ $\theta = 90, 180, 270$ $\therefore \theta = 90°, 180°, 210°, 270°, 330°$	M1 apply hence + factorization A1 solve $\sin 2\theta = 0$ correctly A1 solve $\sin\theta = -0.5$ correctly
5	$\frac{1}{3}(3 + \sqrt{2})^2 h = \frac{1}{3}(29 - 2\sqrt{2})$ $h = \frac{29 - 2\sqrt{2}}{11 + 6\sqrt{2}}$ $= \frac{(29 - 2\sqrt{2})(11 - 6\sqrt{2})}{49}$ $= \frac{319 - 174\sqrt{2} - 22\sqrt{2} + 24}{49}$ $= \frac{343 - 196\sqrt{2}}{49}$ $= (7 - 4\sqrt{2})cm$	M1: forming an equation M1: Calculating $(3 + \sqrt{2})^2$ M1: Rationalising denominator M1: Simplifying after expansion A1

6i	$\frac{2}{\alpha}+\frac{2}{\beta}=\frac{5}{6}$ $\frac{2(\alpha+\beta)}{\alpha\beta}=\frac{5}{6}$ --- (1) $\left(\frac{2}{\alpha}\right)\left(\frac{2}{\beta}\right)=\frac{1}{3}$ $\alpha\beta=12$ --- (2) Sub (2) into (1) $\alpha+\beta=5$ $\frac{\alpha}{\alpha+2}+\frac{\beta}{\beta+2}=\frac{\alpha(\beta+2)+\beta(\alpha+2)}{(\alpha+2)(\beta+2)}$ $=\frac{2\alpha\beta+2(\alpha+\beta)}{\alpha\beta+2(\alpha+\beta)+4}$ $=\frac{17}{13}$	M1: applying concept of sum and product of roots A1: $\alpha\beta$ A1: $\alpha+\beta$ M1 A1
6ii	$\left(\frac{\alpha}{\alpha+2}\right)\left(\frac{\beta}{\beta+2}\right)=\frac{\alpha\beta}{(\alpha+2)(\beta+2)}$ $=\frac{6}{13}$ $\therefore$ Equation : $x^2-\frac{17}{13}x+\frac{6}{13}=0$ $or\ 13x^2-17x+6=0$	M1 A1
7i	When v = 0, $t^2-5t+4=0$ $(t-4)(t-1)=0$ $t=1\quad or\quad t=4$ $a=\frac{dv}{dt}$ $=2t-5$ When $t=1, a=-3m/s^2$	M1 A1: Differentiate correctly A1
7ii	$s=\int t^2-5t+4\,dt$ $=\frac{1}{3}t^3-\frac{5}{2}t^2+4t+c$ When $t=0, s=0\rightarrow c=0$ $\therefore s=\frac{1}{3}t^3-\frac{5}{2}t^2+4t$ When $t=1, s=\frac{11}{6}m$ $t=4, s=-\frac{8}{3}m$ $t=5, s=\ -\frac{5}{6}m$	A1: integrate correctly (look out for +c, unless definite integral)

<table>
<tr><td></td><td>$\therefore Average\ Speed = \dfrac{\frac{11}{6} + \left(\frac{11}{6} + \frac{8}{3}\right) + \left(\frac{8}{3} - \frac{5}{6}\right)}{5}$
$= 1\frac{19}{30} m/s$ or 1.63 m/s</td><td>M1: Calculating distance in 5th sec
M1: formula for average speed
A1</td></tr>
<tr><td>8</td><td>
$y = ab^{x+1}$
$lgy = (x+1)lgb + lga$
$lgy = xlgb + (lgb + lga)$
<table><tr><td>x</td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>lg y</td><td>1.005</td><td>1.010</td><td>1.015</td><td>1.020</td></tr></table></td><td>G1: correct scale
G1: straight line

M1: table of values</td></tr>
<tr><td>8ii</td><td>$\lg b = \dfrac{1.0175 - 1}{3.5}$
$= 0.005$
$b = 1.01\ (3sf)$

$\lg b + \lg a = 1$
$\lg a = 1 - 0.005$
$a = 9.89\ (3sf)$</td><td>M1

A1

M1

A1</td></tr>
<tr><td>8iii</td><td>The values of a and b will remain unchanged.</td><td>B1</td></tr>
<tr><td>9i</td><td>O is the midpoint of AB and W is the midpoint of AC,
By Midpoint Theorem, BC parallel to OW.</td><td>M1
A1</td></tr>
<tr><td>9ii</td><td>$Angle\ AOW = Angle\ ABC\ (corr\ angles, OW//BC)$
$Angle\ ABC = Angle\ CAK (alt\ segment\ theorem)$
$\rightarrow Angle\ AOW = Angle\ CAK$
$Angle\ BAC = Angle\ ACK\ (alt\ angles, AB//CK)$
$\therefore Angle\ AWO$
$= 180° - Angle\ BAC - Angle\ AOW\ (Angle\ sum\ of\ \Delta)$
$= 180° - Angle\ ACK - Angle\ CAK$
$= Angle\ AKC$ (shown)</td><td>M1
M1

A1</td></tr>
</table>

10i	Let $AC = r$ and $BC = h$ $r^2 = 16 - h^2$ $V = \frac{1}{3}\pi r^2 h$ $= \frac{1}{3}\pi(16 - h^2)h$ $= \frac{16}{3}\pi h - \frac{1}{3}\pi h^3$ $\frac{dV}{dh} = \frac{16}{3}\pi - \pi h^2$ $When \frac{dV}{dh} = 0,$ $\frac{16}{3}\pi = \pi h^2$ $h = \frac{4}{\sqrt{3}} \left(rej\ h = -\frac{4}{\sqrt{3}}\ since\ h > 0\right)$ $\frac{d^2V}{dh^2} = -2\pi h$ $= -\frac{8}{\sqrt{3}}\pi\ (< 0)$ $\therefore h = \frac{4}{\sqrt{3}} = \frac{4\sqrt{3}}{3}\ cm$	M1: Finding r/s between h and r M1: finding V in terms of one variable M1: differentiation M1: Stationary point M1: Prove Max A1
10ii	$r^2 = 16 - \left(\frac{4}{\sqrt{3}}\right)^2$ $r = \frac{4\sqrt{2}}{\sqrt{3}}$ $\frac{h}{r} = \frac{\frac{4}{\sqrt{3}}}{\frac{4\sqrt{2}}{\sqrt{3}}}$ $\frac{h}{r} = \frac{1}{\sqrt{2}}$ $\therefore BC: CA = 1: \sqrt{2}$	M1 A1
11i	$\frac{dy}{dx} = \frac{(2x-5)(5-x) - (4-5x+x^2)(-1)}{(5-x)^2}$ $= \frac{10x - 2x^2 + 5x - 25 + 4 - 5x + x^2}{(5-x)^2}$ $= \frac{-x^2 + 10x - 21}{(5-x)^2}$ Since $(5-x)^2 > 0$, for y to be an increasing function, $-x^2 + 10x - 21 > 0$ $x^2 - 10x + 21 < 0$ $(x-3)(x-7) < 0$ $3 < x < 7, x \neq 5$	M1: Applying quotient rule M1 A1

11ii	When $y = 0$ $4 - 5x + x^2 = 0$ $(x-4)(x-1) = 0$ $x = 4 \; or \; x = 1$ $\frac{x^2 - 5x + 4}{5 - x} = \frac{-x(5-x) + 4}{(5-x)}$ $= -x + \frac{4}{5-x}$ Area of shaded region $= \int_0^1 -x + \frac{4}{5-x}\,dx + \left\lvert \int_1^4 -x + \frac{4}{5-x}\,dx \right\rvert$ $= [-0.5x^2 - 4\ln(5-x)]_0^1 + \lvert[-0.5x^2 - 4\ln(5-x)]_1^4\rvert$ $= 0.39257 + \lvert -1.95482 \rvert$ $= 2.35 \; units^2 \; (3sf)$	M1 M1, M1 M1: correct integration A1
12i	Gradient of AB $= -\frac{1}{2}$ Midpoint of AB = (4,2) Eqn of perpendicular bisector of AB: $y - 2 = 2(x - 4)$ $y = 2x - 6$ Sub $y = 2x - 6$ into $y = -\frac{4}{3} + 4$, $2x - 6 = -\frac{4}{3} + 4$ $x = 3$ $\rightarrow y = 0$ $C\;(3, 0)$	M1: Midpoint M1: gradient =2 M1: forming equation M1: Solving simultaneous A1
12ii	Radius $= \sqrt{(3-0)^2 + (0-4)^2}$ $= 5 \; units$ Equation of circle: $(x-3)^2 + y^2 = 25$ Or $x^2 + y^2 - 6x - 16 = 0$	M1: Finding radius A1
12iii	$Angle\ GCF = 90°\ (Angle\ in\ Semicircle)$ $GF^2 = 5^2 + 5^2$ $GF = \sqrt{50}$ $Radius\ of\ C_2 = \frac{1}{2}\sqrt{50}$ $= \frac{5}{2}\sqrt{2}\ units$ $or \quad = 3.54\ units\ (3sf)$	M1 A1

END ☺

Name: **Register Number:** **Class:**

南僑中學

NAN CHIAU HIGH SCHOOL

PRELIMINARY EXAMINATION (2) 2018
SECONDARY FOUR EXPRESS

ADDITIONAL MATHEMATICS **4047/02**
Paper 2 **12 September 2018, Wednesday**

Additional Materials : Writing Paper (8 sheets) **2 hours 30 minutes**

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen on the separate writing papers provided.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
Calculators should be used where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 100.

Setter: Mdm Chua Seow Ling

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \cdots + \binom{n}{r} a^{n-r} b^r + \cdots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)...(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

Answer ALL Questions.

1. **(i)** Given $\frac{3\lg 3x - 2\lg x}{4} = \lg 3$, find the value of x. [3]

(ii) Given $\log_{(x-2)} y = 2$ and $log_y(x + k) = \frac{1}{2}$, find the value of k if k is an integer. [3]

2. **(i)** Show that $\frac{\mathrm{d}}{\mathrm{d}x}\left[ln\left(\frac{\sin x}{1-\cos x}\right)\right] = -\frac{1}{\sin x}$. [4]

(ii) Hence evaluate $\int \sin^2 x + \frac{2}{\sin x}\,\mathrm{d}x$. [4]

3. It is given that $y_1 = -2\cos x + 1$ and $y_2 = \sin\frac{1}{2}x$.

For the interval $0 < x < 2\pi$,

(i) state the amplitude and period of y_1 and of y_2, [2]

(ii) sketch, on the same diagram, the graphs of of y_1 and y_2, [4]

(iii) find the x-coordinate of the points of intersection of the two graphs drawn in **(ii)**, [3]

(iv) hence, find the range of values of x for which $y_1 \leq y_2$. [2]

4. Some liquid is poured onto a flat surface and formed a circular patch. This circular patch is left to dry and its surface area decreases at a constant rate of 4 cm^2/s. The patch remains circular during the drying process. Find the rate of change of the circumference of the circular patch at the instant when the area of the patch is 400 cm^2. [4]

5. **(i)** In the expansion of $\left(2 + \frac{4}{x^4}\right)\left(kx^3 - \frac{2}{x}\right)^{13}$ where k is a constant and $k \neq 0$, find the value of k if there is no coefficient of $\frac{1}{x}$. [5]

(ii) Given the coefficients of $\frac{1}{x}$ and $\frac{1}{x^2}$ in the expansion of $\left(1 - \frac{c}{x}\right)^n$ are -80 and 3000 respectively. Find the value of c and of n where n is a positive integer greater than 2 and c is a constant. [5]

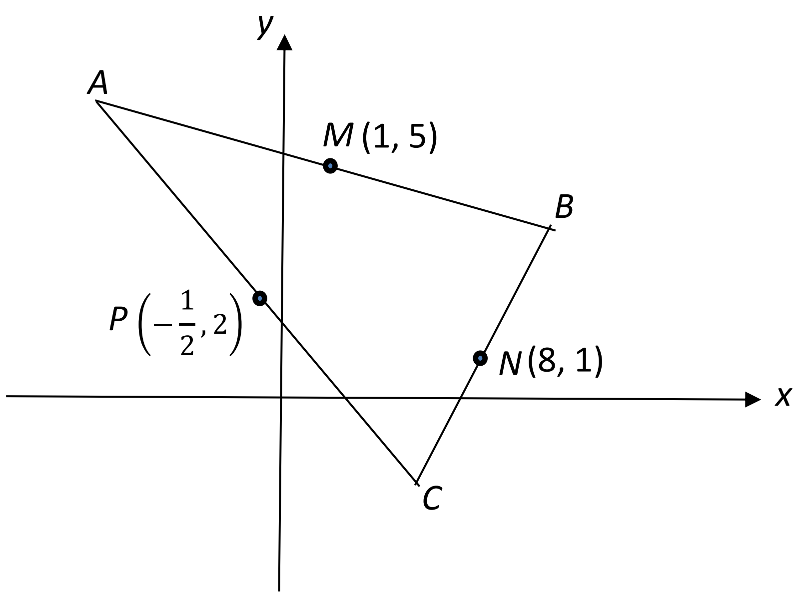
6. Curve A is such that $\frac{dy}{dx} = 27(2x-1)^2$ and curve B is such that $\frac{dy}{dx} = -27(2x-1)^3$, and the y-coordinates of the stationary points for both curves are -4.

(i) Find the coordinates of the stationary points for curve *A* and *B*. [2]

(ii) Determine the nature of the stationary points for curve *A* and *B*. [4]

(iii) Find the equations of curve *A* and *B*. [4]

7. The diagram shows a triangle *ABC*. The mid-points of the sides of the triangle are $M(1, 5)$, $N(8, 1)$ and $P\left(-\frac{1}{2}, 2\right)$.



(i) State and explain which line is parallel to *AB*. [1]

(ii) Find the equation of the line *AB*. [3]

(iii) Find the equation of the line *AC*. [3]

(iv) Show the coordinates of *A* is $\left(-7\frac{1}{2}, 6\right)$. [3]

(v) Find the area of the quadrilateral *AMNP*. [2]

8. Diagram I and II show two types of isosceles triangular cards, ΔOAB with $\angle AOB = \theta$, $OA = OB = 3$ cm and ΔOMN with $OM = ON = 2$ cm. These two types of cards are connected as shown in diagram III where $\angle AON = 120^\circ$.

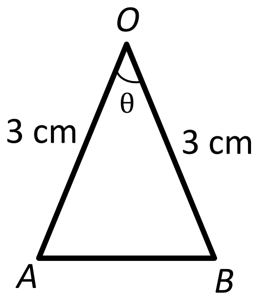


Diagram I

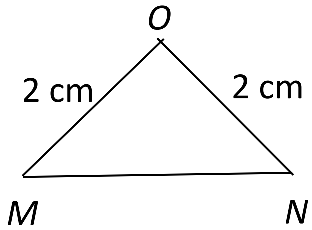


Diagram II

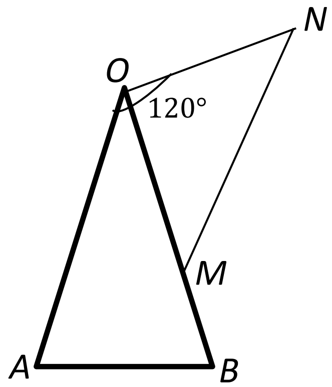


Diagram III

Three sets of cards from diagram III are connected as shown in diagram IV.

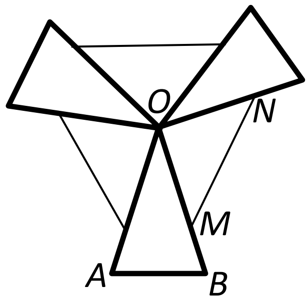


Diagram IV

(i) Show that the area of all the connected cards in diagram IV, A cm^2 is given by $A = \frac{33}{2}\sin\theta + 3\sqrt{3}\cos\theta$. [3]

(ii) Express A in the form $A = R\cos(\theta - \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. [3]

(iii) Find the value of θ for which $A = 15$, where $0^\circ < \theta < 90^\circ$. [3]

(iv) Find the maximum value of A and the corresponding value of θ. [2]

9. In an experiment to study the growth of a certain type of bacteria, the bacteria are injected into a mouse and the mouse's blood samples are collected at various time interval for testing. The blood test result shows that the population, P, of the bacteria is related to the time, t hours, after the injection, by the equation $P = 550 + 200e^{kt}$, where k is a constant. It takes **one day** for the population of bacteria to double.

(i) Find the population of the bacteria at the start of the experiment. [1]

(ii) Find the value of k. [2]

(iii) Find the percentage increase of the population of the bacteria when $t = 30$. [4]

(iv) The line $P = mt + c$ is a tangent to the curve $P = 550 + 200e^{kt}$ at the point where $t = 30$. Find the constant value of m and of c. [3]

(v) At $t = 50$, an antibiotics dosage is injected into the same mouse to stop the growth of bacteria. The dosage is able to kill the bacteria at a constant rate of 25 bacteria per hour. How much time needed for the dosage fully take its effectiveness? Hence sketch the graph of P against t for the whole experiment. [4]

10. A curve has the equation of $y = p(x-2)^2 - (x-3)(x+2)$ where p is a constant and $p \neq 1$.

(i) Find the range of values of p for which curve has a minimum point. [2]

Given that the curve touches the x-axis at point A.

(ii) Show that $p = \frac{25}{16}$. [3]

(iii) Find the coordinates of point A. [4]

(iv) Given that the line $y = mx + 2$ intersects the curve $y = p(x-2)^2 - (x-3)(x+2)$ at two distinct points where one of the points is at point A. Another line of the equation $y = mx + c$, is a tangent to the same curve at point B. Find the value of c where m and c are constants. [5]

End of Paper

Answers

1i) $x = 3$,

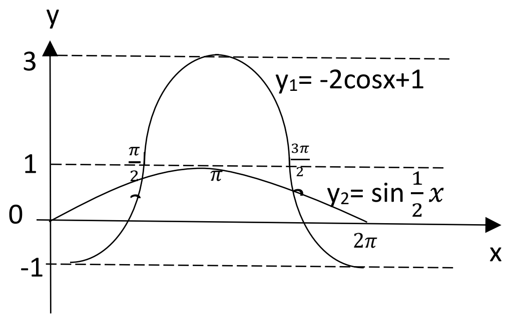
1ii) $k = -2$

2ii) $\frac{1}{2}x - \frac{1}{4}sin2x - 2ln\left(\frac{sinx}{1-cosx}\right) + c$

3ii)

Amplitude of y_1 = 2, Period of y_1 = 2π

Amplitude of y_2 = 1, Period of y_2 = 4π



3iii) x = 1.39, 4.89

3iv) $0 < x \leq 1.39$ Or $4.89 \leq x < 2\pi$

4) - k = $\frac{2}{5}$

5i) k = $\frac{2}{5}$

5ii) c = 5, n = 16

6i) $\left(\frac{1}{2}, -4\right)$

6ii) $\left(\frac{1}{2}, -4\right)$ *is point of inflexion − Curve A*, $\left(\frac{1}{2}, -4\right)$ *is a* max *stationary point − curve B*

6iii) $y_A = \frac{9}{2}(2x-1)^3 - 4$, $y_B = -\frac{27}{8}(2x-1)^4 - 4$

7i) Line PN is parallel to AB (Mid-Point Theorem)

7ii) $y = -\frac{2}{17}x + 5\frac{2}{17}$

7iii) $y = -\frac{4}{7}x + \frac{12}{7}$

7v) A = 27

8ii) $A = \frac{3\sqrt{133}}{2}\cos(\theta - 72.5°)$ $= 17.3\cos(\theta - 72.5°)$

8iii) 42.6°

8iv) Max A = $\frac{3\sqrt{133}}{2}$ =17.3, $\theta = 72.5°$

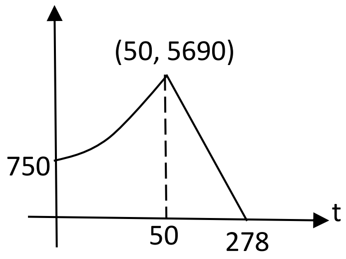
9i)750

9ii) k = $\frac{1}{24}\ln\frac{19}{4}$ = 0.0649

9iii) 160 %

9iv) m =91.1, c = -779

9v)



10i) p >1

10iii) $A\left(\frac{14}{3}, 0\right)$

10iv) c = $\frac{94}{49}$

2018 NCHS A-Math Prelim 2/ Paper 2 solution

(1i)

$$\frac{3\lg 3x - 2\lg x}{4} = \lg 3$$
3lg 3x -2lg x = 4lg3
$$\lg(3x)^3 - lgx^2 = lg3^4$$
$$\frac{(3x)^3}{x^2} = 3^4$$
$$27x = 81$$
x = 3

(1ii)

$$\log_{(x-2)}y = 2$$
$$y = (x-2)^2$$
$$y^{\frac{1}{2}} = x - 2$$
$$log_y(x+k) = \frac{1}{2}$$
$$x + k = y^{\frac{1}{2}}$$
x+ k = x-2
k = -2

$$\frac{1}{log_{(x-2)}y} = \frac{1}{2}$$
$$\log_y(x-2) = \frac{1}{2}$$
$$log_y(x+k) = \frac{1}{2}$$
k = -2

(2i)

$$\left[ln\left(\frac{\sin x}{1-\cos x}\right)\right]$$
$$= lnsinx - ln(1-cosx)$$
$$\frac{d}{dx}\left[ln\left(\frac{\sin x}{1-\cos x}\right)\right] =$$
$$\frac{d}{dx}[lnsinx - ln(1-cosx)]$$
$$= \frac{cosx}{sinx} - \frac{-(-sinx)}{1-cosx}$$
$$= \frac{cosx}{sinx} - \frac{sinx}{1-cosx}$$
$$= \frac{cosx(1-cosx)-sin^2x}{sinx(1-cosx)}$$
$$= \frac{cosx-cos^2x-sin^2x}{sinx(1-cosx)}$$
$$= \frac{cosx-1}{sinx(1-cosx)}$$
$= -\frac{1}{\sin x}$ (shown)

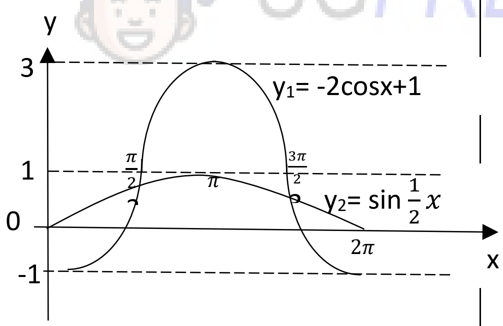
(2ii)

$$\int \sin^2 x + \frac{2}{\sin x}dx$$
$$= \int \frac{1-cos2x}{2} + \frac{2}{sinx}dx$$
$$= \frac{1}{2}x - \frac{1}{4}sin2x - 2ln\left(\frac{sinx}{1-cosx}\right) + c$$

(3i)

Amplitude of y_1 = 2, Period of y_1 = 2π
Amplitude of y_2 = 1, Period of y_2 = 4π

(3ii)



(3iii)

$$-2\cos x + 1 = \sin\frac{1}{2}x$$
$$-2\left(1 - 2sin^2\frac{x}{2}\right) + 1 = sin\frac{1}{2}x$$
$$4sin^2\frac{x}{2} - sin\frac{1}{2}x - 1 = 0$$
$$sin\frac{1}{2}x = 0.6403882$$
$$\alpha = 0.69500$$
$$\frac{1}{2}x = 0.69500, \quad \pi - 0.69500$$
= 0.695 or 2.4466
x = 1.39, 4.89

(3iv)

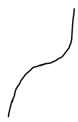
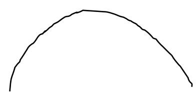
$0 < x \leq 1.39$ Or $4.89 \leq x < 2\pi$

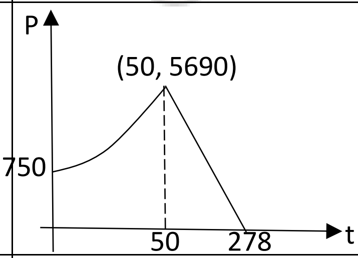
(4)

$$A = \pi r^2$$
$$\frac{dA}{dr} = 2\pi r$$
$$C = 2\pi r$$
$$\frac{dC}{dr} = 2\pi$$
$$\pi r^2 = 400$$
$$r = \frac{20}{\sqrt{\pi}}$$
$$\frac{dA}{dr} \times \frac{dr}{dt} = \frac{dA}{dt}$$
$$2\pi r \times \frac{dr}{dt} = -4$$

(4)

$$\frac{dr}{dt} = \frac{-4}{2\pi r}$$
$$\frac{dC}{dt} = \frac{dC}{dr} \times \frac{dr}{dt}$$
$$\frac{dC}{dt} = 2\pi \times \frac{-4}{2\pi r}$$
$$= \frac{-4}{r}$$
$$= \frac{-4}{\left(\frac{20}{\sqrt{\pi}}\right)}$$
$= -\frac{\sqrt{\pi}}{5}$ cm/s
= - 0.354 cm/s

(5i)	$\left(2+\frac{4}{x^4}\right)\left(kx^3-\frac{2}{x}\right)^{13}$ $=\left(2+\frac{4}{x^4}\right)\left(\frac{1}{x},\quad x^3\right)$ $\left(kx^3-\frac{2}{x}\right)^{13}=\binom{13}{r}(kx^3)^{13-r}\left(-\frac{2}{x}\right)^r+\cdots$ $=\binom{13}{9}(kx^3)^4\left(-\frac{2}{x}\right)^9+\binom{13}{10}(kx^3)^3\left(-\frac{2}{x}\right)^{10}+\cdots$ $=715k^4x^{12}\left(-\frac{512}{x^9}\right)+286k^3x^9\left(\frac{1024}{x^{10}}\right)+\cdots$ $=-366080k^4x^3+\frac{292864}{x}k^3+\cdots$ $=\left(2+\frac{4}{x^4}\right)\left(-366080k^4x^3+\frac{292864}{x}k^3+\cdots\right)$ $=\frac{585728k^3}{x}-\frac{1464320}{x}k^4+\cdots$ $585728k^3-1464320k^4=0$ $k^3(585728-1464320k)=0$ k = $\frac{2}{5}$	(5ii)	$\left(1-\frac{c}{x}\right)^n=\binom{n}{1}\left(-\frac{c}{x}\right)+\binom{n}{2}\left(-\frac{c}{x}\right)^2+\cdots$ $=-\frac{nc}{x}+\frac{n(n-1)}{2}.\frac{c^2}{x^2}+\cdots$ - nc = - 80 nc = 80 $\frac{n(n-1)c^2}{2}=3000$ $n^2c^2-nc^2=6000$ $80^2-80c=6000$ c = 5 , n =16
(6i)	Curve A --- $\frac{dy}{dx}=27(2x-1)^2$ $27(2x-1)^2=0$ $x=\frac{1}{2}$ $\left(\frac{1}{2},-4\right)$	(6i)	Curve B --- $\frac{dy}{dx}=-27(2x-1)^3$ $-27(2x-1)^3=0$ $x=\frac{1}{2}$ $\left(\frac{1}{2},-4\right)$
(6ii)	Curve A --- x = 0.4, x = 0.5, x = 0.6 $\frac{dy}{dx}>0$ $\frac{dy}{dx}=0$ $\frac{dy}{dx}>0$  $\left(\frac{1}{2},-4\right)$ *is point of inflexion*	(6ii)	Curve A --- x = 0.4, x = 0.5, x = 0.6 $\frac{dy}{dx}>0$ $\frac{dy}{dx}=0$ $\frac{dy}{dx}<0$  $\left(\frac{1}{2},-4\right)$ *is a maximum stationary point*
(6iii)	$y_A=\frac{27(2x-1)^3}{3(2)}+c$ c = -4 $y_A=\frac{9}{2}(2x-1)^3-4$	(6iii)	$y_B=\frac{-27(2x-1)^4}{4(2)}+c$ c = -4 $y_B=-\frac{27}{8}(2x-1)^4-4$
(7i)	Line PN is parallel to AB (Mid-Point Theorem)		
(7ii)	$m_{PN}=\frac{2-1}{-\frac{1}{2}-8}=-\frac{2}{17}$, $m_{AB}=-\frac{2}{17}$ $y=-\frac{2}{17}x+c_1$ $5=-\frac{2}{17}(1)+c_1$ $c_1=5\frac{2}{17}$ $y=-\frac{2}{17}x+5\frac{2}{17}$	(7iii)	$m_{MN}=\frac{5-1}{1-8}=-\frac{4}{7}$, $m_{AC}=-\frac{4}{7}$ $y=-\frac{4}{7}x+c_2$ $2=-\frac{4}{7}(-\frac{1}{2})+c_2$ $c_2=\frac{12}{7}$ $y=-\frac{4}{7}x+\frac{12}{7}$
(7iv)	$-\frac{2}{17}x+5\frac{2}{17}=-\frac{4}{7}x+\frac{12}{7}$ $x=-7\frac{1}{2}$ $y=-\frac{4}{7}\left(-\frac{15}{2}\right)+\frac{12}{7}$ $y=6$ $\left(-7\frac{1}{2},6\right)$ Shown	(7v)	A = $\frac{1}{2}\begin{vmatrix}-\frac{15}{2} & -\frac{1}{2} & 8 & 1 & -\frac{15}{2} \\ 6 & 2 & 1 & 5 & 6\end{vmatrix}$ $=\frac{1}{2}\left(\frac{61}{2}-\left(-\frac{47}{2}\right)\right)$ = 27

(8i)	$A = 3\left[\frac{1}{2}(3)(3)sin\theta + \frac{1}{2}(2)(2)\sin(120-\theta)\right]$ $=3\left[\frac{9}{2}sin\theta + (2)(sin120cos\theta - sin\theta cos120)\right]$ $= 3\left[\frac{9}{2}sin\theta + 2(\frac{\sqrt{3}}{2}cos\theta - sin\theta(-\frac{1}{2}))\right]$ $= 3\left[\frac{9}{2}sin\theta + \sqrt{3}cos\theta + sin\theta)\right]$ $= \frac{33}{2}\sin\theta + 3\sqrt{3}\cos\theta$ (shown)	(8ii)	R = $\sqrt{\left(\frac{33}{2}\right)^2 + (3\sqrt{3})^2} = \sqrt{\frac{1197}{4}} = \frac{3\sqrt{133}}{2}$ $\tan\alpha = \frac{\frac{33}{2}}{3\sqrt{3}}$ $\alpha = 72.5198$ $A = \frac{3\sqrt{133}}{2}\cos(\theta - 72.5°)$ $=17.3\cos(\theta - 72.5°)$
(8iii)	$\frac{3\sqrt{133}}{2}\cos(\theta - 72.5198°) = 15$ $\cos(\theta - 72.5198°) = 0.86711$ $\cos\alpha_1 = 0.86711$ $\alpha_1 = 29.8755$ $\theta - 72.5198 = 29.8755, -29.8755$ $\theta = 102.4\ (reject)\ \ or\ \ 42.6°$	(8iv)	Max A = $\frac{3\sqrt{133}}{2}$ =17.3 $\cos(\theta - 72.5198°) = 1$ $\cos\alpha_2 = 1$ $\alpha_2 = 0$ $\theta - 72.5198 = 0$ $\theta = 72.5°$
(9i)	$P = 550 + 200e^{kt}$ P = 550 + 200 = 750	(9ii)	$2(750) = 550 + 200e^{k(24)}$ $e^{24k} = \frac{19}{4}$ 24k = $\ln\frac{19}{4}$ k = $\frac{1}{24}\ln\frac{19}{4}$ = 0.0649
(9iii)	$P = 550 + 200e^{\left(\frac{1}{24}\ln\frac{19}{4}\right)(30)}$ $P = 550 + 200e^{\left(\frac{30}{24}\ln\frac{19}{4}\right)}$ = 1952.4811 $\frac{1952.4811-750}{750}$x100 = 160 %	(9iv)	$\frac{dP}{dt} = 200ke^{kt}$ $\frac{dP}{dt} = 200ke^{k(30)}$ $m = 200\left(\frac{1}{24}ln\frac{19}{4}\right)e^{\left(\frac{30}{24}\ln\frac{19}{4}\right)}$ m = 91.053 m =91.1 P = mt + c 1952.481 = 91.053(30) + c c = -779
(9v)		(9v)	$\frac{5688.169-0}{50-t}$ = -25 t = 277.56 t = 278 278 – 50 = 228 h
(10i)	$y = p(x-2)^2 - (x-3)(x+2)$ $\frac{dy}{dx} = 2p(x-2) - [(x-3)+(x+2)]$ $= 2p(x-2) - 2x + 1$ $\frac{d^2y}{dx^2} = 2p - 2 > 0$ $2p - 2 > 0$, p >1	(10i)	OR $y = (p-1)x^2 + x - 4px + 4p + 6$ (p - 1) > 0 (happy face since it is a min quadratic curve) p >1
(10ii)	$p(x-2)^2 - (x-3)(x+2)$ $p(x^2 - 4x + 4) - (x^2 - x - 6) = 0$ p$x^2 - x^2 - 4px + x + 4p + 6 = 0$ $b^2 - 4ac = (-4p+1)^2 - 4(p-1)(4p+6)$ $=16p^2 - 8p + 1 - 16p^2 - 8p + 24$ = -16p+25 -16p + 25 =0 p = $\frac{25}{16}$ (shown)	(10iii)	y = p$x^2 - x^2 - 4px + x + 4p + 6$ y = p$x^2 - x^2 - 4px + x + 4p + 6$ $y = \frac{25}{16}x^2 - x^2 - 4\left(\frac{25}{16}\right)$x + x + 4$\left(\frac{25}{16}\right)$ + 6 $y = \frac{9}{16}x^2 - \frac{21}{4}x + \frac{49}{4}$ $\frac{9}{16}x^2 - \frac{21}{4}x + \frac{49}{4} = 0$ $9x^2 - 84x + 196 = 0$ $(3x-14)^2 = 0$ $x = \frac{14}{3}$ $A\left(\frac{14}{3}, 0\right)$

(10iv)	$y = mx + 2$ $0 = m\left(\frac{14}{3}\right) + 2$ $m = -\frac{3}{7}$ $y = -\frac{3}{7}x + c$ $\frac{dy}{dx} = -\frac{3}{7}$ $2p(x-2) - 2x + 1 = -\frac{3}{7}$ $2\left(\frac{25}{16}\right)(x-2) - 2x + 1 = -\frac{3}{7}$ $x = \frac{30}{7}$ $y = \frac{9}{16}x^2 - \frac{21}{4}x + \frac{49}{4}$ $y = \frac{9}{16}\left(\frac{30}{7}\right)^2 - \frac{21}{4}\left(\frac{30}{7}\right) + \frac{49}{4}$ $y = \frac{4}{49}$ $y = -\frac{3}{7}x + c$ $\frac{4}{49} = -\frac{3}{7}\left(\frac{30}{7}\right) + c$ $c = \frac{94}{49}$		

SINGAPORE CHINESE GIRLS' SCHOOL
PRELIMINARY EXAMINATION 2018
SECONDARY FOUR
O-LEVEL PROGRAMME

ADDITIONAL MATHEMATICS **4047/01**
Paper 1

Wednesday **1 August 2018** **2 hours**

Additional Materials: Answer Paper
Graph Paper
Cover Page

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved electronic scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 80.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1. A rectangle has a length of $(6\sqrt{3}+3)$ cm and an area of 66 cm^2. Find the perimeter of the rectangle in the form $(a+b\sqrt{3})$ cm, where a and b are integers. [3]

2. On the same axes sketch the curves $y^2 = 225x$ and $y = 15x^3$. [3]

3. **(i)** Find the exact value of 15^x, given that $25^{x+2} = 36 \times 9^{1-x}$. [3]

(ii) Hence, find the value of x, giving your answer to 2 decimal places. [2]

4. **(a)** Given that $\log_3 y - \log_3 x = 1 + \log_3(x+y)$, express y in terms of x. [3]

(b) Solve the equation $\log_3(8-x) + \log_3 x = 2\log_9 15$. [4]

5. The equation of a curve is $y = \dfrac{x-4}{\sqrt{2x+5}}$.

(i) Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ can be expressed in the form $\dfrac{ax+b}{(2x+5)^{\frac{3}{2}}}$ where a and b are constants. [3]

(ii) Given that y is increasing at a rate of 0.4 units per second, find the rate of change of x when $x = 2$. [2]

6. The roots of the quadratic equation $4x^2 + x - m = 0$, where m is a constant, are α and β.

The roots of the quadratic equation $8x^2 + nx + 1 = 0$, where n is a constant, are $\dfrac{1}{\alpha^3}$ and $\dfrac{1}{\beta^3}$.

(i) Show that $m = -8$ and hence find the value of n. [5]

(ii) Find a quadratic equation whose roots are $\alpha + 2$ and $\beta + 2$. [4]

7. **(i)** Show that $\dfrac{2-2\sec^2 x}{(1+\cos x)(1-\cos x)} = -2\sec^2 x$. [3]

(ii) Hence find, for $-\pi \le x \le \pi$, the values of x in radians for which

$$\frac{2-2\sec^2 x}{(1+\cos x)(1-\cos x)} = 4\tan x.$$ [4]

8. The temperature, T °C, of a container of liquid decreases with time, t minutes. Measured values of T and t are given in the table below.

t (min)	10	20	30	40
T (°C)	58.5	41.6	34.7	31.9

It is known that T and t are related by the equation $T = 30 + pe^{-qt}$, where p and q are constants.

(i) On a graph paper, plot $\ln(T-30)$ against t for the given data and draw a straight line graph. [3]

(ii) Use you graph to estimate the value of p and of q. [4]

(iii) Explain why the temperature of the liquid can never drop to 30°C. [1]

9. Given that $y = 2xe^{1-x}$, find

(i) $\dfrac{dy}{dx}$, [2]

(ii) p for which $\dfrac{d^2y}{dx^2} + 2\dfrac{dy}{dx} + pe^{1-x} = 0$, [4]

(iii) the range of values of x for which y is an increasing function. [3]

10. An open rectangular cake tin is made of thin sheets of steel which costs \$2 per 1000 cm^2. The tin has a square base of length x cm, a height of h cm and a volume of 4000 cm^3.

(i) Show that the cost of steel, C, in dollars, for making the cake tin is given by

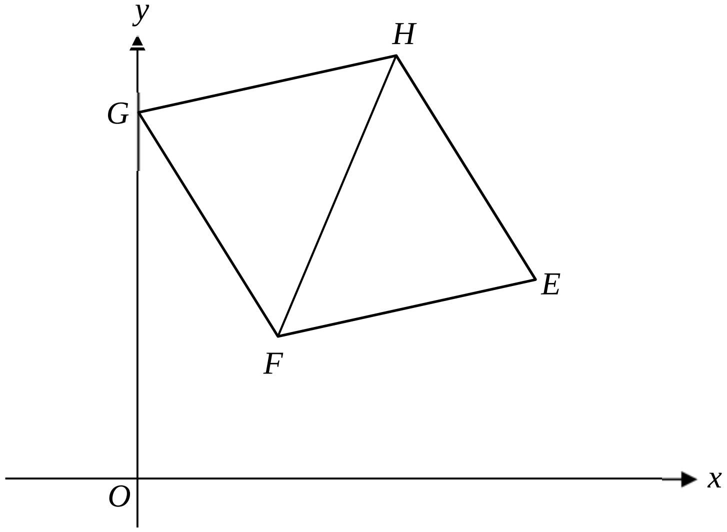
$$C = \frac{x^2}{500} + \frac{32}{x}.$$ [2]

Given that x can vary,

(ii) find the value of x for which C has a stationary value, [3]

(iii) explain why this value of x gives the minimum value of C. [3]

11. The diagram shows a kite *EFGH* with *EF* = *EH* and *GF* = *GH*. The point *G* lies on the *y*-axis and the coordinates of *F* and *H* are (2, 1) and (6, 9) respectively.



The equation of *EF* is $y = \frac{x}{8} + \frac{3}{4}$.

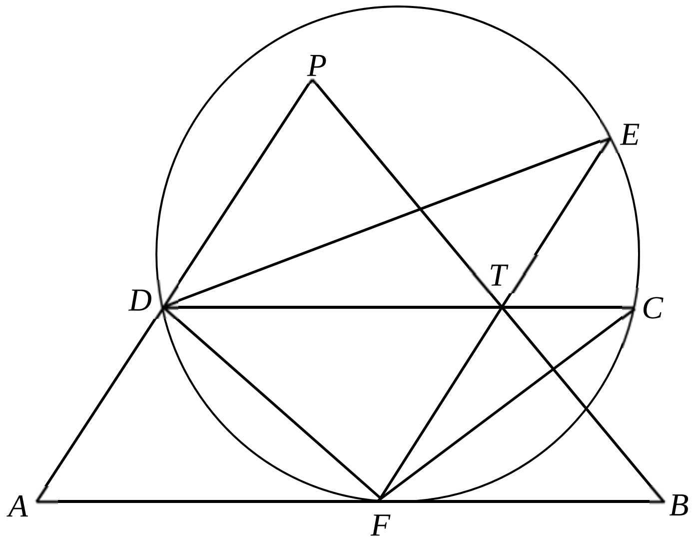
Find

(i) the equation of *EG*, [4]

(ii) the coordinates of *E* and *G*, [3]

(iii) the area of the kite *EFGH*. [2]

12.



The diagram shows a circle passing through points *D*, *E*, *C* and *F*, where *FC* = *FD*.
The point *D* lies on *AP* such that *AD* = *DP*.
DC and *EF* cut *PB* at *T* such that *PT* = *TB*.

(i) Show that *AB* is a tangent to the circle at point *F*. [3]

(ii) By showing that triangle *DFT* and triangle *EFD* are similar, show that

$$DF^2 - FT^2 = FT \times ET\ .$$ [4]

End of Paper 1

[Turn over

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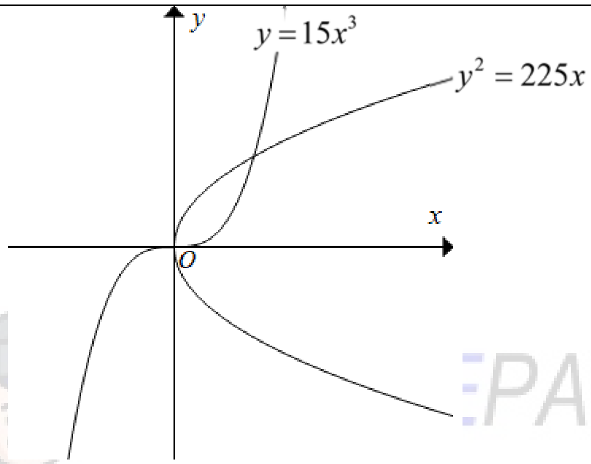
Paper 1

1.		Breadth $= \dfrac{66}{6\sqrt{3}+3}$ or $\dfrac{22}{2\sqrt{3}+1}$ $= \dfrac{66}{6\sqrt{3}+3} \times \dfrac{6\sqrt{3}-3}{6\sqrt{3}-3}$ $\quad \dfrac{22}{2\sqrt{3}+1} \times \dfrac{2\sqrt{3}-1}{2\sqrt{3}-1}$ $= \dfrac{66(6\sqrt{3}-3)}{99}$ $\quad \dfrac{22(2\sqrt{3}-1)}{11}$ $= 4\sqrt{3}-2$ cm Perimeter $= 2(6\sqrt{3}+3+4\sqrt{3}-2)$ $= 20\sqrt{3}+2$ cm
2.		
3.	**(i)**	$25^{x+2} = 36 \times 9^{1-x}$ $(5^{2x})(5^4) = \dfrac{2^2 \times 9^2}{3^{2x}}$ $(5^{2x})(3^{2x}) = \dfrac{2^2 \times 9^2}{25^2}$ $(15^x)^2 = \dfrac{2^2 \times 9^2}{25^2}$ $15^x > 0,\ 15^x = \dfrac{18}{25}$
	(ii)	$15^x = \dfrac{18}{25}$ $x \lg 15 = \lg\left(\dfrac{18}{25}\right)$ $x = \dfrac{\lg\left(\frac{18}{25}\right)}{\lg 15}$ $= -0.12$
4.	**(a)**	$\log_3 y - \log_3 x = 1 + \log_3(x+y)$

[Turn over

		$\log_3 \frac{y}{x} = \log_3 3 + \log_3(x+y)$ $\frac{y}{x} = 3(x+y)$ $y = 3x^2 + 3xy$ $y - 3xy = 3x^2$ $y(1-3x) = 3x^2$ $y = \frac{3x^2}{1-3x}$
	(b)	$\log_3(8-x) + \log_3 x = 2\log_9 15$ $\log_3[x(8-x)] = \frac{2\log_3 15}{\log_3 9}$ $\log_3[x(8-x)] = \frac{2\log_3 15}{2\log_3 3}$ $8x - x^2 = 15$ $x^2 - 8x + 15 = 0$ $(x-3)(x-5) = 0$ $x = 3,\ \ 5$
5.	(i)	$\frac{dy}{dx} = \frac{(2x+5)^{\frac{1}{2}}(1) - \frac{1}{2}(x-4)(2x+5)^{-\frac{1}{2}}(2)}{2x+5}$ $= \frac{(2x+5)^{-\frac{1}{2}}(2x+5-x+4)}{2x+5}$ $= \frac{x+9}{(2x+5)^{\frac{3}{2}}}$
	(ii)	When $x = 2$, $\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$ $0.4 = \frac{2+9}{(4+5)^{\frac{3}{2}}} \times \frac{dx}{dt}$ $\frac{dx}{dt} = 0.4 \times \frac{27}{11}$ $= \frac{54}{55}$ or 0.982 unit per second

6. (i)	$\alpha+\beta=-\frac{1}{4}$ $\alpha\beta=-\frac{m}{4}$ $\frac{1}{(\alpha\beta)^3}=\frac{1}{8}$ $\alpha\beta=2$ $\therefore -\frac{m}{4}=2$ $m=-8$ $\frac{1}{\alpha^3}+\frac{1}{\beta^3}=-\frac{n}{8}$ $\frac{\alpha^3+\beta^3}{\alpha^3\beta^3}=-\frac{n}{8}$ $\alpha^3+\beta^3=-n$ $-n=(\alpha+\beta)\left[(\alpha+\beta)^2-3\alpha\beta\right]$ $n=-\left(-\frac{1}{4}\right)\left[\left(-\frac{1}{4}\right)^2-6\right]$ $=-\frac{95}{64}$
(ii)	Sum of roots $=\alpha+\beta+4$ $=\frac{15}{4}$ Product of roots $=(\alpha+2)(\beta+2)$ $=\alpha\beta+2(\alpha+\beta)+4$ $=2+2\left(-\frac{1}{4}\right)+4$ $=\frac{11}{2}$ New equation: $x^2-\frac{15}{4}x+\frac{11}{2}=0$ or $4x^2-15x+22=0$

7.	**(i)**	$\dfrac{2-2\sec^2 x}{(1+\cos x)(1-\cos x)} = \dfrac{-2(\sec^2 x-1)}{\sin^2 x}$ $= \dfrac{-2\tan^2 x}{\sin^2 x}$ $= \dfrac{-2\left(\dfrac{\sin^2 x}{\cos^2 x}\right)}{\sin^2 x}$ $= \dfrac{-2}{\cos^2 x}$ $= -2\sec^2 x$
	(ii)	$\dfrac{2-2\sec^2 x}{(1+\cos x)(1-\cos x)} = 4\tan x$ $-2\sec^2 x = 4\tan x$ $-\dfrac{1}{\cos^2 x} = \dfrac{2\sin x}{\cos x}$ $-1 = 2\sin x\cos x$ $\sin 2x = -1$ $2x = -\dfrac{\pi}{2}, \dfrac{3\pi}{2}$ $x = -\dfrac{\pi}{4}, \dfrac{3\pi}{4}$
8.	**(i)**	see table below
	(ii)	$T = 30 + pe^{-qt}$ $\ln(T-30) = \ln p - qt$ $\ln p = 4.25$ $p = e^{4.25} = 70.1$ $-q$ = gradient $= \dfrac{0.65-4.25}{40}$ $= -0.09$
	(ii)	Since $e^{-qt} > 0$, $T = 30 + 70e^{-0.09t} > 30$ Hence, $T > 30$ for all values of t.

8. (i)

t (min)	10	20	30	40
$\ln(T-30)$	3.35	2.45	1.55	0.64

9.	(i)	$\frac{dy}{dx} = 2e^{1-x} - 2xe^{1-x}$
	(ii)	$\frac{d^2y}{dx^2} = -2e^{1-x} - 2e^{1-x} + 2xe^{1-x}$ $= -4e^{1-x} + 2xe^{1-x}$ $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + pe^{1-x} = 0$ $-pe^{1-x} = \frac{d^2y}{dx^2} + 2\frac{dy}{dx}$ $= -4e^{1-x} + 2xe^{1-x} + 2(2e^{1-x} - 2xe^{1-x})$ $= -4e^{1-x} + 2xe^{1-x} + 4e^{1-x} - 4xe^{1-x}$ $= -2xe^{1-x}$ $p = 2x$
	(iii)	When $\frac{dy}{dx} > 0$, $2e^{1-x} - 2xe^{1-x} > 0$ $2e^{1-x}(1-x) > 0$ Since $e^{1-x} > 0$ for all x, $1 - x > 0$ $x < 1$
10.	(i)	$x^2h = 4000 \Rightarrow h = \frac{4000}{x^2}$ $C = \frac{2}{1000} \times (x^2 + 4hx)$ $= \frac{2}{1000}\left(x^2 + 4x \times \frac{4000}{x^2}\right)$ $= \frac{x^2}{500} + \frac{32}{x}$
	(ii)	$\frac{dC}{dx} = \frac{x}{250} - \frac{32}{x^2}$ When $\frac{dC}{dx} = 0$, $\frac{x}{250} - \frac{32}{x^2} = 0$ $x^3 = 8000$ $x = 20$
	(iii)	$\frac{d^2C}{dx^2} = \frac{1}{250} + \frac{64}{x^3}$ When $x = 20$, $\frac{d^2C}{dx^2} = \frac{3}{250} > 0$ Since, $\frac{d^2C}{dx^2} > 0$ when $x = 20$, C has a minimum value.

11. **(i)**

Gradient of $FH = \dfrac{9-1}{6-2} = 2$

Gradient of $EG = -\dfrac{1}{2}$

(ii)

Midpoint of $FH \; = \left(\dfrac{2+6}{2}, \dfrac{1+9}{2}\right)$

$= (4, 5)$

Equation of EG: $\quad y - 5 = -\dfrac{1}{2}(x-4)$

$$y = -\frac{x}{2} + 7$$

(iii)

$-\dfrac{x}{2}x + 7 = \dfrac{x}{8} + \dfrac{3}{4}$

$\dfrac{5x}{8} = \dfrac{25}{4} \Rightarrow x = 10$

$y = 2$

Coordinate of E = (10, 2)

$y = -\dfrac{x}{2} + 7$

When $x = 0$, $\quad y = 7$

Coordinate of G = (0, 7)

(iv)

Area of $EFGH$

$= \dfrac{1}{2}\begin{vmatrix} 0 & 2 & 10 & 6 & 0 \\ 7 & 1 & 2 & 9 & 7 \end{vmatrix}$

$= \dfrac{1}{2}\left[(4+90+42) - (14+10+12)\right]$

$= 50 \text{ unit}^2$

<u>Alternative Method</u>

Area of $EFGH \; = \dfrac{1}{2} \times HF \times GE$

$= \dfrac{1}{2} \times \sqrt{4^2 + 8^2} \times \sqrt{10^2 + 5^2}$

$= 50 \text{ units}^2$

12.	**(i)**	*DT* is parallel to *AB*. (Midpoint Theorem) $\angle AFD = \angle TDF$ (alt angles) $= \angle FED$ Since $\angle AFD$ and $\angle FED$ satisfies the alternate segment theorem, *AB* is a tangent at *F*.
	(ii)	$\angle DFE$ is common. $\angle TDF = \angle DCF$ (base angles of an isos triangle) $\angle DCF = \angle DEF$ (angles in the same segment) $\therefore DFT$ and *EFD* are similar triangles (AA) $\frac{DF}{EF} = \frac{FT}{FD}$ $DF^2 = FT \times EF$ $= FT \times (ET + TF)$ $= FT^2 + FT \times ET$ $DF^2 - FT^2 = FT \times ET$

SINGAPORE CHINESE GIRLS' SCHOOL
PRELIMINARY EXAMINATION 2018
SECONDARY FOUR
O-LEVEL PROGRAMME

ADDITIONAL MATHEMATICS **4047/02**
Paper 2

Friday **3 August 2018** **2 hours 30 minutes**

Additional Materials: Answer Paper
Cover Page

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved electronic scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 100.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

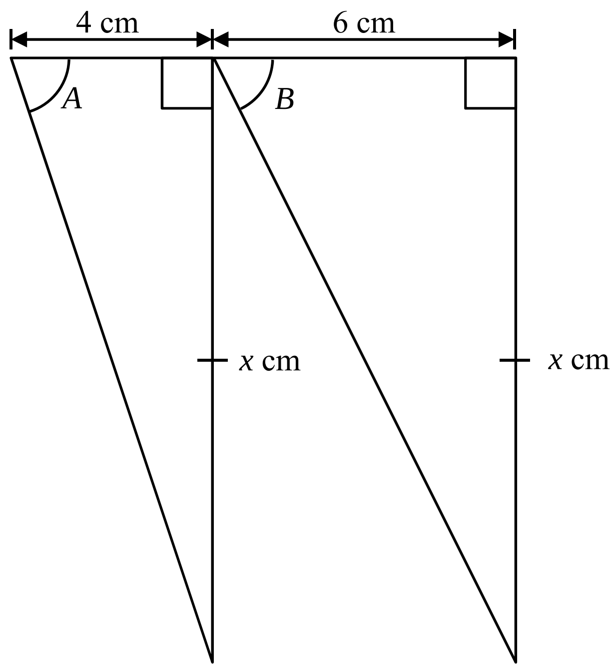
1. **(a)** In the expansion of $(3x-1)(1-kx)^7$ where k is a non-zero constant, there is no term in x^2. Find the value of k. [4]

(b) In the binomial expansion of $\left(\frac{2}{x^3}-x^2\right)^{12}$, in ascending powers of x, find the term in which the power of x first becomes positive. [4]

2. **(a)** Explain why the curve $y=px^2+2x-p$ will always cut the line $y=-1$ at two distinct points for all real values of p. [4]

(b) Find the values of a such that the curve $y=ax^2+x+a$ lies below the x-axis. [4]

3. **(a)** The diagram shows two right-angled triangles with the same height x cm.
One triangle has a base of 4 cm and the other triangle has a base of 6 cm.
Angles A and B are such that $A + B = 135°$.



Find the value of x. [4]

(b) The current y (in amperes), in an alternating current (A.C.) circuit, is given by

$$y=170\sin(kt)\text{, where } t \text{ is the time in seconds.}$$

The period of this function is $\frac{1}{60}$ second.

(i) Find the amplitude of y. [1]

(ii) Find the exact value of k in radians per second. [1]

(iii) For how long in a period is $y > 85$? [3]

4. The function $g(x) = 2x^4 + x^3 + 4x^2 + hx - k$ has a quadratic factor $2x^2 + 3x + 1$.

(i) Find the value of h and of k. [5]

(ii) Determine, showing all necessary working, the number of real roots of the equation $g(x) = 0$. [4]

5. The function f is defined by $f(x) = 4 + 2x - 3x^2$.

(i) Find the value of a, of b and of c for which $f(x) = a + b(x + c)^2$. [4]

(ii) State the maximum value of f(x) and the corresponding value of x. [2]

(iii) Sketch the curve of $y = |f(x)|$ for $-1 \le x \le 2$, indicating on your graph the coordinates of the maximum point. [3]

(iv) State the value(s) of k for which $|f(x)| = k$ has

(a) 1 solution, [1]

(b) 3 solutions. [1]

6. **(i)** Find $\dfrac{d}{dx}\left[(\ln x)^2\right]$. [2]

(ii) Using the result from part **(i)**, find $\displaystyle\int \frac{3x^3 - 5\ln x}{x}\,dx$ and hence show that

$$\int_1^e \frac{3x^3 - 5\ln x}{x}\,dx = e^3 - \frac{7}{2}.$$ [4]

7. **(i)** Show that $\dfrac{d}{dx}(\sec x) = \sec x \tan x$. [2]

(ii) Given that $-\dfrac{\pi}{2} < x < \dfrac{\pi}{2}$, find the value of n for which $y = e^{\tan x}$ is a solution of the equation

$$\frac{d^2y}{dx^2} = (1 + \tan x)^n \frac{dy}{dx}.$$ [7]

8. A circle passes through the points $A(2, 6)$ and $B(5, 5)$, with its centre lying on the line $3y = -x + 5$.

(i) Find the perpendicular bisector of AB. [3]

(ii) Find the equation of the circle. [4]

CD is a diameter of the circle and the point P has coordinates $(-2, -1)$.

(iii) Determine whether the point P lies inside the circle. [2]

(iv) Is angle CPD a right angle? Explain. [1]

9. **(i)** Given that $\dfrac{x^2-4x+1}{x^2-6x+9}=A+\dfrac{B}{x-3}+\dfrac{C}{(x-3)^2}$, where A, B and C are constants, find the value of A, of B and of C. [4]

(ii) Hence, find the coordinates of the turning point on the curve, $y=\dfrac{x^2-4x+1}{x^2-6x+9}$ and determine the nature of this turning point. [6]

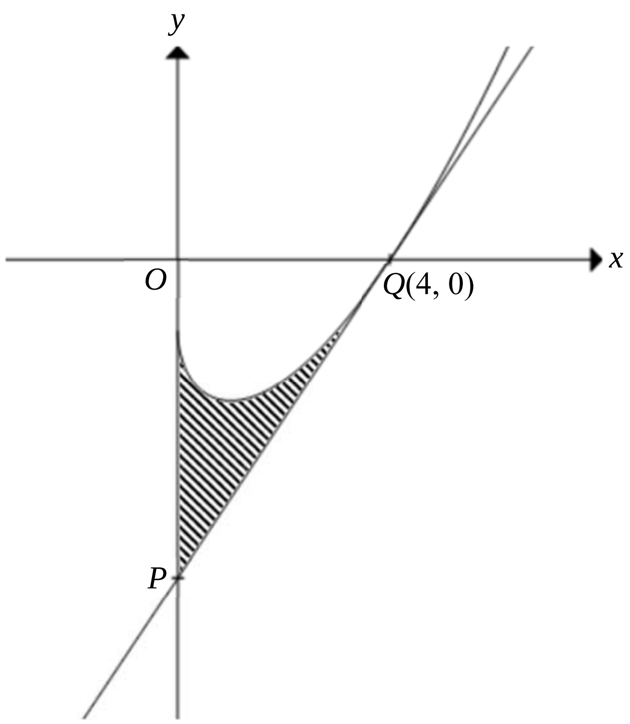
10. A particle starts from rest at O and moves in a straight line with an acceleration of a ms^{-2}, where $a = 2t - 1$ and t is the time in seconds since leaving O.

(i) Find the value of t for which the particle is instantaneously at rest. [4]

(ii) Show that the particle returns to O after $1\frac{1}{2}$ seconds. [4]

(iii) Find the distance travelled in the first 4 seconds. [2]

11. The diagram below shows part of a curve $y=\mathrm{f}(x)$. The curve is such that $\mathrm{f}'(x)=x^{\frac{1}{2}}-x^{-\frac{1}{2}}$ and it passes through the point $Q(4, 0)$. The tangent at Q meets the y-axis at the point P.



(i) Find $\mathrm{f}(x)$. [3]

(ii) Show that the y-coordinate of P is -6. [3]

(iii) Find the area of the shaded region. [4]

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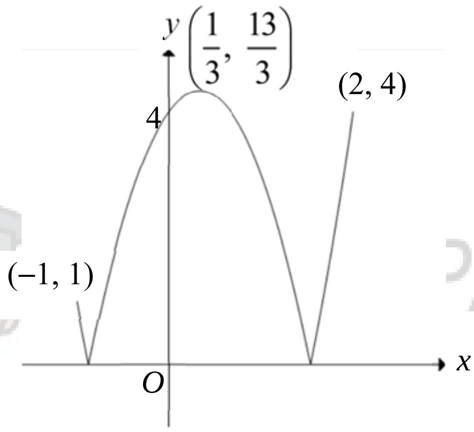
Solution

1.	**(a)**	$(3x-1)(1-kx)^7$ $=(3x-1)(1-7kx+21k^2x^2+...)$ $-21k-21k^2=0$ $-21k(1+k)=0$ $k\neq 0,\ k=-1$
	(b)	$T_{r+1}=\binom{12}{r}\left(\frac{2}{x^3}\right)^{12-r}(-x^2)^r$ $=\binom{12}{r}(2^{12-r})(-1)^r x^{5r-36}$ $5r-36>0$ $r>7.2$ $r=8$ $T_9=\binom{12}{8}(2^4)(-1)^8x^{40-36}$ $=7920x^4$
2.	**(a)**	$px^2+2x-p=-1$ $px^2+2x+1-p=0$ $D=4-4(p)(1-p)$ $=4p^2-4p+4$ $=4(p^2-p+1)$ or $4p^2-4p+1+3$ $=4\left[\left(p-\frac{1}{2}\right)^2+\frac{3}{4}\right]$ $\quad(2p-1)^2+3$ $=4\left(p-\frac{1}{2}\right)^2+3>0$ $\quad(2p-1)^2+3>0$ Since $\left(p-\frac{1}{2}\right)^2\geq 0$ or $(2p-1)^2\geq 0$, the discriminant > 0, the curve will always cut the the line at two distinct points for all real values of *p*.
	(b)	$D=1-4a^2<0$ $D=1-4a^2<0$ and $a<0$ $(1+2a)(1-2a)<0$ or $4a^2-1>0$ $(2a-1)(2a+1)>0$ $a<-\frac{1}{2}$ or $a>\frac{1}{2}$ $\therefore a<-\frac{1}{2}$

3.	**(a)**	$\tan A = \frac{x}{4}$, $\tan B = \frac{x}{6}$ $\tan(A+B) = \tan 135°$ $\frac{\tan A + \tan B}{1 - \tan A \tan B} = -1$ $\frac{x}{4} + \frac{x}{6} = -1 + \left(\frac{x}{4}\right)\left(\frac{x}{6}\right)$ $6x + 4x = -24 + x^2$ $x^2 - 10x - 24 = 0$ $(x-12)(x+2) = 0$ $x = 12, \ -2$ (NA)
	(b)	$y = 170\sin(kt)$
	(i)	Amplitude = 170 or 170 A
	(ii)	$k = 2\pi \div \frac{1}{60}$ $= 120\pi$
	(iii)	When $y = 85$, $170\sin(120\pi t) = 85$ $\sin(120\pi t) = \frac{85}{170} = \frac{1}{2}$ $120\pi t = \frac{\pi}{6}, \ \frac{5\pi}{6}$ $t = \frac{1}{720}, \ \frac{5}{720}$ Duration $= \frac{5}{720} - \frac{1}{720}$ $= \frac{1}{180}$ seconds

4.	**(i)**	$g(x)=2x^4+x^3+4x^2+hx-k$ $2x^2+3x+1=(2x+1)(x+1)$ $g\left(-\frac{1}{2}\right)=0$ $2\left(-\frac{1}{2}\right)^4+\left(-\frac{1}{2}\right)^3+4\left(-\frac{1}{2}\right)^2+h\left(-\frac{1}{2}\right)-k=0$ $1-\frac{h}{2}-k=0$ $k+\frac{h}{2}=1 \quad \ldots\ldots(1)$ $g(-1)=0$ $2-1+4-h-k=0$ $h+k=5 \quad \ldots\ldots(2)$ $\frac{h}{2}=4$ $h=8$ $k=-3$ **<u>Alternative method</u>** $2x^4+x^3+4x^2+hx-k=(2x^2+3x+1)(x^2+bx-k)$ Comparing coefficient of x^3, $1=2b+3$ $b=-1$ Comparing coefficient of x^2, $4=-2k+3b+1$ $k=-3$ Comparing coefficient of x, $h=b-3k$ $h=8$
	(ii)	Let $g(x)=(2x^2+3x+1)(x^2+bx+3)$ Comparing coefficient of x, $8=9+b$ $b=-1$ $g(x)=(2x^2+3x+1)(x^2-x+3)$ $\text{g}(x)=0$ $(2x+1)(x+1)(x^2-x+3)=0$ $x=-\frac{1}{2},\ x=-1$ or $x^2-x+3=0$ $b^2-4ac=1-12<0$ $=-11<0$ No real roots. Hence, $\text{g}(x)=0$ has only 2 real roots

5. (i)	$\text{f}(x) = 4 + 2x - 3x^2$ $= -3\left(x^2 - \frac{2x}{3}\right) + 4$ $= -3\left[\left(x - \frac{1}{3}\right)^2 - \frac{1}{9}\right] + 4$ $= -3\left(x - \frac{1}{3}\right)^2 + \frac{13}{3}$ $a = \frac{13}{3}$, $b = -3$, $c = -\frac{1}{3}$
(ii)	Max value $= \frac{13}{3}$ or $4\frac{1}{3}$ at $x = \frac{1}{3}$
(iii)	
(iv)(a)	$k = \frac{13}{3}$
(b)	$1 < k \leq 4$

6.	(i)	$\frac{\mathrm{d}}{\mathrm{d}x}(\ln x)^2 = 2\ln x\left(\frac{1}{x}\right)$ $= \frac{2\ln x}{x}$
	(ii)	$\int \frac{3x^3 - 5\ln x}{x}\mathrm{d}x = \int 3x^2\mathrm{d}x - \int \frac{5\ln x}{x}\mathrm{d}x$ $= x^3 - \frac{5}{2}(\ln x)^2 + C$ $\int_1^e \frac{3x^3 - 5\ln x}{x}\mathrm{d}x = \left[x^3 - \frac{5}{2}(\ln x)^2\right]_1^e$ $= e^3 - \frac{5}{2}(\ln e)^2 - 1$ $= e^3 - \frac{7}{2}$
7.	**(i)**	$\frac{\mathrm{d}}{\mathrm{d}x}(\sec x) = \frac{\mathrm{d}}{\mathrm{d}x}[(\cos x)^{-1}]$ $= (-1)(\cos x)^{-2}(-\sin x)$ $= \frac{1}{\cos x} \times \frac{\sin x}{\cos x}$ $= \sec x \tan x$
	(ii)	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}}{\mathrm{d}x}(e^{\tan x})$ $= \sec^2 x e^{\tan x}$ $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \frac{\mathrm{d}}{\mathrm{d}x}(\sec^2 x e^{\tan x})$ $= e^{\tan x}(2\sec x)(\sec x \tan x) + \sec^2 x(\sec^2 x e^{\tan x})$ $= \sec^2 x e^{\tan x}(2\tan x + \sec^2 x)$ $= (1 + 2\tan x + \tan^2 x)\frac{\mathrm{d}y}{\mathrm{d}x}$ $= (1 + \tan x)^2 \frac{\mathrm{d}y}{\mathrm{d}x}$ $\therefore n = 2$

8.	**(i)**	Midpoint of AB $=\left(\frac{2+5}{2},\frac{6+5}{2}\right)$ $=\left(\frac{7}{2},\frac{11}{2}\right)$ Gradient of AB $=\frac{5-6}{5-2}$ $=-\frac{1}{3}$ Gradient of perpendicular bisector = 3 Equation of perpendicular bisector, $y-\frac{11}{2}=3\left(x-\frac{7}{2}\right)$ $y=3x-5$
	(ii)	From (i) $y=3x-5$ ……(1) The centre also lies on $3y=-x+5$ ……(2) Substitute (1) into (2), $3(3x-5)=-x+5$ x = 2 y = 1 Centre of circle, (2, 1) Radius of circle $=\sqrt{(2-5)^2+(1-5)^2}$ $=\sqrt{25}$ = 5 units Equation of circle, $(x-2)^2+(y-1)^2=25$ Or $x^2+y^2-4x-2y-20=0$
	(iii)	Distance between the Centre and P $=\sqrt{(2+2)^2+(1+1)^2}$ $=2\sqrt{5}$ units < 5 units $\therefore P$ lies inside the circle. If angle CPD = 90°, P should lie on the circle. (Right angle in a semicircle) Hence, angle CPD cannot be 90°

9.	**(i)**	$\frac{x^2-4x+1}{x^2-6x+9}$ Using Long Division, $\frac{x^2-4x+1}{x^2-6x+9}=1+\frac{2x-8}{(x-3)^2}$ Let $\frac{2x-8}{(x-3)^2}=\frac{B}{x-3}+\frac{C}{(x-3)^2}$. $\frac{2x-8}{(x-3)^2}=\frac{B(x-3)+C}{(x-3)^2}$ $2x-8=B(x-3)+C$ Comparing coefficient of x, $B = 2$ Let $x=3$, $6 - 8 = C$ $C = -2$ $A = 1$
	(ii)	$\frac{dy}{dx}=\frac{d}{dx}\left[1+2(x-3)^{-1}-2(x-3)^{-2}\right]$ $=-2(x-3)^{-2}-2(-2)(x-3)^{-3}$ $=-2(x-3)^{-3}(x-3-2)$ $=-\frac{2(x-5)}{(x-3)^3}$ or $\frac{10-2x}{(x-3)^3}$ or $-\frac{2}{(x-3)^2}+\frac{4}{(x-3)^3}$ When $\frac{dy}{dx}=0$, $-\frac{2(x-5)}{(x-3)^3}=0$ $x = 5$ When $x=5$, $y=\frac{3}{2}$ Turning point, $\left(5,\ \frac{3}{2}\right)$.
		<u>**Alternative Method**</u> When $\frac{dy}{dx}=0$, $-\frac{2}{(x-3)^2}+\frac{4}{(x-3)^3}=0$ $\frac{4}{(x-3)^3}=\frac{2}{(x-3)^2}$ $2(x-3)^2=(x-3)^3$ Since $x \neq 3$, $2=x-3$ $x = 5$ When $x=5$, $y=\frac{3}{2}$ Turning point, $\left(5,\ \frac{3}{2}\right)$.

	$\frac{d^2y}{dx^2} = \frac{-2(x-3)^3 + 2(3)(x-5)(x-3)^2}{(x-3)^6}$ $= \frac{-2(x-3)+6(x-5)}{(x-3)^4}$ $= \frac{4x-24}{(x-3)^4}$ or $\frac{4}{(x-3)^3} - \frac{12}{(x-3)^4}$ When $x = 5$, $\frac{d^2y}{dx^2} = -\frac{1}{4} < 0$ $\left(5, \frac{3}{2}\right)$ is maximum point. **Alternative method** (table below) $\left(5, \frac{3}{2}\right)$ is maximum point.

x	5^-	5	5^+
$\frac{dy}{dx}$	+	0	–
Slope	/	–	\

$\left(5, \frac{3}{2}\right)$ is maximum point.

10. (i)	$v = \int (2t-1)\mathrm{d}t$		
	$= t^2 - t + C$	**M1**	
	When $t = 0$, $v = 0$, $C = 0$		
	$\therefore v = t^2 - t$	**A1**	**A0 (If no C)**
	When $v = 0$, $t^2 - t = 0$	**M1**	
	$t(t^2 - 1) = 0$		
	$t = 0$ (NA), 1	**A1**	
(ii)	$s = \int (t^2 - t)\mathrm{d}t$		
	$= \frac{t^3}{3} - \frac{t^2}{2} + D$	**M1**	
	When $t = 0$, $s = 0$, $D = 0$		
	$\therefore s = \frac{t^3}{3} - \frac{t^2}{2}$	**A1**	**A0 (If no C)**
	When $s = 0$, $\frac{t^3}{3} - \frac{t^2}{2} = 0$	**M1**	
	$2t^3 - 3t^2 = 0$		
	$t^2(2t-3) = 0$		
	$t = 0, \frac{3}{2}$		
	Hence, the particle returns to *O* after $1\frac{1}{2}$ seconds.	**A1**	**Answer with conclusion**
	<u>Alternative method</u>		
	When $t = \frac{3}{2}$, $s = \frac{1.5^3}{3} - \frac{1.5^2}{2}$	**M1**	
	$= 0$		
	Hence, the particle returns to *O* after $1\frac{1}{2}$ seconds.	**A1**	**Answer with conclusion**
(iii)	When $t = 1$, $s = \frac{1^3}{3} - \frac{1^2}{2} = -\frac{1}{6}$		
	When $t = 4$, $s = \frac{4^3}{3} - \frac{4^2}{2} = 13\frac{1}{3}$		
	Distance travelled		
	$= 13\frac{1}{3} + 2\left(\frac{1}{6}\right)$	**M1**	
	$= 13\frac{2}{3}$ m	**A1**	
			(10 marks)

11.	**(i)**	$f'(x)=x^{\frac{1}{2}}-x^{-\frac{1}{2}}$		
		$f(x)=\int(x^{\frac{1}{2}}-x^{-\frac{1}{2}})\,dx$	**M1**	
		$=\frac{2}{3}x^{\frac{3}{2}}-2x^{\frac{1}{2}}+C$	**A1**	
		At (4, 0), $\frac{2}{3}(4)^{\frac{3}{2}}-2(4)^{\frac{1}{2}}+C=0$		
		$C=-\frac{4}{3}$		
		$f(x)=\frac{2}{3}x^{\frac{3}{2}}-2x^{\frac{1}{2}}-\frac{4}{3}$	**A1**	
	(ii)	At Q, $f'(x)=\frac{dy}{dx}=4^{\frac{1}{2}}-4^{-\frac{1}{2}}$		
		$=\frac{3}{2}$	**B1**	
		Equation of PQ, $y=\frac{3}{2}(x-4)$	**M1**	
		$y=\frac{3}{2}x-6$		
		$\therefore$ at P, $y=-6$	**A1**	
	(iii)	Area of shaded region		
		$=\frac{1}{2}\times4\times6+\int_0^4\left(\frac{2}{3}x^{\frac{3}{2}}-2x^{\frac{1}{2}}-\frac{4}{3}\right)dx$ or $=\frac{1}{2}\times4\times6-\left\lvert\int_0^4\left(\frac{2}{3}x^{\frac{3}{2}}-2x^{\frac{1}{2}}-\frac{4}{3}\right)dx\right\rvert$	**M1**	**Difference**
		$=12+\left[\frac{\frac{2}{3}x^{\frac{5}{2}}}{\frac{5}{2}}-\frac{2x^{\frac{3}{2}}}{\frac{3}{2}}-\frac{4}{3}x\right]_0^4$	**M1**	**Integral + limits**
		$=12+\left[\frac{4}{15}x^{\frac{5}{2}}-\frac{4}{3}x^{\frac{3}{2}}-\frac{4}{3}x\right]_0^4$		
		$=12+\left[\frac{4}{15}(4)^{\frac{5}{2}}-\frac{4}{3}(4)^{\frac{3}{2}}-\frac{4}{3}(4)\right]$		
		$=12-\frac{112}{15}$	**M1**	
		$=\frac{68}{15}$ unit^2 or $4\frac{8}{15}$ unit^2 or 4.53 unit^2	**A1**	
				(10 marks)

End of Paper 2

SECONDARY 4
2018 Preliminary Examinations

ADDITIONAL MATHEMATICS
Paper 1

4047/1

10 September 2018 (Monday) **2 hours**

CANDIDATE NAME	**Solutions**
CLASS	
INDEX NUMBER	

READ THESE INSTRUCTIONS FIRST
Do not turn over the page until you are told to do so.
Write your name, class and index number in the spaces provided above.

Write in dark blue or black pen on both sides of the paper. You may use a pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid/tape.

INFORMATION FOR CANDIDATES
Answer **all** the questions.
Write your answers on the separate writing paper provided.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of a scientific calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your answer scripts securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is **80**.

For Examiner's Use		
Q1	3	
Q2	6	
Q3	5	
Q4	5	
Q5	5	
Q6	6	
Q7	6	
Q8	8	
Q9	9	
Q10	9	
Q11	9	
Q12	9	
Total	/80	

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!\,(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for $\Delta\ ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

[Turn over

1. Given that $a=\sqrt{2}-\sqrt{3}$, find the value of a^2, leaving your answer in exact form. Hence, or otherwise, and without the use of a calculator, find the exact value of $2a^4-16a^2+5$. [3]

2. A curve, for which $\frac{dy}{dx}=kx^2-8$, has a gradient of -4 at $x=2$.

(i) State the value of k. [1]

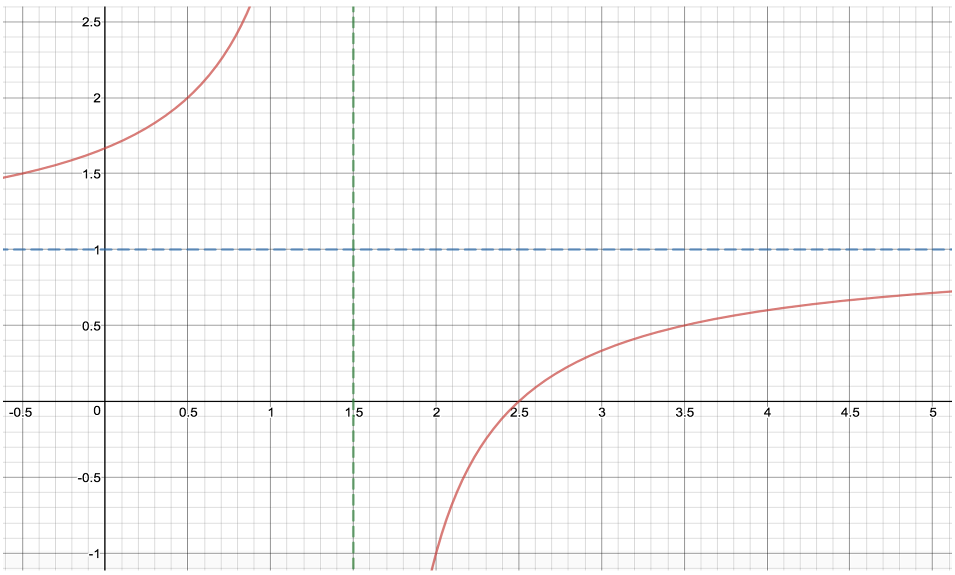
With this value of k, find

(ii) the equation of the normal at point $P(3,\ -2)$, [2]

(iii) the equation of the curve y. [3]

3. (i) Sketch the graph of $y^2=9x$. [2]

(ii) You were going through your old notes and happen to come across the following graph sketched on a piece of paper. It brought back some memories of your time in SST because you had to draw that graph in a Mathematics quiz. However, the equation of the function is missing from the graph. You decided to complete the equation before putting the graph back into the pile.



Given that the y-intercept of the graph is $\frac{5}{3}$ and that the equation is of the curve is of the form $y=\frac{k}{(x-h)}+c$, where h, k, c are constants that need to be determined, find the value of h, of k and of c. [3]

[Turn over

4. Express $\dfrac{2x^2+x+1}{(x+1)(x-2)}$ in partial fractions. [5]

5. **Answer the whole of this question on a piece of graph paper.**

Variables x and y are known to be related by an equation of the form $y = a\sqrt{x} + \dfrac{b}{\sqrt{x}}$, where a and b are constants. The table shows experimental values of the two variables.

x	1.0	1.5	2.0	2.5	3.0	3.5
y	2.4	3.9	5.1	6.4	7.4	8.3

(i) Plot $y\sqrt{x}$ against x and draw a straight-line graph. [3]

(ii) Use the graph to estimate the values of a and of b. [2]

6. Given that the roots of the quadratic equation $2x^2+x+6=0$ are α and β.

(i) Find the quadratic equation whose roots are $\left(\alpha+\dfrac{1}{2\beta}\right)$ and $\left(\beta+\dfrac{1}{2\alpha}\right)$. [4]

(ii) Explain why the value for $\alpha-\beta$ is undefined. [2]

7. (i) Prove the following trigonometric identity:

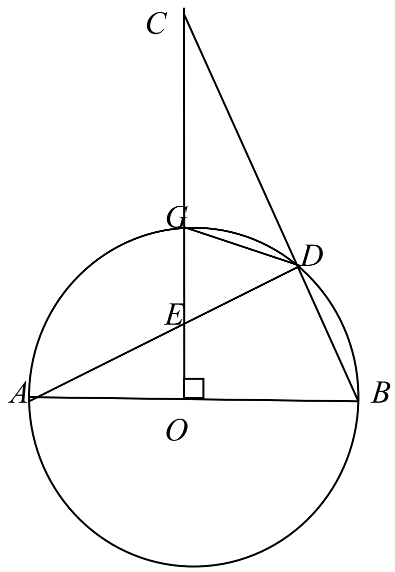
$$\left(\frac{1-\cos\theta}{1+\cos\theta}\right) \equiv (\operatorname{cosec}\theta-\cot\theta)^2.$$ [3]

(ii) Hence, for $-\pi \le \theta \le \pi$, solve the equation

$$(\operatorname{cosec}\theta-\cot\theta)^2 = 5.$$ [3]

[Turn over

8.



AB is a diameter of the circle with centre *O*. *C* is a point on *OG* produced and *CB* intersects the circle at *D*. *OG* is perpendicular to *AB* and *OG* intersects the chord *AD* at *E*,

(i) Prove that $AE \times ED = OE \times EC$. [4]

(ii) Explain why *C* is at an equal distance from *A* and *B*. [2]

(iii) Explain why a circle with *BC* as a diameter passes through *O*. [2]

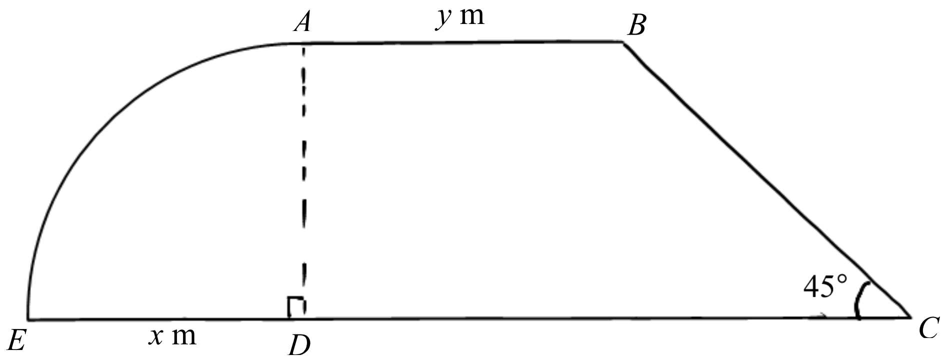
9. The straight line $3x - y + 5 = 0$ and the curve $x^2 + y^2 - 2x - 6y + 5 = 0$ intersect at two points, *A* and *B*.

(i) Find the coordinates of *A* and of *B*. [3]

(ii) Find the equation of the perpendicular bisector of *AB*. [3]

(iii) Find the coordinates of the centre of the circle $x^2 + y^2 - 2x - 6y + 5 = 0$ and determine whether the point $(1, 1)$ lies within, outside or on the circumference of the circle. [3]

10. A piece of wire of length 680m is bent to form an enclosure consisting of a trapezium *ABCD* and a quadrant *ADE* with $AB = y$ m, $DE = x$ m and $B\hat{C}D = 45°$.



(i) Show that the area A m^2 of the enclosure is given by

$$A = 340x - \frac{\sqrt{2}+1}{2}x^2.$$ [4]

(ii) Find the value of *x,* correct to 2 decimal places, for which there is a stationary value for *A* and determine whether it is a maximum or a minimum. [5]

11. A particle starts from a point *O* and moves in a straight line so that its velocity, *v* m/s, is given by $v = (3t + 5)(t - 5)$ where *t* is the time in seconds after leaving *O*.

Find,

(i) the time(s) when the particle is at rest, [2]

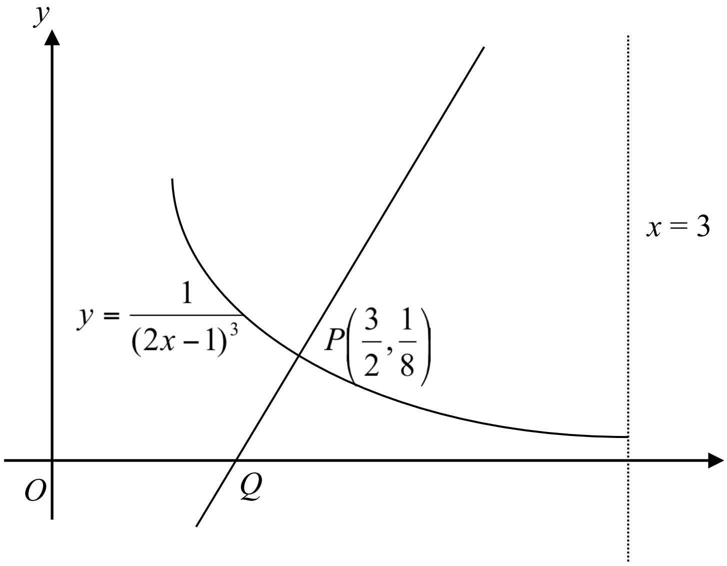
(ii) the time when the particle passes through *O* again, [3]

(iii) the distance travelled during the third second, [2]

(iv) the time interval during which the velocity is decreasing. [2]

[Turn over

12.



In the diagram above the line PQ is normal to the curve $y = \frac{1}{(2x-1)^3}$ at the point $P\left(\frac{3}{2}, \frac{1}{8}\right)$.

(i) Find the length of OQ. [4]

(ii) Find the area bounded by the line PQ, the curve $y = \frac{1}{(2x-1)^3}$ and the line $x = 3$. [5]

END OF PAPER

[Turn over

① $a = \sqrt{2} - \sqrt{3}$

$$a^2 = (\sqrt{2} - \sqrt{3})^2$$

$$= 2 - 2\sqrt{2}\sqrt{3} + 3$$

$$= 5 - 2\sqrt{6}$$

$$\therefore 2a^4 - 16a^2 + 5 = 2(a^2)^2 - 16(a^2) + 5$$

$$= 2[5 - 2\sqrt{6}]^2 - 16(5 - 2\sqrt{6}) + 5$$

$$= 2[25 - 2(5)(2\sqrt{6}) + 24]^2 - 80 + 32\sqrt{6} + 5$$

$$= 2[49 - 20\sqrt{6}] - 75 + 32\sqrt{6}$$

$$= 23 - 8\sqrt{6}$$ #

② $\frac{dy}{dx} = kx^2 - 8$

(i) @ $x = 2$, $\frac{dy}{dx} = -4$

$-4 = k(2)^2 - 8$

$k = 1$ ⧣

(ii) $\frac{dy}{dx} = x^2 - 8$

@ $x = 3$, $\frac{dy}{dx} = 3^2 - 8$

$= 1$

∴ the equation of the normal

$y - (-2) = \frac{-1}{1}(x - 3)$

$y = -x + 1$ ⧣

(iii) $y = \int x^2 - 8 \, dx$

$= \frac{1}{3}x^3 - 8x + C$

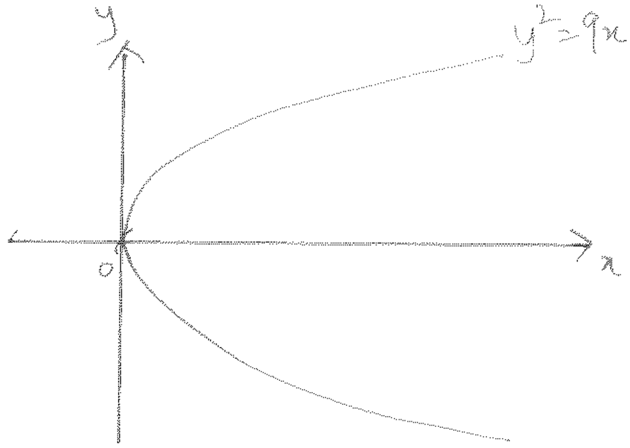
@ P(3, -2)

$-2 = \frac{1}{3}(3)^3 - 8(3) + C$

$C = 13$

∴ $y = \frac{1}{3}x^3 - 8x + 13$ ⧣

3) (i)



(ii) asymptotes @ $x = 3/2$

$y = 1$

$\therefore h = 3/2$

$c = 1$

$\therefore y = \dfrac{k}{x - 3/2} + 1$

@ $(0, \frac{5}{3})$

$\frac{5}{3} = \dfrac{k}{0 - 3/2} + 1$

$k = -1$ //

$\therefore y = \dfrac{-1}{x - 3/2} + 1$ //

4) $\frac{2x^2+x+1}{(x+1)(x-2)} = \frac{2x^2+x+1}{x^2-x-2}$

$= \frac{2(x^2-x-2)+3x+5}{(x+1)(x-2)}$

$= 2 + \frac{3x+5}{(x+1)(x-2)}$

$= 2 + \frac{B}{x+1} + \frac{C}{x-2}$

$= 2 + \frac{B(x-2)+C(x+1)}{(x+1)(x-2)}$

$= 2 + \frac{(B+C)x+(-2B+C)}{(x+1)(x-2)}$

OR

$\frac{2x^2+x+1}{(x+1)(x-2)} = \frac{2x^2+x+1}{x^2-x-2}$

$$\begin{array}{r} 2 \\ x^2-x-2 \overline{)\ 2x^2+x+1} \\ -(2x^2-2x-4) \\ \hline 3x+5 \end{array}$$

$\therefore \frac{2x^2+x+1}{(x+1)(x-2)} = 2 + \frac{3x+5}{(x+1)(x-2)}$

$\therefore$ by comparing coefficients

$A = 2$ #

$B + C = 3$ ①

$-2B + C = 5$ ②

① − ②

$B+C-(-2B+C) = 3-5$

$3B = -2$

$B = -2/3$ #

$\therefore -\frac{2}{3} + C = 3$

$C = \frac{11}{3}$ #

5

(i) $y = a\sqrt{x} + \frac{b}{\sqrt{x}}$

$\therefore y\sqrt{x} = ax + b$

$\therefore Y = aX + b$

$X = x$	1	1.5	2	2.5	3	3.5
$Y = y\sqrt{x}$	2.4	4.777	7.212	10.119	12.817	15.528

(ii) from the graph:

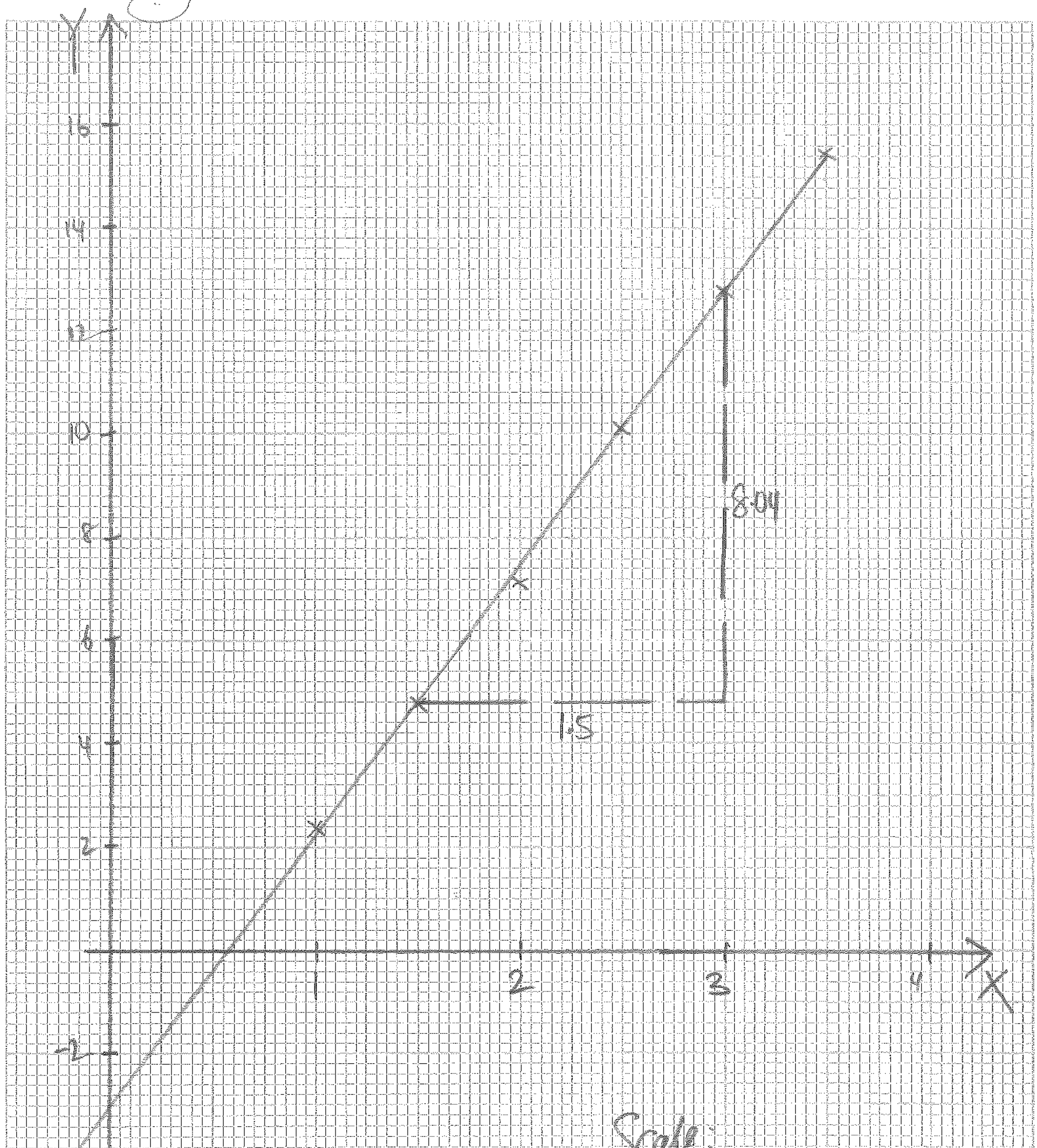
$b = -3$ #

$a = \frac{8.04}{1.5}$

$= 5.36$ #

Name ______ Index No. ______

Subject ______ Class ______ Date ______



Scale:
X: 4cm to 1 unit
Y = 2cm to 2 units

20 cm × 25 cm

(6) $2x^2+x+6=0$

$\therefore \alpha+\beta = \frac{-1}{2}$

$\alpha\beta = \frac{6}{2} = 3$

$\therefore \left(\alpha+\frac{1}{2\beta}\right)+\left(\beta+\frac{1}{2\alpha}\right) = (\alpha+\beta)+\frac{1}{2\alpha}+\frac{1}{2\beta}$

$= (\alpha+\beta) + \frac{\beta+\alpha}{2\alpha\beta}$

$= \left(-\frac{1}{2}\right) + \frac{-1/2}{2(3)}$

$= \frac{-7}{12}$

& $\left(\alpha+\frac{1}{2\beta}\right)\left(\beta+\frac{1}{2\alpha}\right) = \alpha\beta+\frac{1}{2}+\frac{1}{2}+\frac{1}{4\alpha\beta}$

$= \alpha\beta + \frac{1}{4(\alpha\beta)} + 1$

$= 3 + \frac{1}{4(3)} + 1$

$= \frac{49}{12}$

$\therefore$ the equation

$x^2 - \left(\frac{-7}{12}\right)x + \frac{49}{12} = 0$

$x^2 + \frac{7}{12}x + \frac{49}{12} = 0$ # or $12x^2+7x+49=0$ #

(ii) $(\alpha-\beta)^2 = (\alpha+\beta)^2 - 4\alpha\beta$

$= \left(\frac{-1}{2}\right)^2 - 4(3)$

$= \frac{-47}{4}$

< 0

& $\because (\alpha-\beta)^2 \geq 0$ for all values of α & β,

$\therefore \alpha-\beta$ is undefined #

(OR)

$2x^2+x+6=0$

roots $= \frac{-1 \pm \sqrt{-47}}{4}$

$= \frac{-1+\sqrt{-47}}{4}$ or $\frac{-1-\sqrt{-47}}{4}$

$\therefore \alpha-\beta = \frac{-1+\sqrt{-47}}{4} - \frac{-1-\sqrt{-47}}{4}$

$= \frac{-1+\sqrt{-47}+1+\sqrt{-47}}{4}$

$= \frac{2\sqrt{-47}}{4}$

$\therefore (\alpha-\beta)$ is undefined because no real values exist for $\sqrt{-47}$. #

(7)

(i) from RHS:

$$(\operatorname{cosec}\theta - \cot\theta)^2 = \left(\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right)^2$$

$$= \frac{(1-\cos\theta)^2}{\sin^2\theta}$$

$$= \frac{(1-\cos\theta)^2}{1-\cos^2\theta}$$

$$= \frac{(1-\cos\theta)^2}{(1+\cos\theta)(1-\cos\theta)}$$

$$= \frac{1-\cos\theta}{1+\cos\theta}$$

= LHS (proven) #

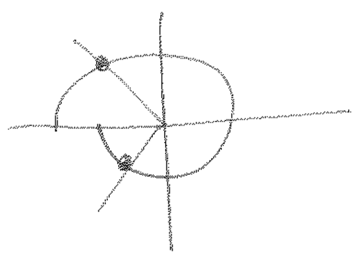
(ii) $(\operatorname{cosec}\theta - \cot\theta)^2 = 5$

$$\frac{1-\cos\theta}{1+\cos\theta} = 5$$

$$5 + 5\cos\theta = 1 - \cos\theta$$

$$6\cos\theta = -4$$

$$\cos\theta = \frac{-2}{3}$$

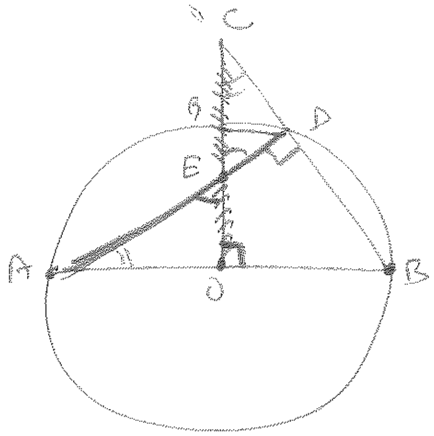


ref $\angle$ = 0.84107 rad (to 5DP)

$\therefore \theta = (-\pi + 0.84107)$ & $(\pi - 0.84107)$

= −2.30 rad & 2.30 rad (to 3SF) #

(8)



(i) $\angle ADB = 90^\circ$ ($\angle$s in a semicircle)

& $\triangle AEO$ is similar $\triangle CED$

- $\because \angle AEO = \angle CED$ (Vertically opposite angles)
- $\angle EAO = 90^\circ - \angle AEO$ ($\angle$ sum of triangle)

$= 90^\circ - \angle CED$

$= \angle ECD$

OR $\angle AOE = 90^\circ$

$= \angle ADB$ (angles in semi circle)

$= \angle CDE$ (supplementary angles)

$\therefore \frac{AE}{CE} = \frac{EO}{ED} = \frac{AO}{CD}$

$\Rightarrow AE \times ED = OE \times EC$ (proven) #

(ii) $\because$ OG is perpendicular to AB (given)

& OG passes through the centre, ie its equidistant from A, B

$\therefore$ all points along OG will be equidistant from A, B

& $\because$ C lies on OG produced

$\Rightarrow$ C will be equal distance from A & B (shown) #

8 iii) ∵ ∡COB = 90° (given)

∴ ∵ angles in semicircle = 90°

∴ there is a circle, with CB as its diameter that passes through point O.

(9) $3x - y + 5 = 0$

$y = 3x + 5$ ①

& $x^2 + y^2 - 2x - 6y + 5 = 0$ ②

Subst ① into ②

$x^2 + (3x+5)^2 - 2x - 6(3x+5) + 5 = 0$

$x^2 + 9x^2 + 30x + 25 - 2x - 18x - 30 + 5 = 0$

$10x^2 + 10x = 0$

$10x(x+1) = 0$

$\therefore x = 0$ or $x = -1$

@ $x = 0$, $y = 3(0) + 5$
$= 5$

& @ $x = -1$, $y = 3(-1) + 5$
$= 2$

$\therefore A(0,5)$ # & $B(-1,2)$ #

(ii) gradient (AB) $= \dfrac{2-5}{-1-0} = 3$

midpoint (AB) $= \left(\dfrac{0+(-1)}{2}, \dfrac{5+2}{2}\right) = \left(\dfrac{-1}{2}, \dfrac{7}{2}\right)$

$\therefore$ the equation of the perpendicular bisector of AB

$y - \dfrac{7}{2} = \dfrac{-1}{3}\left(x - \left(\dfrac{-1}{2}\right)\right)$

$y = \dfrac{-1}{3}x + \dfrac{10}{3}$ #

(iii) $x^2+y^2-2x-6y+5=0$

$$x^2-2x \qquad +y^2-6y \qquad +5=0$$

$$x^2-2x+1^2-1^2 \quad +y^2-2(3y)+3^2-3^2 \quad +5=0$$

$$(x-1)^2+(y-3)^2 = 5$$

@ (1,1)

$$\because (1-1)^2+(1-3)^2 = 4$$

$$< 5$$

$\therefore$ point (1,1) lies inside the circumference of the circle. #

(a) $\overset{\frown}{AB} = \frac{1}{4}[2\pi(x)]$ & $BC^2 = x^2 + x^2$ (Pythagoras' Theorem)

$= \frac{\pi}{2}x$

$BC = \sqrt{2x^2}$ $(BC > 0)$

$= x\sqrt{2}$

$\therefore\ x + y + x + x\sqrt{2} + y + \frac{\pi}{2}x = 680$

$2y + (2 + \frac{\pi}{2} + \sqrt{2})x = 680$

$y = \frac{680 - (2 + \frac{\pi}{2} + \sqrt{2})x}{2}$ (1)

& $\text{Area} = \frac{1}{4}[\pi(x)^2] + \frac{1}{2}(y + y + x)(x)$

$= \frac{\pi}{4}x^2 + \frac{x}{2}(2y + x)$

$= \frac{\pi}{4}x^2 + \frac{x}{2}\left[2\left(\frac{680 - (2 + \frac{\pi}{2} + \sqrt{2})x}{2}\right) + x\right]$

$= \frac{\pi}{4}x^2 + \frac{x}{2}\left[680 - (2 + \frac{\pi}{2} + \sqrt{2})x + x\right]$

$= \frac{\pi}{4}x^2 + 340x - \frac{x^2}{2}(2 + \frac{\pi}{2} + \sqrt{2}) + \frac{x^2}{2}$

$= 340x + \left[\frac{\pi}{4} - \frac{1}{2}(2 + \frac{\pi}{2} + \sqrt{2}) + \frac{1}{2}\right]x^2$

$= 340x + \left[-\frac{1}{2} - \frac{\sqrt{2}}{2}\right]x^2$

$= 340x - \left(\frac{1 + \sqrt{2}}{2}\right)x^2$ (shown) #

(ii) $\text{Area} = 340x - \left(\frac{1+\sqrt{2}}{2}\right)x^2$

$$\frac{d\text{Area}}{dx} = 340 - 2\left(\frac{1+\sqrt{2}}{2}\right)x$$

$$= 340 - (1+\sqrt{2})x$$

$$\frac{d^2\text{Area}}{dx^2} = -(1+\sqrt{2})$$

$$< 0$$

∴ area will be maximized at stationary point.

∴ maximum value of x @ $\frac{d\text{Area}}{dx} = 0$

$$\therefore 340 - (1+\sqrt{2})x = 0$$

$$x = \frac{340}{1+\sqrt{2}}$$

$$= 140.83\text{m (to 2DP)}$$ #

(11) $v = (3t+5)(t-5) = 3t^2 - 10t - 25$

(i) @ $v = 0$

$(3t+5)(t-5) = 0$

$\therefore t = -\frac{5}{3}$ & $t = 5$ sec #

(na $\because t > 0$)

(ii) $S = \int 3t^2 - 10t - 25 \, dt$

$= t^3 - 5t^2 - 25t + C$

@ $t = 0$, $S = 0$, $\therefore C = 0$

$\therefore S = t^3 - 5t^2 - 25t$

$= t(t^2 - 5t - 25)$

@ $S = 0$

$t = 0$ or $t^2 - 5t - 25 = 0$

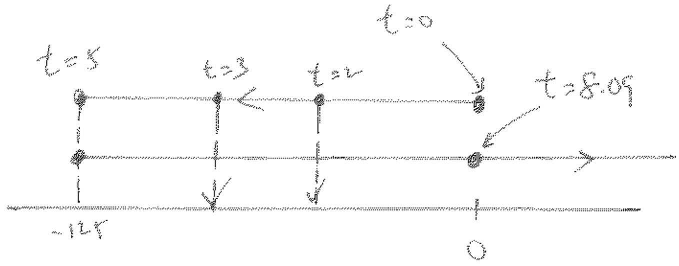
$t = \frac{5 \pm \sqrt{125}}{2}$

$= \frac{5 \pm 5\sqrt{5}}{2}$

$= 8.09$ sec or -3.09 (to 3SF)
(na)

$\therefore$ particle will pass through the origin again @ $t = 8.09$ sec #

(iii)

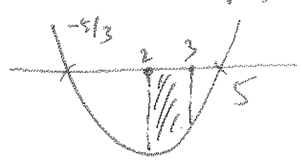


@ $t = 2$, $S = 2(2^2 - 5(2) - 25) = -62$ m

@ $t = 3$, $S = 3(3^2 - 5(3) - 25) = -93$ m

$\therefore$ distance travelled $= 31$ m #

(OR) Distance $= \int_3^2 3t^2 - 10t - 25 \, dt$



$= \left[t^3 - 5t^2 - 25t \right]_3^2$

$= [-62] - [-93]$

$= 31$ m #

(iv) Velocity decreasing $\Rightarrow a < 0$

$\therefore a =$ acceleration

$= \frac{dv}{dt}$

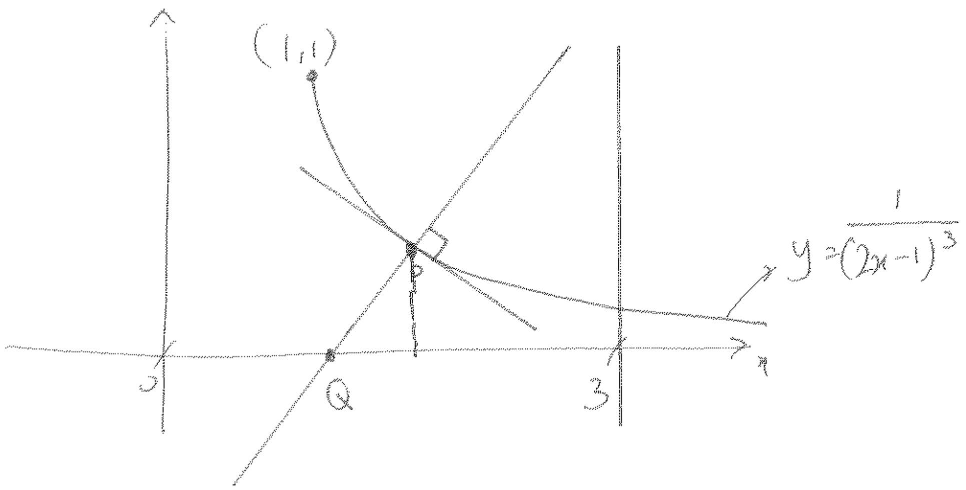
$= 6t - 10$

$\therefore 6t - 10 < 0$

$t < \frac{5}{3}$

$\therefore 0 < t < \frac{5}{3}$ #

(12)



(i) $y = (2x-1)^{-3}$

$\therefore \frac{dy}{dx} = -3(2x-1)^{-4} \cdot [2]$

$= \frac{-6}{(2x-1)^4}$

@ $x = 3/2$,

gradient $= \frac{dy}{dx} = \frac{-6}{[2(\frac{3}{2})-1]^4}$

$= \frac{-3}{8}$

$\therefore$ the equation of the normal @ P

$y - \frac{1}{8} = \frac{8}{3}(x - \frac{3}{2})$

$y = \frac{8}{3}x - \frac{31}{8}$

(ii) @ $y = 0$

$x = \frac{93}{64}$

$\therefore Q(\frac{93}{64}, 0)$

$\therefore OQ = \frac{93}{64}$ units #

(iii)

$$\text{Area} = \frac{1}{2}\left(\frac{3}{2}-\frac{93}{64}\right)\left(\frac{1}{8}\right) + \int_{3/2}^{3} (2x-1)^{-3}\,dx$$

$$= \frac{3}{1024} + \left[\frac{(2x-1)^{-2}}{(-2)(2)}\right]_{3/2}^{3}$$

$$= \frac{3}{1024} + \left[\frac{-1}{4(2x-1)^2}\right]_{3/2}^{3}$$

$$= \frac{3}{1024} + \left[\frac{-1}{4[2(3)-1]^2} - \frac{-1}{4[2(\frac{3}{2})-1]^2}\right]$$

$$= \frac{3}{1024} + \left[\frac{-1}{100} + \frac{1}{16}\right]$$

$$= \frac{1419}{25600}$$

$= 0.0554$ units^2 (to 3SF) (shown) //

SCHOOL OF SCIENCE AND TECHNOLOGY, SINGAPORE

SECONDARY 4
2018 Preliminary Examinations

ADDITIONAL MATHEMATICS
Paper 2

4047/2

12 September 2018 (Wednesday)

2 hours 30 minutes

CANDIDATE NAME			
CLASS		INDEX NUMBER	

READ THESE INSTRUCTIONS FIRST

Do not turn over the page until you are told to do so.
Write your name, class and index number in the spaces above.
Write in dark blue or black pen in the writing papers provided.

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INFORMATION FOR CANDIDATES

Answer **all** the questions.

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At the end of the examination, fasten all your answer scripts securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is **100**.

For Examiner's Use		
Q1	4	
Q2	6	
Q3	6	
Q4	6	
Q5	7	
Q6	8	
Q7	8	
Q8	9	
Q9	10	
Q10	10	
Q11	13	
Q12	13	
Total		/100

This document consists of **8** printed pages including the cover page.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for Δ ABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

1 Find the value of the constant k for which $y = x^2e^{1-2x}$ is a solution of the equation

$$\frac{d^2y}{dx^2} - \frac{2y}{x^2} = k\left(\frac{dy}{dx} + y\right).$$

[4]

2 **(i)** Find $\frac{d}{dx}(\frac{\ln x}{x})$. **[2]**

(ii) Hence find $\int \frac{\ln x}{x^2} dx$. **[3]**

The curve $y = f(x)$ is such that $f(x) = \frac{\ln x}{x}$, for $x > 0$.

(iii) Explain why the curve $y = f(x)$ has only one stationary point. **[1]**

3 The expression $2x^3 + ax^2 + bx - 35$, where a and b are constants, has a factor of $2x - 7$ and leaves a remainder of -36 when divided by $x + 1$.

(i) Find the value of a and of b. **[4]**

(ii) Using the values of a and b found in part **(i)**, explain why the equation $2x^3 + ax^2 + bx - 35 = 0$ has only one real root. **[2]**

4 As part of his job in a restaurant, John learned to cook a hot pot of soup late at night so that there would be enough for sale the next day. While refrigeration was essential to preserve the soup overnight, the soup was too hot to be put directly in the refrigerator when it was ready at 100 °C. The soup subsequently cooled in such a way that its temperature, x °C after t minutes, was given by the expression $x = 20 + Ae^{-kt}$, where A and k are constants.

(i) Explain why $A = 80$. **[1]**

(ii) When $t = 15$, the temperature of the soup is 58 °C.
Find the value of k. **[2]**

(iii) Deduce the temperature of the soup if it is left unattended for a long period of time, giving a reason for your answer. **[1]**

(iv) For the soup to be refrigerated, its temperature should be less than 35 °C. What is the shortest possible time, correct to the nearest minute, that John has to wait before he could refrigerate the soup? **[2]**

5 **(a)** The function f is defined, for all values of x, by

$$f(x) = x^2(3-4x).$$

Find the range of values of x for which f is an increasing function. **[3]**

(b) A particle moves along the curve $y = \dfrac{16}{(3-4x)^2}$ in such a way that the y-coordinate of the particle is increasing at a constant rate of 0.03 units per second. Find the exact y-coordinate of the particle at the instant that the x-coordinate of the particle is decreasing at 0.12 units per second. **[4]**

6 **(a) (i)** Sketch the graph of $y = 10^x$. **[1]**

(ii) Given that $\dfrac{4^x}{2^{x+2}} = \dfrac{3}{5^x}$, find the value of 10^x. **[2]**

(b) Solve the equation $\log_2\sqrt{5x+1} + 2\log_9 3 = \log_4(2x-3) + \log_3 27$. **[5]**

7 The population of a herd of deer can be modelled by the function $D = 400 + 40\sin\left(\frac{\pi}{6}t\right)$, where D is the deer population in week t of the year for $0 \le t \le 24$.

Using the model,

(i) state the amplitude of the function, **[1]**

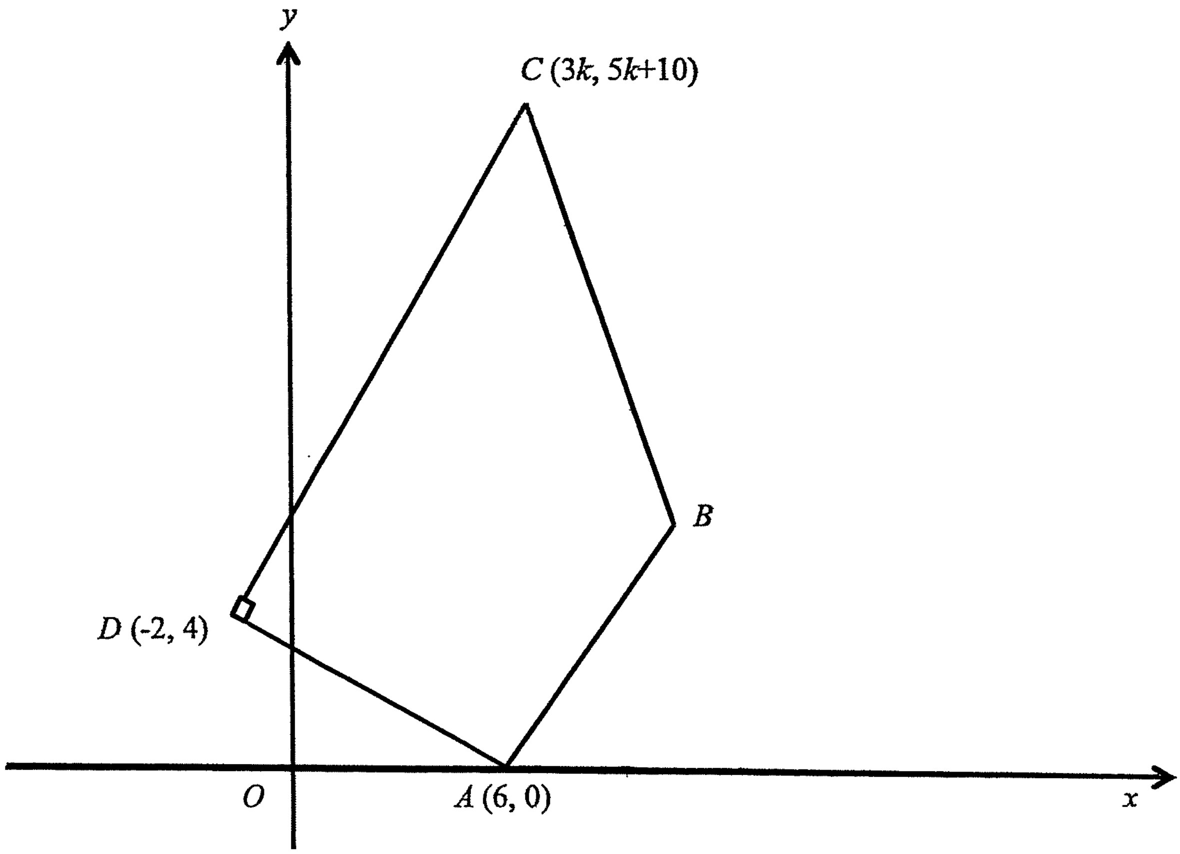
(ii) state the period of the function, **[1]**

(iii) find the maximum and minimum values of D, **[1]**

(iv) sketch the function $D = 400 + 40\sin\left(\frac{\pi}{6}t\right)$ for $0 \le t \le 24$. **[2]**

(v) estimate the number of weeks for $0 \le t \le 24$ that the population is greater than 420. **[3]**

8



The diagram shows a quadrilateral $ABCD$ in which A is (6, 0), C is $(3k, 5k + 10)$ and D is (-2, 4). The equation of line AB is $y = 2x - 12$ and angle $ADC = 90°$.

(i) Find the value of k. [3]

Given that the perpendicular bisector of CD passes through B, find

(ii) the coordinates of B, [4]

(iii) the area of the quadrilateral $ABCD$. [2]

[Turn over

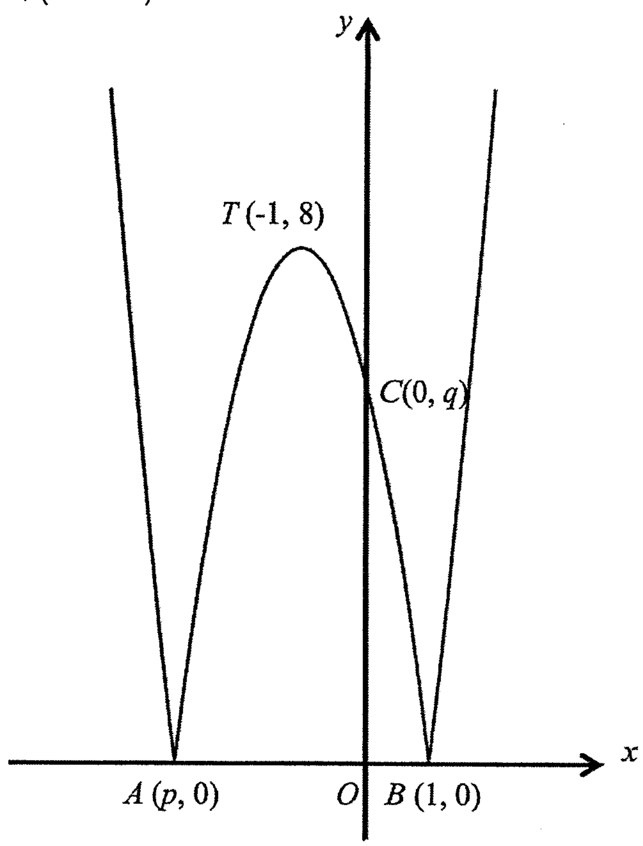
9 **(a)** The first three terms in the binomial expansion of $(1+px)^n$ are $1-48x+960x^2$.
Find the value of p and of n. **[4]**

(b) In the expansion of $\left(2x^2+\dfrac{a}{x}\right)^8$, where a is a non-zero real number, the ratio of the coefficient of the 3rd term to that of the 5th term is 5 : 2.

(i) Find the possible values of a. **[4]**

(ii) Explain whether the term independent of x exists for the expansion of $\left(2x^2+\dfrac{a}{x}\right)^8$. **[2]**

10



The diagram shows part of the curve $y=|ax^2+bx+c|$ where $a<0$.
The curve touches the x-axis at A (p, 0) and at B (1, 0).
The curve touches the y-axis at C (0, q) and has a maximum point at T (-1, 8).

(i) Explain why $p = -3$. **[1]**

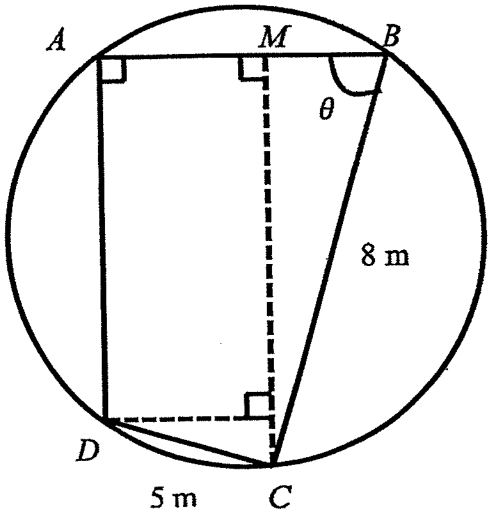
(ii) Determine the value of a, b, c and q. **[4]**

(iii) State the range of values of r for which the line $y=r$ intersects the curve $y=|ax^2+bx+c|$ at four distinct points. **[1]**

(iv) In the case where $r = 2$, find the exact x-coordinates of all points of intersection of the line $y=r$ and the curve $y=|ax^2+bx+c|$. **[4]**

[Turn over

11



The diagram shows a circular garden. A farmer decides to fence part of the garden. He puts fences around the perimeter $ABCD$ such that $BC = 8$ m, $CD = 5$ m, angle $DAB = 90^0$ and angle $ABC = \theta$ where $0^0 < \theta < 90^0$.

(i) Given that CM is perpendicular to AB, express CM and AB in terms of θ. **[4]**

(ii) Show that L m, the length of fencing needed for perimeter $ABCD$, is given by
$$L = 13 + 3\cos\theta + 13\sin\theta.$$ **[2]**

(iii) Express L in the form $13 + R\cos(\theta - \alpha)$ where $R > 0$ and α is an acute angle. **[4]**

(iv) Given that the farmer uses exactly 26.2 m of fencing, find the possible values of θ. **[3]**

12 **(a)** It is given that $\int f(x)dx = k\ cos2x - sin3x + c$, where c is a constant of integration, and that $\int_0^{\frac{\pi}{6}} f(x)dx = \frac{1}{3}$.

(i) Show that $k = -2\frac{2}{3}$. **[1]**

(ii) Find f (x). **[2]**

(b) A curve has the equation $y = g(x)$, where $g(x) = 2\sin^2 x - \sin 2x$ for $0 \le x \le \pi$.

(i) Find the x-coordinates of the stationary points of the curve. **[3]**

(ii) Use the second derivative test to determine the nature of each of these points. **[3]**

(iii) Given that $\int g(x)dx = ax + b\sin x\cos x + \cos^2 x + k$, where k is a constant of integration, find the value of a and of b. **[4]**

END OF PAPER

SECONDARY 4
2018 Preliminary Examinations

ADDITIONAL MATHEMATICS
Paper 2 **4047/2**

12 September 2018 (Wednesday) **2 hours 30 minutes**

CANDIDATE NAME	**Solutions**

CLASS		INDEX NUMBER	

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where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!\,(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for Δ *ABC*

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

[Turn over

1 Find the value of the constant k for which $y = x^2 e^{1-2x}$ is a solution of the equation

$$\frac{d^2y}{dx^2} - \frac{2y}{x^2} = k\left(\frac{dy}{dx} + y\right).$$

[4]

Solution
$y = x^2 e^{1-2x}$ $\frac{dy}{dx} = x^2(-2e^{1-2x}) + e^{1-2x}(2x)$ $= -2x^2e^{1-2x} + 2xe^{1-2x}$ $= -2y + 2x\dot{e}^{1-2x} = -2y + \frac{2y}{x}$ $\frac{d^2y}{dx^2} = -2\frac{dy}{dx} + 2x(-2e^{1-2x}) + 2e^{1-2x}$ $= -2\frac{dy}{dx} - 4xe^{1-2x} + 2e^{1-2x}$ $= -2\frac{dy}{dx} - \frac{4y}{x} + \frac{2y}{x^2}$ $\frac{d^2y}{dx^2} - \frac{2y}{x^2}$ $= -2\frac{dy}{dx} - \frac{4y}{x}$ $= -2\frac{dy}{dx} - 2(\frac{dy}{dx} + 2y)$ $= -4\frac{dy}{dx} - 4y$ $= -4(\frac{dy}{dx} + y)$ ***k* = -4**

2 **(i)** Find $\frac{d}{dx}(\frac{\ln x}{x})$. **[2]**

Solution

$$\frac{d}{dx}(\frac{\ln x}{x})$$

$$= \frac{x(\frac{1}{x}) - \ln x}{x^2}$$

$$= \frac{1 - \ln x}{x^2}$$

(ii) Hence find $\int \frac{\ln x}{x^2} dx$. **[3]**

Solution

From (i),

$$\int \frac{1-\ln x}{x^2} dx = \frac{\ln x}{x} + C$$

$$\int \frac{1}{x^2} dx - \int \frac{\ln x}{x^2} dx = \frac{\ln x}{x} + C$$

$$-\frac{1}{x} - \int \frac{\ln x}{x^2} dx = \frac{\ln x}{x} + C$$

$$\int \frac{\ln x}{x^2} dx = \frac{-1}{x} - \frac{\ln x}{x} + D$$

The curve $y = \mathrm{f}(x)$ is such that $\mathrm{f}(x) = \frac{\ln x}{x}$, for $x > 0$.

(iii) Explain why the curve $y = \mathrm{f}(x)$ has only one stationary point. **[1]**

Solution

$\mathrm{f}(x) = \frac{\ln x}{x}$

$\mathrm{f}'(x) = \frac{1 - \ln x}{x^2}$

For stationary point to exist, $\mathrm{f}'(x) = 0$

$1 - \ln x = 0$

$\ln x = 1$

$x = \mathrm{e}$

For $x > 0$, $y = \mathrm{f}(x)$ has only 1 stationary point at $x = \mathrm{e}$.

[Turn over

3 The expression $2x^3 + ax^2 + bx - 35$, where a and b are constants, has a factor of $2x - 7$ and leaves a remainder of -36 when divided by $x + 1$.

(i) Find the value of a and of b. **[4]**

(ii) Using the values of a and b found in part **(i)**, **explain** why the equation $2x^3 + ax^2 + bx - 35 = 0$ has only one real root. **[2]**

Solution
(i) $f(x) = 2x^3 + ax^2 + bx - 35$ $f(\frac{7}{2}) = 0$ $2(\frac{7}{2})^3 + a(\frac{7}{2})^2 + b(\frac{7}{2}) - 35 = 0$ $\frac{343}{4} + \frac{49}{4}a + \frac{7b}{2} - 35 = 0$ $\frac{49a}{4} + \frac{7b}{2} = \frac{-203}{4}$ $49a + 14b = -203 - - - - - - - (1)$ $f(-1) = -36$ $2(-1)^3 + a(-1)^2 + b(-1) - 35 = -36$ $-2 + a - b - 35 = -36$ $a - b = 1 - - - - - - - - - - - - (2)$ $49(1+b) + 14b = -203$ $49 + 63b = -203$ $63b = -252$ $b = -4$ $a = b + 1 = -4 + 1 = -3$ **(ii)** $2x^3 - 3x^2 - 4x - 35 = 0$ $2x^3 + ax^2 + bx - 35 = (2x-7)(x^2 + 2x + 5) = 0$ For $x^2 + 2x + 5$, since $(2)^2 - 4(1)(20) < 0$ and the coefficient of x^2 is always positive, $x^2 + 2x + 5$ is always positive.

4 As part of his job in a restaurant, John learned to cook a hot pot of soup late at night so that there would be enough for sale the next day. While refrigeration was essential to preserve the soup overnight, the soup was too hot to be put directly in the refrigerator when it was ready at $100\,°C$. The soup subsequently cools in such a way that its temperature, $x\,°C$ after t minutes, is given by the expression $x = 20 + Ae^{-kt}$, where A and k are constants.

(i) **Explain** why $A = 80$. **[1]**

Solution
Since the soup is ready at $100\,°C$ initially, At $t = 0$, $x = 20 + Ae^{0} = 100$ $A = 80$

(ii) When $t = 15$, the temperature of the soup is $58\,°C$.

Find the value of k. **[2]**

Solution
$58 = 20 + 80e^{-k(15)}$ $38 = 80e^{-15k}$ $e^{-15k} = \frac{38}{80}$ $-15k = \ln\frac{38}{80}$ $k = 0.0496$

(iii) Deduce the temperature of the soup if it is left unattended for a long period of time, **giving a reason** for your answer. **[1]**

Solution
For $x = 20 + 80e^{-kt}$, as $t \to \infty, e^{-kt} \to 0$ Temperature of the soup approaches $20\,°C$ if it is left unattended for a long period of time.

4 (iv) For the soup to be refrigerated, its temperature should be less than $35\,°\mathrm{C}$.
What is the shortest possible time, correct to the nearest minute that John has to wait before he can refrigerate the soup? **[2]**

Solution
$20+80e^{-(\frac{\ln\frac{38}{80}}{-15})t}=35$ $80e^{\frac{\ln\frac{38}{80}}{15}t}=15$ $e^{\frac{\ln\frac{38}{80}}{15}t}=\frac{15}{80}$ $\frac{\ln\frac{38}{80}}{15}t=\ln\frac{15}{80}$ $t=33.7$ Shortest possible time = 34 minutes

5 **(a)** The function f is defined, for all values of x, by

$$f(x) = x^2(3-4x).$$

Find the values of x for which f is an increasing function. **[3]**

Solution
$f(x)=3x^2-4x^3$ $f'(x)=6x-12x^2$ For f to be an increasing function, $f'(x)>0$ $6x-12x^2>0$ $6x(1-2x)>0$ $0<x<\frac{1}{2}$

[Turn over

5 (b) A particle moves along the curve $y = \dfrac{16}{(3-4x)^2}$ in such a way that the y-coordinate of the particle is increasing at a constant rate of 0.03 units per second. Find the exact y-coordinate of the particle at the instant that the x-coordinate of the particle is decreasing at 0.12 units per second. **[4]**

Solution

$$y = \frac{16}{(3-4x)^2} = 16(4-4x)^{-2}$$

$$\frac{dy}{dx} = -32(3-4x)^{-3}(-4) = \frac{128}{(3-4x)^3}$$

$$\frac{dy}{dt} = 0.03$$

$$\frac{dy}{dt} = \frac{dy}{dx} x \frac{dx}{dt}$$

$$0.03 = \frac{128}{(3-4x)^3}(-0.12)$$

$$(3-4x)^3 = -512$$

$$3-4x = -8$$

$$-4x = -11$$

$$x = \frac{11}{4}$$

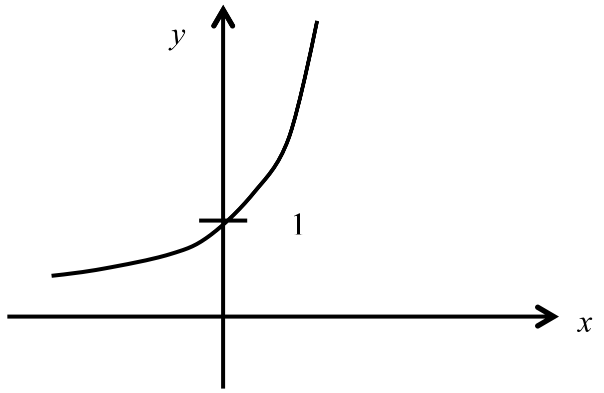
$$y = \frac{16}{(3-4(\frac{11}{4}))^2} = \frac{1}{4}$$

6 **(a) (i)** Sketch the graph of $y = 10^x$. **[1]**

(ii) Given that $\frac{4^x}{2^{x+2}} = \frac{3}{5^x}$, find the value of 10^x. **[2]**

Solution

(i)



(ii) $\frac{4^x}{2^{x+2}} = \frac{3}{5^x}$

$\frac{2^{2x}}{2^{x+2}} = \frac{3}{5^x}$

$2^{2x-(x+2)} 5^x = 3$

$2^{x-2} 5^x = 3$

$\frac{2^x}{4}(5^x) = 3$

$10^x = 12$

(b) Solve the equation $\log_2 \sqrt{5x+1} + 2\log_9 3 = \log_4(2x-3) + \log_3 27$. **[5]**

Solution

$\log_2 \sqrt{5x+1} + 2\log_9 3 = \log_4(2x-3) + \log_3 27$

$\log_2 \sqrt{5x+1} = \log_4(2x-3) + 3 - 1$

$\log_2 \sqrt{5x+1} = \frac{l\text{og}_2(2x-3)}{\log_2 2^2} + \log_2 2^2$

$\log_2 \sqrt{5x+1} = \log_2(2x-3)^{\frac{1}{2}} + \log_2 4$

$\sqrt{5x+1} = 4\sqrt{2x-3}$

$5x+1 = 16(2x-3)$

$5x+1 = 32x - 48$

$27x = 49$

$x = \frac{49}{27}$

or x=1.81 (3sf)

7 The population of a herd of deer can be modelled by the function $D = 400 + 40\sin\left(\frac{\pi}{6}t\right)$, where D is the deer population in week t of the year for $0 \le t \le 24$.

Using the model,

(i) state the amplitude of the function, **[1]**

(ii) state the period of the function, **[1]**

(iii) find the maximum and minimum values of D, **[2]**

(iv) sketch the function $D = 400 + 40\sin\left(\frac{\pi}{6}t\right)$ for $0 \le t \le 24$. **[2]**

(v) estimate the number of weeks for $0 \le t \le 24$ that the population is greater than 420. **[3]**

Solution

(i) $D = 400 + 40\sin\left(\frac{\pi}{6}t\right)$ for $0 \le t \le 24$

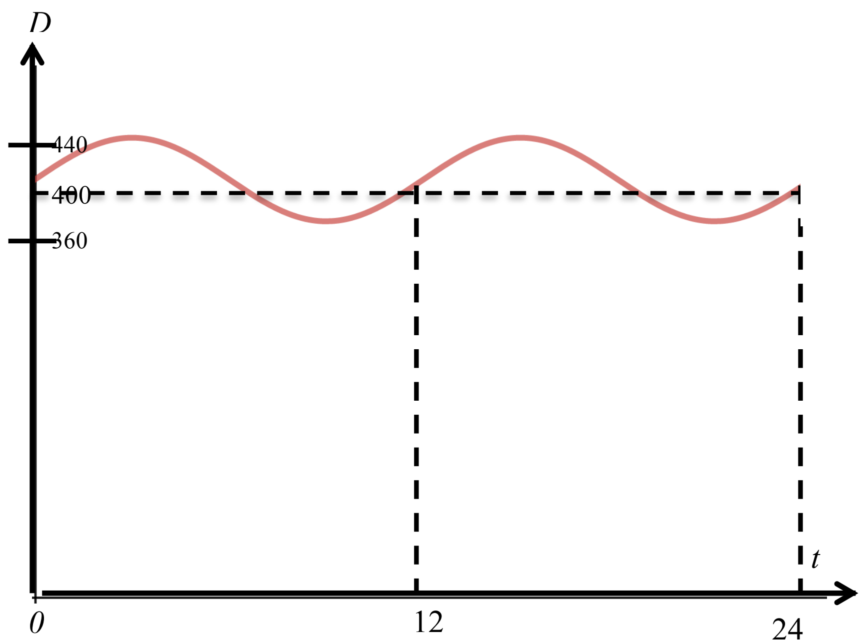
Amplitude = 40

(ii) Period = $\frac{2\pi}{\frac{\pi}{6}}$ =12

(iii) Maximum D = 400 + 40 = 440

Minimum D = 400 – 40 = 360

(iv)



(v) $400 + 40\sin\left(\frac{\pi}{6}t\right) = 420$

[Turn over

$40\sin(\frac{\pi}{6}t)=20$

$\sin(\frac{\pi}{6}t)=0.5$

Basic angle = $\frac{\pi}{6}$

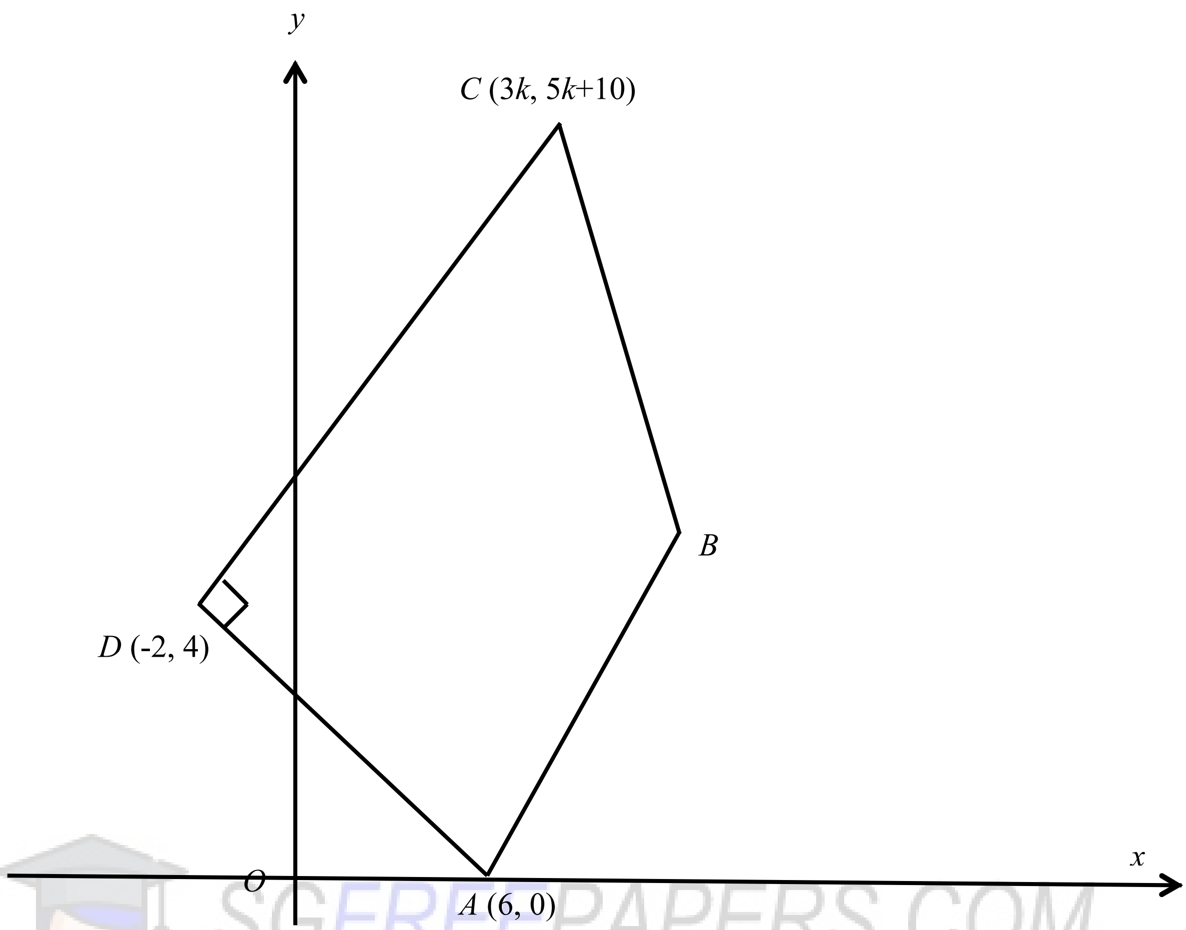
$$\frac{\pi}{6}t = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}$$

t = 1, 5, 13, 17

No of weeks = (5-1) + (17-13) = 8

[Turn over

8



The diagram shows a quadrilateral *ABCD* in which *A* is (6, 0), *C* is (3*k*, 5*k* + 10) and *D* is (-2, 4). The equation of line *AB* is $y = 2x - 12$ and angle $ADC = 90°$.

(i) Find the value of *k*. **[3]**

Solution
Gradient of line *AD* = $\frac{4-0}{-2-6} = -\frac{1}{2}$ Gradient of line *CD* = 2 $\frac{5k+10-4}{3k+2} = 2$ $5k + 6 = 6k + 4$ $k = 2$

Given that the perpendicular bisector of *CD* passes through *B*, find

[Turn over

(ii) the coordinates of *B*, **[4]**

Solution
Midpoint of line *CD* = $(\frac{6-2}{2}, \frac{24}{2})$ = (2, 12) Gradient of perpendicular bisector of *CD* = $-\frac{1}{2}$ Equation of perpendicular bisector of *CD*: $y-12=\frac{-1}{2}(x-2)$ $y=-\frac{1}{2}x+13$ To find intersection point between equation of line *AB* with perpendicular bisector of *CD*: solve simultaneously $y=-\frac{1}{2}x+13$ $y=2x-12$ $-\frac{1}{2}x+13=2x-12$ $2.5x=25$ $x=10,$ $y=8$ *B* = (10, 8)

(iii) the area of the quadrilateral *ABCD*. **[2]**

Solution
Area of *ABCD* $=\frac{1}{2}\begin{vmatrix} 6 & 10 & 6 & -2 & 6 \\ 0 & 8 & 20 & 4 & 0 \end{vmatrix}$ $=\frac{1}{2}\lvert 48+200+24-(48-40+24)\rvert$ =120 units^2

9 **(a)** The first three terms in the binomial expansion of $(1+px)^n$ are $1-48x+960x^2$. Find the value of p and of n. **[4]**

Solution

$$(1+px)^n = \binom{n}{0}(px)^0 + \binom{n}{1}(px)^1 + \binom{n}{2}(px)^2 + \cdots$$
$$= 1 + npx + \frac{n(n-1)}{2}p^2x^2 + \cdots$$

Comparing coefficients of

x ------- $np = -48$

x^2 ------- $\frac{n(n-1)}{2}p^2 = 960$

Solving by substitution: $p = \frac{-48}{n}$

$\frac{n(n-1)}{2}(\frac{-48}{n})^2 = 960$

$\frac{n-1}{n} = \frac{5}{6}$

$6n - 6 = 5n$

$n = 6$

$p = \frac{-48}{6} = -8$

(b) In the expansion of $\left(2x^2 + \frac{a}{x}\right)^8$, where a is a non-zero real number, the ratio of the coefficient of the 3rd term to that of the 5th term is 5 : 2.

(i) Find the possible values of a. **[4]**

Solution

General Term, $T_{r+1} = \binom{8}{r}(2x^2)^{8-r}(\frac{a}{x})^r$

$T_3 = \binom{8}{2}(2x^2)^6(\frac{a}{x})^2 = \binom{8}{2}(2)^6(a)^2(x)^{10}$

$T_5 = \binom{8}{4}(2x^2)^4(\frac{a}{x})^4 = \binom{8}{4}(2)^4(a)^4(x)^4$

$\frac{28(64)a^2}{70(16)a^4} = \frac{5}{2}$

$3584a^2 = 5600a^4$

$5600a^4 - 3584a^2 = 0$

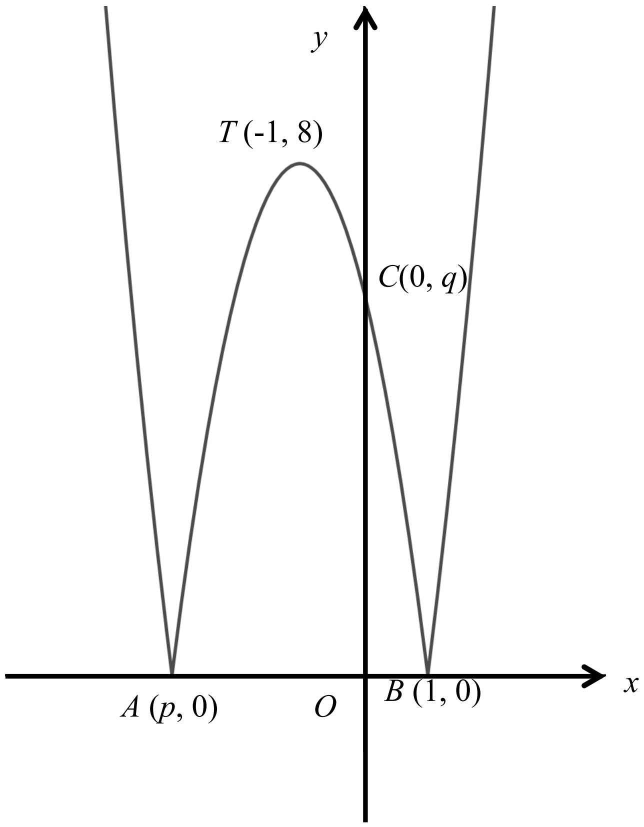
$a^2(5600a^2 - 3584) = 0$

$a = 0$ (Rejected) or $a^2 = \frac{3584}{5600}$ ==> $a = \pm\frac{4}{5}$

(ii) **Explain** whether the term independent of x exists for the expansion of $\left(2x^2 + \frac{a}{x}\right)^8$. **[2]**

Solution
For term independent of x, power of $x = 0$ Considering the terms in x of the general term, $(x^2)^{8-r}(x)^{-r} = x^{16-3r}$ Supposing $16 - 3r = 0$, $r = \frac{16}{3}$ (not a positive integer/whole number) Term independent of x does not exist.

10



The diagram shows part of the curve $y = |ax^2 + bx + c|$ where $a < 0$.
The curve touches the x-axis at A (p, 0) and at B (1, 0).
The curve touches the y-axis at C (0, q) and has a maximum point at T (-1, 8).

(i) **Explain** why $p = -3$. **[1]**

Solution
The curve is symmetrical about the line $x = -1$. x-coord of $A = p = -1 -2 = -3$

(ii) Determine the value of each of a, b, c and q. **[4]**

Solution
$y = \|m(x+3)(x-1)\|$ At x = -1, y =8 8 = $\|m(2)(-2)\|$ m = 2 or -2 For $y = \|ax^2 + bx + c\|$ where $a < 0$, a = -2 $y = \|-2x^2 + bx + c\|$ $-2x^2 + bx + c$ $= -2(x-1)(x+3)$ $= -2(x^2 + 2x - 3)$ b = -4, c = 6 At x = 0, y =6. Therefore q = 6.

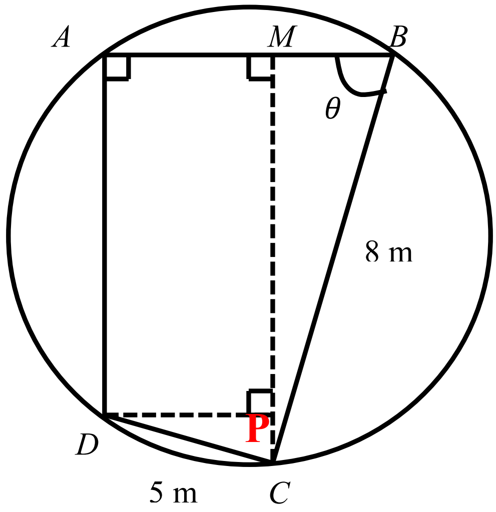
(iii) State the set of values of r for which the line $y = r$ intersects the curve $y = |ax^2 + bx + c|$ at four distinct points. **[1]**

Solution
$0 < r < 8$

(iv) In the case where $r = 2$, find the exact x-coordinates of all points of intersection of the line $y = r$ and the curve $y = |ax^2 + bx + c|$. **[4]**

Solution
Line: $y = 2$ Curve: $y = \|-2x^2 - 4x + 6\|$ $-2x^2 - 4x + 6 = 2$ $-2x^2 - 4x + 6 = -2$ $2x^2 + 4x - 4 = 0$ or $2x^2 + 4x - 8 = 0$ $x^2 + 2x - 2 = 0$ $x^2 + 2x - 4 = 0$ $x = \frac{-2 \pm \sqrt{2^2 - 4(1)(-2)}}{2(1)}$ or $x = \frac{-2 \pm \sqrt{2^2 - 4(1)(-4)}}{2(1)}$ $x = \frac{-2 \pm \sqrt{12}}{2}$ or $x = \frac{-2 \pm \sqrt{20}}{2}$ $x = -1 \pm \sqrt{3}$ $x = -1 \pm \sqrt{5}$

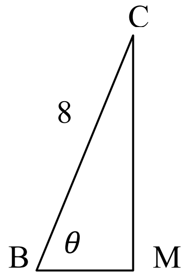
11

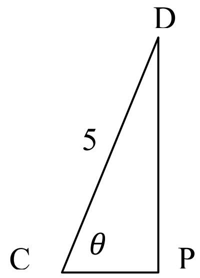


The diagram shows a circular garden. A farmer decides to fence part of the garden. He puts fences around the perimeter *ABCD* such that *BC* = 8 m, *CD* = 5 m, angle *DAB* = 90^0 and angle *ABC* = θ where $0^0 < \theta < 90^0$.

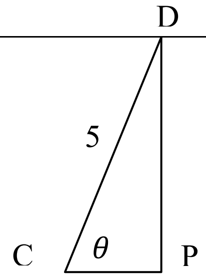
(i) Given that *CM* is perpendicular to *AB*, express *CM* and *AB* in terms of θ. **[4]**

Solution
$\sin\theta = \frac{CM}{8}$ $CM = 8sin\theta$ $\cos\theta = \frac{BM}{8}$ $BM = 8cos\theta$ $\sin\theta = \frac{DP}{5}$ $DP = 5sin\theta$ $AB = 5sin\theta + 8cos\theta$





(ii) Show that L m, the length of fencing needed for perimeter $ABCD$, is given by
$L = 13 + 3\cos\theta + 13\sin\theta$. **[2]**

Solution
$\cos\theta = \frac{CP}{5}$ $CP = 5cos\theta$ $MP = 8sin\theta - 5cos\theta = AD$  Perimeter $ABCD$ $= 5sin\theta + 8cos\theta + 8 + 5 + 8sin\theta - 5cos\theta$ $= 13 + 3cos\theta + 13sin\theta$

(iii) Express L in the form $13 + R\cos(\theta - \alpha)$ where $R > 0$ and α is an acute angle. **[4]**

Solution
$L = 13 + \sqrt{3^2 + 13^2}\cos(\theta - \alpha)$ $\quad tan\alpha = \frac{13}{3}$ $= 13 + \sqrt{178}\cos(\theta - 77.0°)$ $\quad \alpha = 77.0°$

(iv) Given that the farmer uses exactly 26.2 m of fencing, find the possible values of θ.**[3]**

Solution
$13 + \sqrt{178}\cos(\theta - 77.0°) = 26.2$ $\sqrt{178}\cos(\theta - 77.0°) = 13.2$ $\cos(\theta - 77.0°) = \frac{13.2}{\sqrt{178}}$ Basic Angle = 8.4° $\theta - 77.0° = 8.4°, -8.4°$ $\theta = 85.4°, 68.6°$

12 **(a)** It is given that $\int f(x)dx = k\cos 2x - \sin 3x + c$, where c is a constant of integration,

and that $\int_0^{\frac{\pi}{6}} f(x)\,dx = \frac{1}{3}$.

(i) Show that $k = -2\frac{2}{3}$. **[1]**

Solution
$[k\cos 2x - \sin 3x]_0^{\frac{\pi}{6}} = \frac{1}{3}$ $k\cos\frac{\pi}{3} - \sin\frac{\pi}{2} - (k\cos 0) = \frac{1}{3}$ $\frac{k}{2} - 1 - k = \frac{1}{3}$ $-\frac{k}{2} = \frac{4}{3}$ $k = -\frac{8}{3} = -2\frac{2}{3}$

(ii) Find f (x). **[2]**

Solution
$f(x) = \frac{d}{dx}(-2\frac{2}{3}\cos 2x - \sin 3x)$ $= -2\frac{2}{3}(-2\sin 2x) - 3\cos 3x$ $= \frac{16}{3}\sin 2x - 3\cos 3x$

(b) A curve has the equation $y = g(x)$, where $g(x) = 2\sin^2 x - \sin 2x$ for $0 \le x \le \pi$.

(i) Find the x-coordinates of the stationary points of the curve. **[3]**

Solutions
$y = 2\sin^2 x - \sin 2x$ $\frac{dy}{dx} = 4\sin x\cos x - 2\cos 2x = 0$ $2\sin 2x - 2\cos 2x = 0$ $\sin 2x = \cos 2x$ $\tan 2x = 1$ Basic Angle = $\frac{\pi}{4}$ $2x = \frac{\pi}{4}, \frac{5\pi}{4}$ $x = \frac{\pi}{8}, \frac{5\pi}{8}$

(ii) Use the second derivative test to determine the nature of each of these points. **[3]**

Solution
$\frac{d^2y}{dx^2} = 4\cos 2x - 2(-2\sin 2x)$ $= 4\cos 2x + 4\sin 2x$ At $x = \frac{\pi}{8}$, $\frac{d^2y}{dx^2} = 4\cos\frac{\pi}{4} + 4\sin\frac{\pi}{4} > 0$ Minimum point at $x = \frac{\pi}{8}$. At $x = \frac{5\pi}{8}$, $\frac{d^2y}{dx^2} = 4\cos\frac{10\pi}{8} + 4\sin\frac{10\pi}{8} < 0$ Maximum point at $x = \frac{5\pi}{8}$.

(iii) Given that $\int g(x)dx = ax + b\sin x\cos x + \cos^2 x + k$, where k is a constant of integration, find the value of a and of b. **[4]**

Solutions
$\int 2\sin^2 x - \sin 2x \, dx$ $= \int 1 - cos2x - sin2x \, dx$ $= x - \frac{sin2x}{2} + \frac{cos2x}{2} + C$ $= x - \frac{2sinxcosx}{2} + \frac{2cos^2x - 1}{2} + C$ $= x - sinxcosx + cos^2x - \frac{1}{2} + C$ $a = 1$, $b = -1$

END OF PAPER

Name: ________________________ (   )   Class: ___________

PRELIMINARY EXAMINATION
GENERAL CERTIFICATE OF EDUCATION ORDINARY LEVEL

ADDITIONAL MATHEMATICS **4047/01**

Paper 1 **Thursday 16 August 2018**

2 hours

Additional Materials: Answer Paper

READ THESE INSTRUCTIONS FIRST

Write your name, class, and index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use paper clips, highlighters, glue, or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Paper provided
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, staple all your work together with this cover sheet.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is **80**.

FOR EXAMINER'S USE

Q1		Q6		Q11	
Q2		Q7			
Q3		Q8			
Q4		Q9			
Q5		Q10			

/ 80

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!\,r!} = \frac{n(n-1)........(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1.$$

$$\sec^2 A = 1 + \tan^2 A.$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A.$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

1 Express $\dfrac{3x^3+2x^2+4x-1}{x^3+x^2}$ in partial fractions. [4]

2 A cylinder has a radius of $(1+2\sqrt{2})$ cm and its volume is $\pi(84+21\sqrt{2})\,\text{cm}^3$.
Find, **without using a calculator**, the exact length of the height of the cylinder in the form $(a+b\sqrt{2})$ cm, where a and b are integers. [5]

3 **(i)** Sketch the graph of $y=4-3\sin 2x$ for $0\le x\le \pi$. [3]

(ii) State the range of values of k for which $4-3\sin 2x=k$ has two roots for $0\le x\le \pi$. [2]

4 **Solutions to this question by accurate drawing will not be accepted.**

PQRS is a parallelogram in which the coordinates of the points *P* and *R* are $(-5, 8)$ and $(6, -2)$ respectively. Given that *PQ* is perpendicular to the line $y=-\dfrac{1}{2}x+3$ and *QR* is parallel to the *x* axis, find

(i) the coordinates of *Q* and of *S*, [5]

(ii) the area of *PQRS*. [2]

5 **(i)** Differentiate $\dfrac{\ln x}{x}$ with respect to *x*. [3]

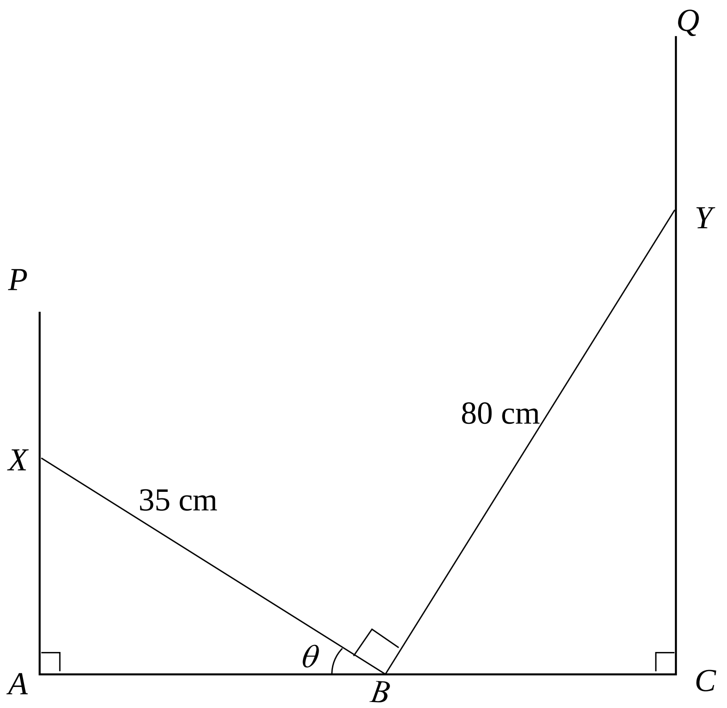
(ii) Hence find $\displaystyle\int \frac{\ln x^2}{x^2}\,dx$. [4]

6 **(i)** Show that $\dfrac{2}{\tan\theta + \cot\theta} = \sin 2\theta$. [3]

(ii) Hence find the value of *p*, giving your answer in terms of π, for which

$$\int_0^p \frac{4}{\tan 2x + \cot 2x}\,\mathrm{d}x = \frac{1}{4}, \text{ where } 0 < p < \frac{\pi}{4}.$$ [4]

7



In the diagram *XBY* is a structure consisting of a beam *XB* of length 35 cm attached at *B* to another beam *BY* of length 80 cm so that angle $XBY = 90^\circ$. Small rings at *X* and *Y* enable *X* to move along the vertical wire *AP* and Y to move along the vertical wire *CQ*. There is another ring at *B* that allows *B* to move along the horizontal line *AC*. Angle $ABX = \theta$ and θ can vary.

(i) Show that $AC = (35\cos\theta + 80\sin\theta)$ cm. [2]

(ii) Express *AC* in the form of $R\sin(\theta + \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. [4]

(iii) Tom claims that the length of *AC* is 89cm. Without measuring, Mary said that this was not possible. Explain how Mary came to this conclusion. [1]

8 **(a)** Find the range of values of *p* for which $px^2 + 4x + p > 3$ for all real values of *x*. [5]

(b) Find the range of values of *k* for which the line $5y = k - x$ does not intersect the curve $5x^2 + 5xy + 4 = 0$. [5]

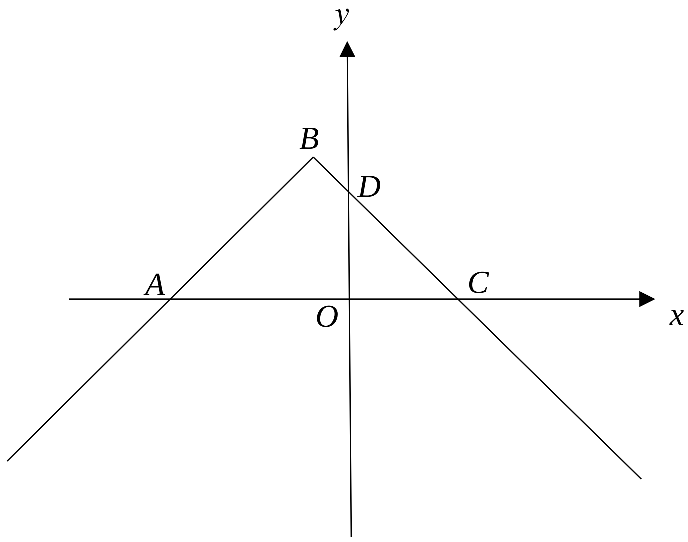
9 The diagram shows part of the graph of $y = 4 - |x+1|$.

(i) Find the coordinates of the points *A*, *B*, *C* and *D*. [5]

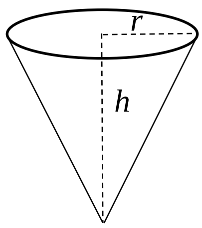
(ii) Find the number of solutions of the equation $4 - |x+1| = mx + 3$ when

(a) $m = 2$ **(b)** $m = -1$ [2]

(iii) State the range of values of *m* for which the equation $4 - |x+1| = mx + 3$ has two solutions. [1]



10 The diagram shows a cone of radius *r* cm and height *h* cm. It is given that the volume of the cone is 10π cm^3.



(i) Show that the curved surface area, *A* cm^2, of the cone, is $A = \dfrac{\pi\sqrt{r^6 + 900}}{r}$. [3]

(ii) Given that *r* can vary, find the value of *r* for which *A* has a stationary value. [4]

(iii) Determine whether this value of *A* is a maximum or a minimum. [2]

11 The equation of a curve is $y = x(2-x)^3$.

(i) Find the range of values of *x* for which *y* is an increasing function. [5]

(ii) Find the coordinates of the stationary points of the curve. [3]

(iii) Hence, sketch the graph of $y = x(2-x)^3$. [3]

St Nicholas Girls School Additional Mathematics Preliminary Examination Paper I 2018

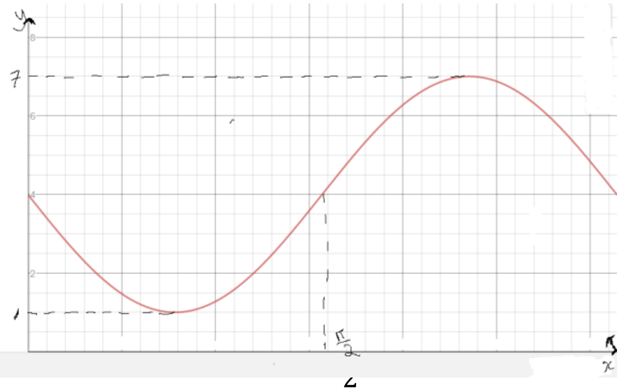
Answers

Paper 1

1. $3 + \frac{5}{x} - \frac{1}{x^2} - \frac{6}{x+1}$

2. $(12 - 3\sqrt{2}\ )$ cm

3 (i)



(ii) $1 < k < 4$ or $4 < k < 7$

4 (i) $Q(-10,\ -2)$, $S(11, 8)$ (ii) 160 units^2

5 (i) $\frac{1-\ln x}{x^2}$ (ii) $2\left(-\frac{1}{x} - \frac{\ln x}{x}\right) + c$

6 (ii) $\frac{\pi}{12}$

7 (ii) $5\sqrt{305}\sin(\theta + 23.6^\circ)$ cm or $87.3\sin(\theta + 23.6^\circ)$ cm

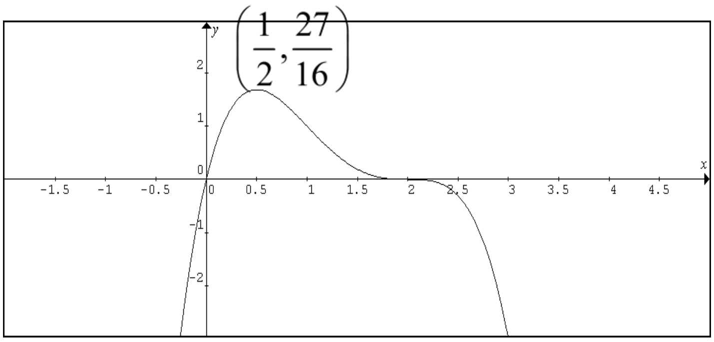
(iii) The maximum value of AC=87.3cm <89 cm

8 (a) $p > 4$ (b) $-8 < k < 8$

9 (i) $A(-5,0)$, B(–1,4), C(3,0), D(0,3) (ii) (a) 1 (b) infinite (iii) 2

10 (ii) 2.77 (iii) minimum

11 (i) $x < \frac{1}{2}$ (ii) $(2,0)$ $\left(\frac{1}{2}, \frac{27}{16}\right)$ (iii)



Name: ________________________ (   )   Class: ___________

PRELIMINARY EXAMINATION
GENERAL CERTIFICATE OF EDUCATION ORDINARY LEVEL

ADDITIONAL MATHEMATICS   4047/01

Paper 1   **Marking Scheme**   **Thursday 16 August 2018**

2 hours

Additional Materials: Answer Paper

READ THESE INSTRUCTIONS FIRST

Write your name, class, and index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use paper clips, highlighters, glue, or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Paper provided
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, staple all your work together with this cover sheet.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is **80**.

FOR EXAMINER'S USE

Q1		Q6		Q11	
Q2		Q7			
Q3		Q8			
Q4		Q9			
Q5		Q10			

/ 80

This document consists of **5** printed pages.

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[Turn over

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!\,r!} = \frac{n(n-1)........(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1.$$

$$\sec^2 A = 1 + \tan^2 A.$$

$$\text{cosec}^2 A = 1 + \cot^2 A.$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 Express $\dfrac{3x^3+2x^2+4x-1}{x^3+x^2}$ in partial fractions. [4]

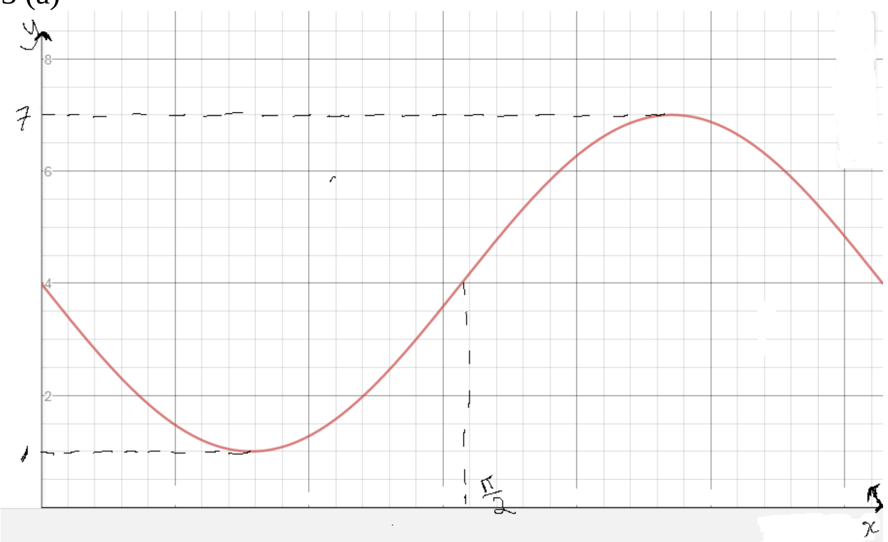
1 Long division: $x^3+x^2 \overline{)\,3x^3+2x^2+4x-1}$ quotient 3; $3x^3+3x^2$; remainder $-x^2+4x-1$ $\dfrac{3x^3+2x^2+4x-1}{x^2+x^3}=3+\frac{-x^2+4x-1}{x^2(x+1)}$ $\dfrac{-x^2+4x-1}{x^2(x+1)}=\dfrac{A}{x}+\dfrac{B}{x^2}+\dfrac{c}{x+1}$ $-x^2+4x-1=Ax(x+1)+B(x+1)+cx^2$ Let $x=-1$ $\quad -1-4-1=c$ $c=-6$ Let $x=0$ $\quad$ B= –1 $-x^2+4x-1=Ax(x+1)-1(x+1)-6x^2$ Let $x=1$ $\quad -1+4-1=2A-2-6$ $A=5$ $\dfrac{3x^3+2x^2+4x-1}{x^2+x^3}=3+\frac{5}{x}-\frac{1}{x^2}-\frac{6}{x+1}$ **[4]**	 M1√ M1√ M1√ A1
If • $\dfrac{3x^3+2x^2+4x-1}{x^2+x^3}=\frac{A}{x}+\frac{B}{x^2}+\frac{c}{x+1}$ • $\dfrac{3x^3+2x^2+4x-1}{x^2+x^3}=3+\frac{Ax+B}{x^2}+\frac{c}{x+1}$ • $\dfrac{3x^3+2x^2+4x-1}{x^2+x^3}=\frac{Ax+B}{x^2}+\frac{c}{x+1}$	 Max 3m 3m 2m

2 A cylinder has a radius of $(1+2\sqrt{2})$ cm and its volume is $\pi(84+21\sqrt{2})$ cm^3.
Find, **without using a calculator**, the exact length of the height of the cylinder in the form $(a+b\sqrt{2})$ cm, where a and b are integers. [5]

2. $\pi(84+21\sqrt{2}) = \pi(1+2\sqrt{2})^2 \times h$	B1	
$h = \dfrac{84+21\sqrt{2}}{(1+2\sqrt{2})^2}$	M1	expansion
$h = \dfrac{84+21\sqrt{2}}{1+4\sqrt{2}+8}$	M1√	Conjugate surd
$h = \dfrac{(84+21\sqrt{2})(4\sqrt{2}-9)}{(4\sqrt{2}+9)(4\sqrt{2}-9)}$	M1√	For either expansion
$h = \dfrac{756-336\sqrt{2}+189\sqrt{2}-168}{81-32}$		
$h = \dfrac{588-147\sqrt{2}}{49}$		
$h = (12-3\sqrt{2})$ cm **[5]**	A1	No unit, overall -1m

3 **(i)** Sketch the graph of $y = 4 - 3\sin 2x$ for $0 \le x \le \pi$. [3]

(ii) State the range of values of k for which $4 - 3\sin 2x = k$ has two roots for $0 \le x \le \pi$. [2]

3 (a)  [3]	B1 any one pt B1 2nd pt The 2 pts must be differernt nature B1 perfect	• sine shape • -ve sine shape • 1 cycle • Amplitude • shift +4 up ignoring no labelling of axes
3 (b) $1<k<4$ or $4<k<7$ [2] **Alternative** **[5]** $1<k<7, k \ne 4$	B1 + B1 B1 + B1	

4 **Solutions to this question by accurate drawing will not be accepted.**

PQRS is a parallelogram in which the coordinates of the points *P* and *R* are $(-5, 8)$ and $(6, -2)$ respectively. Given that *PQ* is perpendicular to the line $y=-\frac{1}{2}x+3$ and *QR* is parallel to the *x* axis, find

(i) the coordinates of *Q* and of *S*, [5]

(ii) the area of *PQRS*. [2]

1(i)	Since *QR* parallel to the *x* axis, $y_Q=-2$. Since *PQ* is perpendicular to the line $y=-\frac{1}{2}x+3$, gradient of *PQ* = 2 $\frac{(-2)-(8)}{x_Q-(-5)}=2$ $-10=2x_Q+10$ $x_Q=-10$ $Q(-10,\ -2)$ Midpoint of *PR* = Midpoint of *QS* or by inspection $\left(\frac{(-5)+(6)}{2},\ \frac{(8)+(-2)}{2}\right)=\left(\frac{(-10)+x_s}{2},\ \frac{(-2)+y_s}{2}\right)$ $1=-10+x_s$ $6=-2+y_s$ $x_s=11$ $y_s=8$ *S*(11, 8) [5]	B1 B1 (⊥ gradient) M1 A1 B1
(ii)	Area of *PQRS* $=\frac{1}{2}\begin{vmatrix}-5 & -10 & 6 & 11 & -5\\ 8 & -2 & -2 & 8 & 8\end{vmatrix}$ $=\frac{1}{2}\left\|(10+20+48+88)-(-80-12-22-40)\right\|$ or (5+11)(8+2) $=\frac{1}{2}\left\|320\right\|$ [2] $=160$ units^2 **[7]**	√M1 A1 no unit overall -1m

5 **(i)** Differentiate $\dfrac{\ln x}{x}$ with respect to x. [3]

(ii) Hence find $\displaystyle\int \frac{\ln x^2}{x^2}\,\mathrm{d}x$. [4]

(i)	$\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\dfrac{\ln x}{x}\right) = \dfrac{x\left(\frac{1}{x}\right) - \ln x}{x^2}$	B1 +B1	Either $v\frac{du}{dx}$ $or u\frac{dv}{dx}$ with the use of quotient rule /product rule perfect
	$= \dfrac{1 - \ln x}{x^2}$ [3]	A1	
(ii)	$\displaystyle\int \frac{1 - \ln x}{x^2}\,\mathrm{d}x = \frac{\ln x}{x}$	M1	Integration is the reverse process of differentiation
	$\displaystyle\int \frac{1}{x^2}\,\mathrm{d}x - \int \frac{\ln x}{x^2}\,\mathrm{d}x = \frac{\ln x}{x}$ $\displaystyle\int x^{-2}\,\mathrm{d}x - \frac{\ln x}{x} = \int \frac{\ln x}{x^2}\,\mathrm{d}x$	M1	Making $\displaystyle\int \frac{\ln x}{x^2}\,\mathrm{d}x$ the subject or split the expression
	$\dfrac{x^{-1}}{-1} - \dfrac{\ln x}{x} = \displaystyle\int \frac{\ln x}{x^2}\,\mathrm{d}x$ $\displaystyle\int \frac{\ln x}{x^2}\,\mathrm{d}x = -\frac{1}{x} - \frac{\ln x}{x}$	B1	Integration of x^{-2}
	$\displaystyle\int \frac{\ln x^2}{x^2}\,\mathrm{d}x = 2\int \frac{\ln x}{x^2}\,\mathrm{d}x$ $= 2\left(-\dfrac{1}{x} - \dfrac{\ln x}{x}\right) + c$ [4] **[7]**	A1	With c

[Turn over

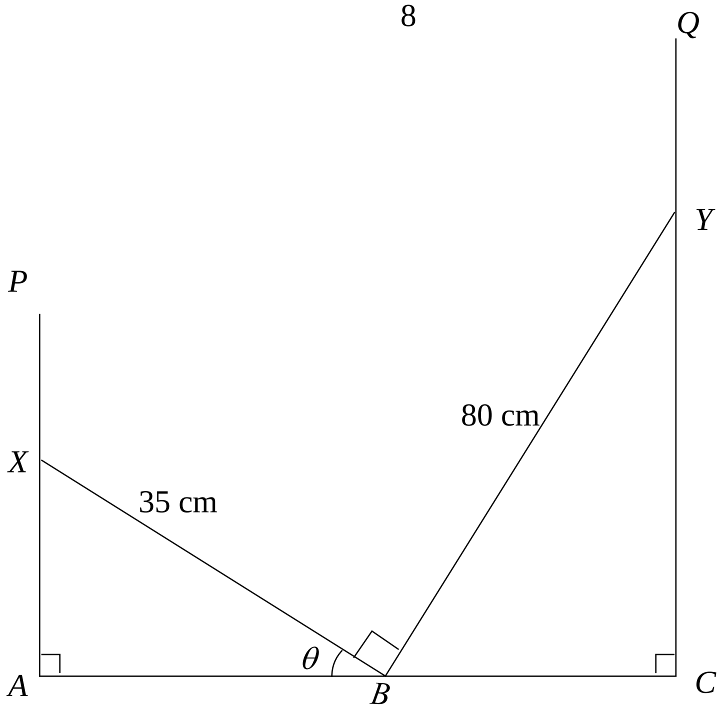
6 **(i)** Show that $\dfrac{2}{\tan\theta+\cot\theta}=\sin 2\theta$. [3]

(ii) Hence find the value of p, giving your answer in terms of π, for which

$\displaystyle\int_0^p \frac{4}{\tan 2x+\cot 2x}\,dx=\frac{1}{4}$, where $0 < p < \dfrac{\pi}{4}$. [4]

(i)	$\dfrac{2}{\tan\theta+\cot\theta}=2\div\left(\dfrac{\sin\theta}{\cos\theta}+\dfrac{\cos\theta}{\sin\theta}\right)$ $=2\div\left(\dfrac{\sin^2\theta+\cos^2\theta}{\cos\theta\sin\theta}\right)$ $=2\div\left(\dfrac{1}{\cos\theta\sin\theta}\right)$ $=2\sin\theta\cos\theta$ $=\sin 2\theta$ [3]	B1 M1 M1	change to sin and cos combine terms for identity …to the end. (must show "1")
(ii)	$\displaystyle\int_0^p \frac{4}{\tan 2x+\cot 2x}\,\mathrm{dx}$ $\displaystyle=2\int_0^p \sin 4x\,\mathrm{dx}$ $=2\left[-\dfrac{\cos 4x}{4}\right]_0^p$ $=\left(-\dfrac{1}{2}\cos 4p\right)-\left(-\dfrac{1}{2}\cos 0\right)$ $=-\dfrac{1}{2}\cos 4p+\dfrac{1}{2}$ $\displaystyle\int_0^p \frac{4}{\tan 2x+\cot 2x}\,dx=\frac{1}{4}$ $-\dfrac{1}{2}\cos 4p+\dfrac{1}{2}=\dfrac{1}{4}$ $-\dfrac{1}{2}\cos 4p=-\dfrac{1}{4}$ $\cos 4p=\dfrac{1}{2}$ $4p=\dfrac{\pi}{3}$ $p=\dfrac{\pi}{12}$ [4] **[7]**	B1 M1 M1 A1	 integrate their sinkx for substitution in their integral

7

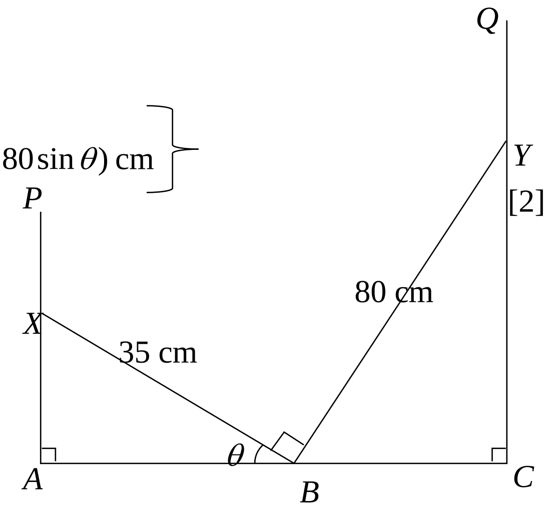


In the diagram *XBY* is a structure consisting of a beam *XB* of length 35 cm attached at *B* to another beam *BY* of length 80 cm so that angle $XBY = 90^\circ$. Small rings at *X* and *Y* enable *X* to move along the vertical wire *AP* and Y to move along the vertical wire *CQ*. There is another ring at *B* that allows *B* to move along the horizontal line *AC*. Angle $ABX = \theta$ and θ can vary.

(i) Show that $AC = (35\cos\theta + 80\sin\theta)$ cm. [2]

(ii) Express *AC* in the form of $R\sin(\theta+\alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. [4]

(iii) Tom claims that the length of *AC* is 89cm. Without measuring, Mary said that this was not possible. Explain how Mary came to this conclusion. [1]

7 (i)	$AB = 35\cos\theta$ $\angle YBC = 90^\circ - \theta$ $\angle BYC = \theta$ $BC = 80\sin\theta$ $AC = (35\cos\theta + 80\sin\theta)$ cm  [2]	B1 either AB or BC B1 -1m overall for no unit
7 (ii)	$R\sin(\theta+\alpha) = R\sin\theta\cos\alpha + R\cos\theta\sin\alpha$ $AC = 35\cos\theta + 80\sin\theta$ $R\sin\alpha = 35$ $R\cos\alpha = 80$ $R^2\cos^2\alpha + R^2\sin^2\alpha = 80^2 + 35^2$ $R = \sqrt{80^2+35^2}$ $R^2 = 7625$ $R = 87.3$ *or* $5\sqrt{305}$ $\frac{R\sin\alpha}{R\cos\alpha} = \frac{35}{80}$ $\tan\alpha = \frac{35}{80}$ $\alpha = 23.6^\circ$ $AC = 35\cos\theta + 80\sin\theta$ $35\cos\theta + 80\sin\theta = 5\sqrt{305}\sin(\theta + 23.6^\circ)$ cm or $87.3\sin(\theta + 23.6^\circ)$ cm [4]	B1 M1 for R M1 for $\tan\alpha = \frac{35}{80}$ A1
7 (iii)	The maximum value of *AC*=87.3cm Therefore it is not possible for the length to be more than that. **<u>Alternative</u>** $5\sqrt{305}\sin(\theta + 23.6^\circ) = 89$ $\sin(\theta + 23.6^\circ) = \frac{89}{5\sqrt{305}}$ No Solution Therefore it is not possible for the length to be more than that. [1] **[7]**	DB1 DB1

[Turn over

8 **(a)** Find the range of values of p for which $px^2+4x+p>3$ for all real values of x. [5]

(b) Find the range of values of k for which the line $5y=k-x$ does not intersect the curve $5x^2+5xy+4=0$. [5]

(a)	$px^2+4x+p>3$ for all real values of x $px^2+4x+p-3>0$ for all real values of x, D<0 $4^2-4(p)(p-3)<0$ $16-4p^2+12p<0$ $4p^2-12p-16>0$ $p^2-3p-4>0$ $(p-4)(p+1)>0$ $p<-1,\ p>4$ NA As $p>0$ [5]		M1 M1 M1 DA1+DA1	D<0 with substitution For b^2-4ac For factorisation Upon correct factorisation Ignore"and" and no p>0
(b)	$5y=k-x$ $5x^2+5xy+4=0$ $5x^2+5x\left(\frac{k-x}{5}\right)+4=0$ $5x^2+kx-x^2+4=0$ $4x^2+kx+4=0$ $k^2-4(4)(4)<0$	$5(k-5y)^2+5(k-5y)y+4=0$ $5k^2-50ky+125y^2+5ky-25y^2+4=0$ $100y^2-45ky+5k^2+4=0$ $(-45k)^2-400(5k^2+4)<0$ $2025k^2-2000k^2-1600<0$ $k^2-64<0$ $(k-8)(k+8)<0$	M1 M1 +M1√ M1	For substitution D<0 with substitution For b^2-4ac factorisation
	$-8<k<8$ [5] **[10]**		DA1	Upon correct factorisation

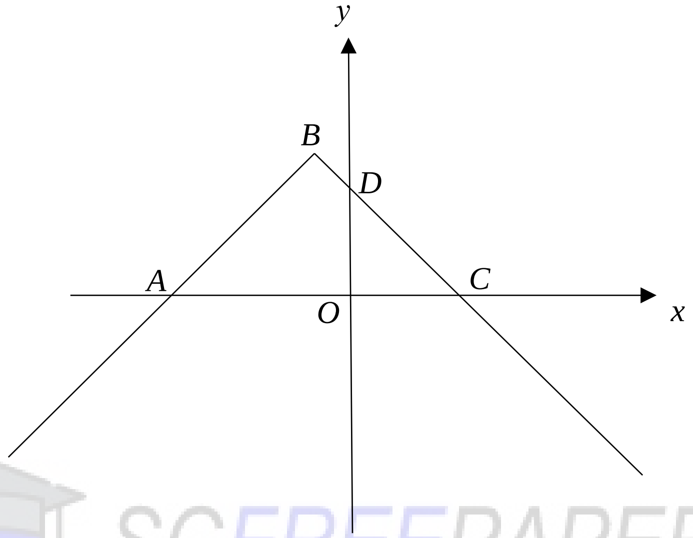
9 The diagram shows part of the graph of $y = 4 - |x+1|$.

(i) Find the coordinates of the points *A*, *B*, *C* and *D*. [5]

(ii) Find the number of solutions of the equation $4 - |x+1| = mx + 3$ when

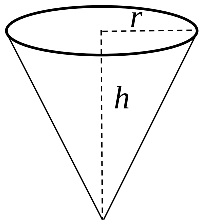
(a) $m = 2$ **(b)** $m = -1$ [2]

(iii) State the range of values of *m* for which the equation $4 - |x+1| = mx + 3$ has two solutions. [1]



(i)	B(−1,4), D(0,3) $4 - \|x+1\| = 0$ $\|x+1\| = 4$ $x + 1 = \pm 4$ $x + 1 = 4$ or $x + 1 = -4$ $x = 3$ or $x = -5$ *A*(−5,0) *C*(3,0) [5]	A1+A1 B1 A1 +A1
(ii) (a) (b)	$4 - \|x+1\| = mx + 3$ When *m* =2, the number of solutions is 1 When *m*=− 1, the number of solutions is infinite [2]	 A1 A1
(iii)	When $-1 < m < 1$, the number of solutions is 2 [1] **[8]**	A1

10 The diagram shows a cone of radius r cm and height h cm. It is given that the volume of the cone is 10π cm^3.



(i) Show that the curved surface area, A cm^2, of the cone, is $A = \dfrac{\pi\sqrt{r^6 + 900}}{r}$. [3]

(ii) Given that r can vary, find the value of r for which A has a stationary value. [4]

(iii) Determine whether this value of A is a maximum or a minimum. [2]

10(i)	Volume = $\frac{1}{3}\pi r^2 h = 10\pi$ $h = \frac{30}{r^2}$ $l^2 = r^2 + h^2$ $= r^2 + \left(\frac{30}{r^2}\right)^2$ $l = \sqrt{r^2 + \frac{900}{r^4}}$ $A = \pi r l = \pi r\sqrt{r^2 + \frac{900}{r^4}}$ $A = \pi r\sqrt{\frac{(r^6+900)}{r^4}}$ $A = \frac{\pi r\sqrt{(r^6+900)}}{r^2}$ $A = \frac{\pi\sqrt{(r^6+900)}}{r}$ [3]	B1 M1 A1	If put cm^2 -1m over all
(ii)	$u = \pi\sqrt{r^6+900}$, $v = r$ $\frac{du}{dr} = \frac{1}{2} \times \pi \times (r^6+900)^{-\frac{1}{2}} \times 6r^5$ $\frac{dv}{dr} = 1$ $\frac{du}{dr} = 3\pi r^5 (r^6+900)^{-\frac{1}{2}}$ $\frac{dA}{dr} = \frac{3\pi r^6 (r^6+900)^{-\frac{1}{2}} - \pi(r^6+900)^{\frac{1}{2}}}{r^2}$ When $\frac{dA}{dr} = 0$ $\frac{\pi(r^6+900)^{-\frac{1}{2}}[3r^6 - r^6 - 900]}{r^2} = 0$ $\frac{\pi[3r^6 - r^6 - 900]}{r^2\,(r^6+900)^{\frac{1}{2}}} = 0$ $2r^6 - 900 = 0$ $r^6 = 450$ $r = 2.77$ [4]	B1 B1 M1 A1	Either $u\frac{dv}{dx}$ or $v\frac{du}{dx}$ With the use of quotient rule or product rule Perfect $\frac{dA}{dr} = 0$ with substitution With cm -1m overall

<table>
<tr>
<td>(iii)</td>
<td>
<table>
<tr><td>r</td><td>r < 2.768</td><td>r = 2.768</td><td>r > 2.768</td></tr>
<tr><td>$\frac{dA}{dr}$</td><td>-</td><td>0</td><td>+</td></tr>
<tr><td>Sketch</td><td>\</td><td>–</td><td>/</td></tr>
</table>
A is a minimum when r = 2.77
[2]
[9]
</td>
<td>M1

DA1</td>
<td>For subst with + r

Upon correct $\frac{dA}{dr}$</td>
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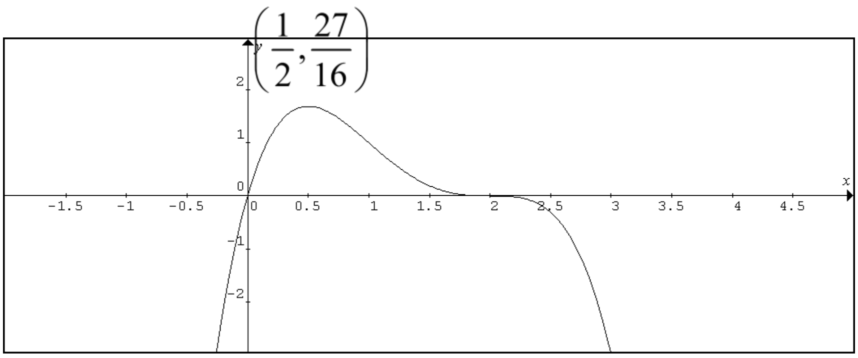
11 The equation of a curve is $y = x(2-x)^3$.

(i) Find the range of values of x for which y is an increasing function. [5]

(ii) Find the coordinates of the stationary points of the curve. [3]

(iii) Hence, sketch the graph of $y = x(2-x)^3$. [3]

$y = x(2-x)^3$ $\frac{dy}{dx} = (2-x)^3(1) - 3x(2-x)^2$ $= (2-x)^2[2-x-3x]$ $= (2-x)^2(2-4x)$ when $\frac{dy}{dx} > 0$, $2-4x > 0$ $-4x > -2$ $x < \frac{1}{2}$ Note: should not see this $2-x>0$ $-x>-2$ $x<2$ No [A1] [5]	B1 +B1 A1 M1 A1	Either $v\frac{du}{dx}$ or $u\frac{dv}{dx}$ and the use of product rule Perfect for $\frac{dy}{dx} > 0$ with substitution
when $\frac{dy}{dx} = 0$, $(2-x)^2(2-4x) = 0$ $x = 2$, $x = \frac{1}{2}$ $y = 2(2-2)^3$ $y = \frac{1}{2}\left(2-\frac{1}{2}\right)^3$ $= 0$ $= \frac{27}{16}$ Ans $(2,0)$ $\left(\frac{1}{2}, \frac{27}{16}\right)$ [3]	M1 A1+A1	$\frac{dy}{dx} = 0$ with substitution If $(2-x)(2-4x) = 0$ don't penalise]
 [3] **[11]**	B1√ B1√ B1	their max pt $\left(\frac{1}{2}, \frac{27}{16}\right)$ (2,0) their pt of inflexion (0,0) -1m for less than perfect

Name: __________________________ (   )   Class: _____________

PRELIMINARY EXAMINATION
GENERAL CERTIFICATE OF EDUCATION ORDINARY LEVEL

ADDITIONAL MATHEMATICS 4047/02

Paper 2

Friday 17 August 2018

2 hours 30 minutes

Additional Materials: Answer Paper
Graph Paper

READ THESE INSTRUCTIONS FIRST

Write your name, class, and index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use paper clips, highlighters, glue, or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Paper provided
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, staple all your work together with this cover sheet.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is **100**.

FOR EXAMINER'S USE

Q1		Q5		Q9	
Q2		Q6		Q10	
Q3		Q7		Q11	
Q4		Q8		Q12	

/ 100

This document consists of **5** printed pages.

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Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)........(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1.$$

$$\sec^2 A = 1 + \tan^2 A.$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A.$$

$$\sin (A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

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Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

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1 **(i)** On the same axes sketch the curves $y^2 = 64x$ and $y = -x^2$. [2]

(ii) Find the equation of the line passing through the points of intersection of the two curves. [4]

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The roots of the equation $x^2 + qx + 27 = 0$, where q is a constant, are α^3 and β^3 .
Find the value of p and of q. [6]

3 **(a)** Given that $3^{2x-2} \times 5^{-2x} = 27^x \div 5^{x+1}$, evaluate the exact value of 15^x. [3]

(b) Given that $\log_x y = 64 \log_y x$, express y in terms of x. [4]

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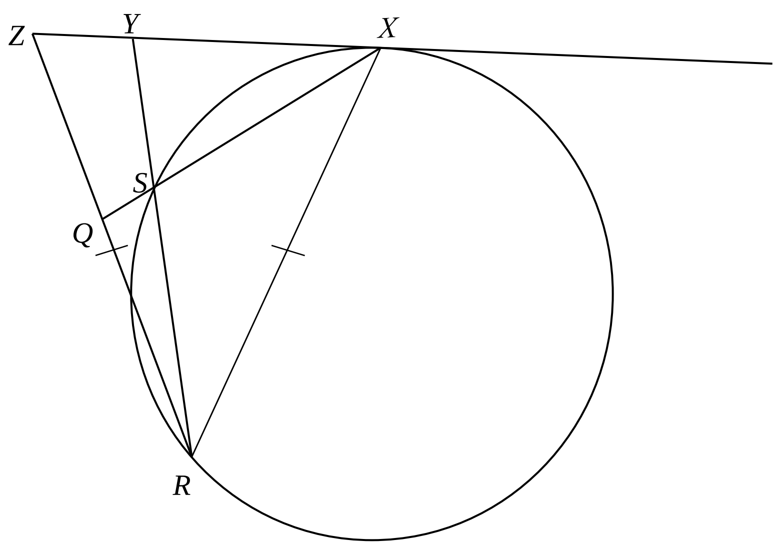
(ii) The first three terms in the expansion, in ascending powers of x, of $(2 + 3x^2)(1 - \frac{x^2}{2})^n$ are $2 - px^2 + 2x^4$, where p is an integer. Find the value of n and of p. [5]

5



In the figure, XYZ is a straight line that is tangent to the circle at X.
XQ bisects $\angle RXZ$ and cuts the circle at S. RS produced meets XZ at Y and $ZR = XR$.
Prove that

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Measured values of A and t are given in the table below.

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A (°C)	49.1	40.2	32.9	26.9

(i) Plot ln A against t for the given data and draw a straight line graph. [2]

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(i) Find the coordinates of the centre of the circle and the radius of the circle. [3]

The highest point of the circle is *A*.

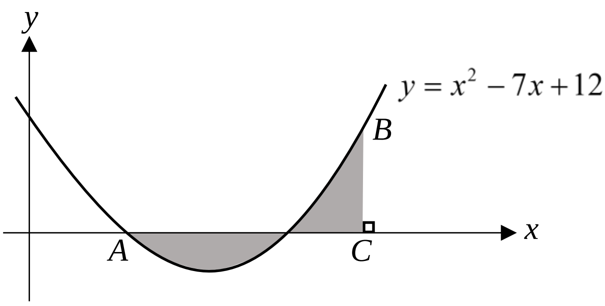
(ii) State the equation of the tangent to the circle at *A*. [1]

(iii) Determine whether the point $(0, -7)$ lies within the circle. [2]

The equation of a chord of the circle is $y = 7x - 14$.

(iv) Find the length of the chord. [5]

11



The diagram shows part of the curve of $y = x^2 - 7x + 12$ passing through the point *B* and meeting the *x*-axis at the point *A*.

(i) Find the gradient of the curve at *A*. [4]

The normal to the curve at *A* intersects the curve at *B*.

(ii) Find the coordinates of *B*. [4]

The line *BC* is perpendicular to the *x*-axis.

(iii) Find the area of the shaded region. [4]

12 A particle *P* moves in a straight line, so that, *t* seconds after passing through a fixed point *O*, its velocity, *v* m s^{-1}, is given by $v = \cos t - \sin 2t$, where $0 \le t \le \frac{\pi}{2}$. Find

(i) in terms of π, the values of *t*, when *P* is at instantaneous rest, [5]

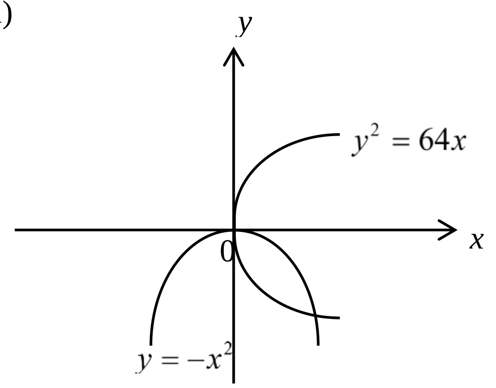
(ii) the distance travelled by *P* from $t = 0$ to $t = \frac{\pi}{2}$, [6]

(iii) an expression for the acceleration of *P* in terms of *t*. [1]

St Nicholas Girls School Additional Mathematics Preliminary Examination Paper II 2018

Answers

1 (i)



(ii) $y = -4x$

2 $p = 3, q = -10$

3 (a) $\frac{5}{9}$ (b) $y = x^8$, $y = x^{-8}$

4 (i) $1 - n\left(\frac{x^2}{2}\right) + \frac{n(n-1)}{8}x^4 + \cdots$ (ii) $n = 8,\quad p = 5$

6 (i) $a = -8, b = 2$ (ii) $x = 2$, $x = \frac{1 \pm \sqrt{7}}{3}$

7 (b) $\frac{\pi}{3}, \frac{5\pi}{3}$

8 (ii) $A_0 = 59.7$, $k = 0.1$ (iii) 6.93

9 (i) $y = \frac{1}{2}e^{2x} + 2x - \frac{1}{2}e^{-2x} + 3$ (ii) $\frac{1}{2}\ln 2$

10 (i) Centre = $(-2,\ -3)$, Radius $= 5$ units (ii) $y = 2$

(iii) The distance of the point from the centre of the cicle $= \sqrt{20} < \sqrt{25}$ radius of the circle, so the point lies within the circle.

(iv) $5\sqrt{2}$ units

11 (i) -1 (ii) $B(5, 2)$ (iii) 1sq unit.

12 (i) $\frac{\pi}{2}$, $\frac{\pi}{6}$ (ii) $\frac{1}{2}$m (iii) $-\sin t - 2\cos 2t$

Name: ________________________ (   )   Class: ____________

PRELIMINARY EXAMINATION
GENERAL CERTIFICATE OF EDUCATION ORDINARY LEVEL

ADDITIONAL MATHEMATICS   4047/02

Paper 2   **Friday 17 August 2018**

Marking Scheme

2 hours 30 minutes

Additional Materials: Answer Paper
Graph Paper

READ THESE INSTRUCTIONS FIRST

Write your name, class, and index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use paper clips, highlighters, glue, or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Paper provided
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, staple all your work together with this cover sheet.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is **100**.

FOR EXAMINER'S USE

Q1		Q5		Q9	
Q2		Q6		Q10	
Q3		Q7		Q11	
Q4		Q8		Q12	

/ 100

This document consists of **5** printed pages.

圣尼各拉女校
CHIJ ST. NICHOLAS GIRLS' SCHOOL
Girls of Grace • Women of Strength • Leaders with Heart

[Turn over

Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!\,r!} = \frac{n(n-1)\ldots\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1.$$

$$\sec^2 A = 1 + \tan^2 A.$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A.$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

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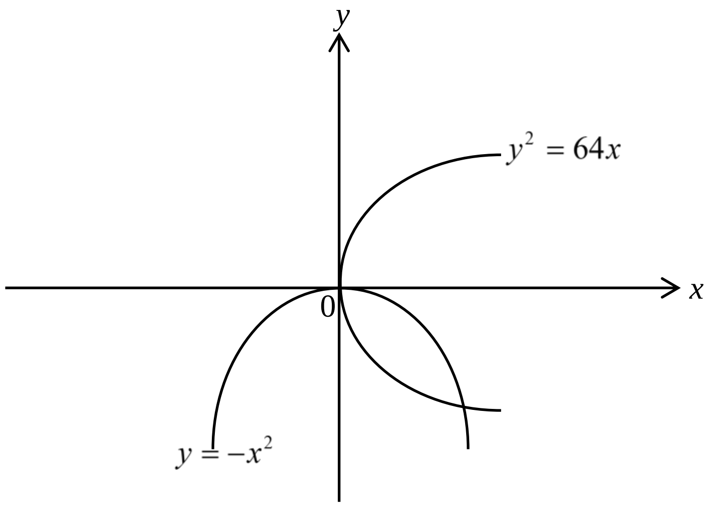
(i)



B1 +B1

-1 mark if no label

[2]

(ii)	$y^2 = 64x$ ------- (1) $y = -x^2$ ------- (2) Sub (2) into (1), $(-x^2)^2 = 64x$ $x^4 = 64x$ $x^4 - 64x = 0$ $x(x^3 - 64) = 0$ $x = 0$ or $x^3 - 64 = 0$ $y = 0$ $x^3 = 64$ $x = 4$ $y = -16$ $m = \frac{-16-0}{4-0}$ $= -4$ $y = -4x$ [4] **[6]**	M1 B1+B1 DA1	Solving Simultaneous Equations Either 1 pairs of *x* values or *y* values. [or 1m for each pair of *x* and *y* values] Must have (–4,16)

2 The roots of the equation $x^2 + 2x + p = 0$, where p is a constant, are α and β.

The roots of the equation $x^2 + qx + 27 = 0$, where q is a constant, are α^3 and β^3.

Find the value of p and of q. [6]

2	$x^2 + 2x + p = 0$ $\quad x^2 + qx + 27 = 0$		
	$\alpha + \beta = -2$ $\quad \alpha^3 + \beta^3 = -q$	B1	For both sum of roots or first pair of sum & product of roots.
	$\alpha\beta = p$ $\quad \alpha^3\beta^3 = 27$	B1	For both product of roots or 2nd pair of product and sum of roots
	$\alpha\beta = 3$		
	$p = 3$	A1	
	$(\alpha + \beta)(\alpha^2 - \alpha\beta + \beta^2) = -q$ or $(\alpha + \beta)^3 - 3\alpha^2\beta + 3\beta^2\alpha = -q$	B1	For $\alpha^3 + \beta^3$
	$(\alpha + \beta)[(\alpha + \beta)^2 - 2\alpha\beta - \alpha\beta] = -q$ or $(\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta) = -q$		
	$(-2)[4 - 9] = -q$ or $(-2)^3 - 3p(-2) = -q$	M1√	
	$q = -10$ **[6]**	A1	

3 **(a)** Given that $3^{2x-2} \times 5^{-2x} = 27^x \div 5^{x+1}$, evaluate the exact value of 15^x. [3]

(b) Given that $\log_x y = 64\log_y x$, express y in terms of x. [4]

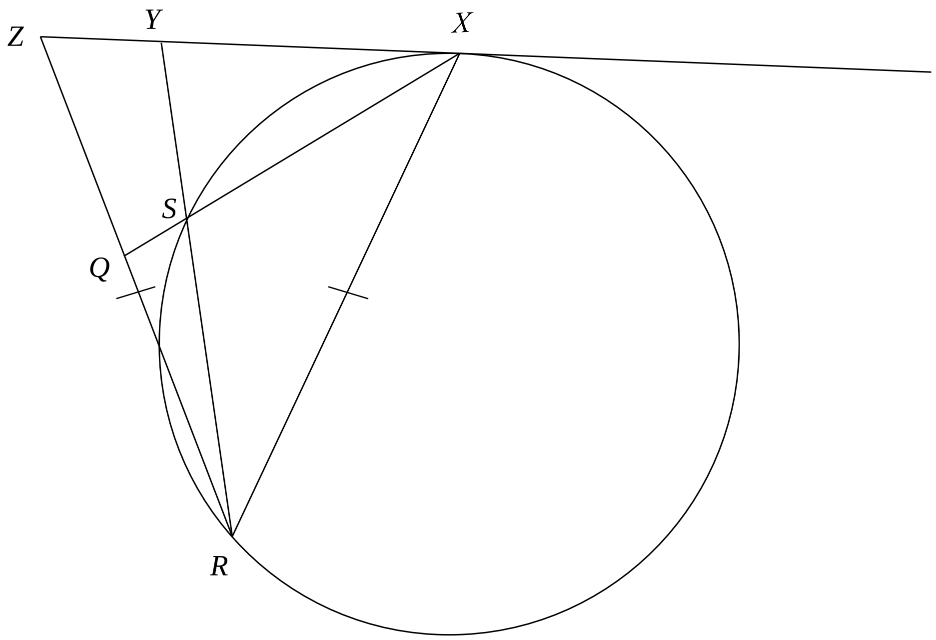
(a)	$3^{2x-2} \times 5^{-2x} = 27^x \div 5^{x+1}$ Method (i) $3^{2x-2} \times 5^{-2x} = 3^{3x} \times 5^{-1-x}$ $\frac{3^{2x-2}}{3^{3x}} = \frac{5^{-1-x}}{5^{-2x}}$ $3^{2x-2-3x} = 5^{-1-x+2x}$ $3^{-x-2} = 5^{x-1}$ $3^{-x} \times 3^{-2} = 5^x \times 5^{-1}$ $3^x \times 5^x = 5^{-1} \div 3^{-2}$	M1 M1√	applying index Law correctly on either LHS or RHS grouping and making power of *x* on one side
	Method (ii) $3^{2x} \times 3^{-2} \times 5^{-2x} = 3^{3x} \times 5^{-x} \times 5^{-1}$ $3^x \times 5^x = 5^{-1} \div 3^{-2}$	M1 M1√	Applying index law grouping and making power of *x* on one side
	$15^x = \frac{5}{9}$ [3]	A1	
(b)	$\log_x y = 64\log_y x$ $\log_x y = \frac{64 log_x x}{log_x y}$ $(log_x y)^2 = 64$ $log_x y = \pm 8$ $y = x^8, \quad y = x^{-8}$ [4] **[7]**	B1 M1√ A1+A1	change of base

4 **(i)** Write down, and simplify, the first three terms in the expansion of $(1-\frac{x^2}{2})^n$,in ascending powers of x, where n is a positive integer greater than 2. [2]

(ii) The first three terms in the expansion, in ascending powers of x, of $(2+3x^2)(1-\frac{x^2}{2})^n$ are $2 - px^2 + 2x^4$, where p is an integer. Find the value of n and of p. [5]

(i)	$\left(1-\frac{x^2}{2}\right)^n = 1 - n\left(\frac{x^2}{2}\right) + {}^nC_2\left(\frac{x^4}{4}\right) + \cdots \ldots\ldots\ldots$ $\left(1-\frac{x^2}{2}\right)^n = 1 - n\left(\frac{x^2}{2}\right) + \frac{n(n-1)}{8}x^4 + \cdots \ldots\ldots\ldots$	M1 B1	Or any two terms 1m, perfect 2m [2]
(ii)	$(2+3x^2)(1-\frac{x^2}{2})^n = (2+3x^2)(1-\frac{nx^2}{2}+\frac{n(n-1)}{8}x^4+\cdots.)$ $= 2 - nx^2 + \frac{n(n-1)}{4}x^4 + 3x^2 - \frac{3n}{2}x^4 + \cdots \ldots\ldots..$ $= 2-(n-3)x^2 + \left(\frac{n^2-7n}{4}\right)x^4 + \cdots ....$ $= 2 - px^2 + 2x^4 + \cdots ....$ $\frac{n^2-7n}{4} = 2$ $n^2 - 7n - 8 = 0$ $(n-8)(n+1) = 0$ $n = 8, n = -1$(NA) $-n + 3 = -p$ $-8 + 3 = -p$ $p = 5$	 M1√ M1√ DA1 M1√ A1	 factorisation Upon correct factorisation [5] **[7]**

5



In the figure, *XYZ* is a straight line that is tangent to the circle at *X*.

XQ bisects $\angle RXZ$ and cuts the circle at *S*. *RS* produced meets *XZ* at Y and *ZR* = *XR*.

Prove that

(a) *SR* = *SX*, [3]

(b) a circle can be drawn passing through *Z*, *Y*, *S* and *Q*. [4]

(a)	$\angle ZXQ = \angle SRX$ (Alternate Segment Theorem) $\angle ZXQ = \angle QXR$ (XQ is the angle bisector of $\angle RXZ$) $\angle QXR = \angle SRX$ By base angles of isosceles triangles, SR=SX [3]	B1 B1 B1
(b)	Let $\angle QXR$ be x $\angle RSX = 180^\circ - 2x$ (Isosceles Triangle) $\angle YSQ = 180^\circ - 2x$ (Vertically Opposite Angles) $\angle RZX = \angle ZXR = 2x$ (Base angles of Isosceles Triangle) $\angle RZX + \angle YSQ = 180^\circ - 2x + 2x = 180^\circ$ Since opposite angles are supplementary in cyclic quadrilaterals, a circle that passes through Z, Y, S and Q can be drawn **Alternative** [4] Similar but use of tangent secant theorem. **[7]**	B1 B1 B1 B1

6 The expression $3x^3 + ax^2 + bx + 4$, where a and b are constants, has a factor of $x - 2$ and leaves a remainder of -9 when divided by $x + 1$.

(i) Find the value of a and of b. [4]

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(i)	$f(x) = 3x^3 + ax^2 + bx + 4$ x-2 is a factor $f(2) = 0$ $3(8) + 4a + 2b + 4 = 0$ $4a + 2b + 28 = 0$ $2a + b + 14 = 0$---------(1) $f(-1) = -9$ $-3 + a - b + 4 = -9$ $a - b = -10$---------(2) (1)+(2) $3a = -24$ $a = -8$ Sub into (2) $-8 - b = -10$ $b = 2$ [4]	 M1 M1 A1 A1
(ii)	$x - 2 \,\overline{)\, 3x^3 - 8x^2 + 2x + 4}$ quotient $3x^2 - 2x - 2$ $3x^3 - 6x^2$ $-2x^2 + 2x$ $-2x^2 + 4x$ $-2x + 4$ $-2x + 4$ $3x^3 - 8x^2 + 2x + 4 = 0$ $(x - 2)(3x^2 - 2x - 2) = 0$ $x - 2 = 0$ $3x^2 - 2x - 2 = 0$ $x = \frac{2 \pm \sqrt{(-2)^2 - 4 \times 3 \times -2}}{2 \times 3}$ $x = \frac{2 \pm \sqrt{28}}{6}$ $x = \frac{2(1 \pm \sqrt{7})}{6}$ $x = 2$ $x = \frac{1 \pm \sqrt{7}}{3}$ [4] **[8]**	 B1 M1√ A1 +A1

7 **(a)** Prove that $\sec\theta + 1 = \dfrac{\tan\theta\sin\theta}{1-\cos\theta}$. [4]

(b) Hence or otherwise, solve $\dfrac{\tan\theta\sin\theta}{1-\cos\theta} = \dfrac{3}{4}\sec^2\theta$ for $0 \le \theta \le 2\pi$. [4]

(a)	$RHS = \dfrac{\tan\theta\sin\theta}{1-\cos\theta}$ $= \dfrac{\frac{\sin\theta}{\cos\theta}\sin\theta}{1-\cos\theta}$ $= \dfrac{\frac{\sin^2\theta}{\cos\theta}}{1-\cos\theta}$ $= \dfrac{1-\cos^2\theta}{(1-\cos\theta)\cos\theta}$ $= \dfrac{(1-\cos\theta)(1+\cos\theta)}{(1-\cos\theta)\cos\theta}$ $= \dfrac{1+\cos\theta}{\cos\theta}$ $= \dfrac{1}{\cos\theta} + 1$ $= \sec\theta + 1$ [4]	B1 B1 B1 B1	change *tan* change sin² to cos² identity $a^2 - b^2$ split and bring to answer
(b)	$\dfrac{\tan\theta\sin\theta}{1-\cos\theta} = \dfrac{3}{4}\sec^2\theta$ $1 + \sec\theta = \dfrac{3}{4}\sec^2\theta$ $3\sec^2\theta - 4\sec\theta - 4 = 0$ $(\sec\theta - 2)(3\sec\theta + 2) = 0$ $\sec\theta = 2$ or $\sec\theta = -\dfrac{2}{3}$ $\cos\theta = \dfrac{1}{2}$ or $\cos\theta = -\dfrac{3}{2}$ (No Solution) $\theta = \dfrac{\pi}{3}, \dfrac{5\pi}{3}$ = 1.05, 5.24 [4] **[8]**	B1 M1 DA1+ DA1	substitution factorization 1st DA1 for change to cos & no soln Upon correct factorisation

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Measured values of A and t are given in the table below.

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(i) Plot ln A against t for the given data and draw a straight line graph. [2]

(ii) Use your graph to estimate the value of A_0 and of k. [4]

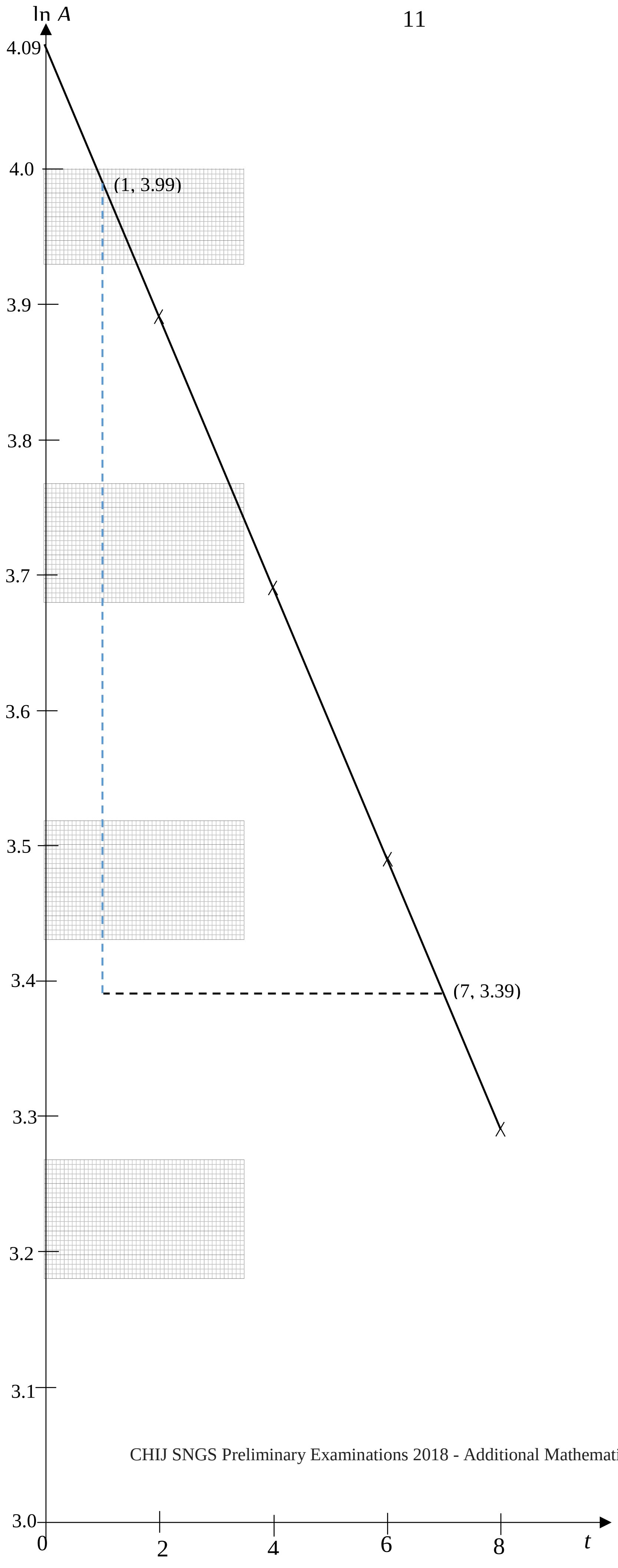
(iii) Assuming that the model is still appropriate, estimate the number of hours for the temperature of the object to be halved. [2]

8 **(i)** B1 for correct points, values & correct axes.

B1 best fit line . [2]

t	2	4	6	8
ln A	3.89	3.69	3.49	3.29

(ii)	$A = A_0 e^{-kt}$ $\ln A = -kt + \ln A_0$ $-k = \text{gradient}$ $-k = \dfrac{3.39 - 3.99}{7 - 1}$ $k = 0.1 \pm 0.02$ $\ln A_0 = 4.09$ $A_0 = e^{4.09}$ $A_0 = 59.7$ (3s.f.) ± 4 [4]	M1 A1 M1 A1	gradient vertical intercept
(iii)	$\frac{1}{2}A_0 = 29.865$ Or $\frac{1}{2}A_0 = A_0 e^{-kt}$ $\ln 29.865 = 3.396$ OR $\dfrac{1}{2} = e^{-0.1t}$ From the graph, $t = 6.9$ $t = 6.93$ (3s.f.) [2] **[8]**	√M1 A1 ±0.5	


ln A
4.09
4.0
3.9
3.8
3.7
3.6
3.5
3.4
3.3
3.2
3.1
3.0
(1, 3.99)
(7, 3.39)
0
2
4
6
8
t

9 The curve $y = \mathrm{f}(x)$ passes through the point $(0,3)$ and is such that $\mathrm{f}'(x) = \left(e^x + \frac{1}{e^x}\right)^2$.

(i) Find the equation of the curve. [4]

(ii) Find the value of x for which $\mathrm{f}''(x) = 3$. [4]

(i)	$y = \int \left(e^x + \frac{1}{e^x}\right)^2 \mathrm{d}x$	M1	knowing $y = \int \mathrm{f}(x)\mathrm{d}x$
	$= \int e^{2x} + 2 + e^{-2x}\,\mathrm{d}x$		
	$= \frac{e^{2x}}{2} + 2x + \frac{e^{-2x}}{-2} + c$	B1	ignore no + c
	at $(0,3)$, $3 = \frac{1}{2}e^0 + 2(0) - \frac{1}{2}e^0 + c$	M1	for ubstitution
	$c = 3$		
	$y = \frac{1}{2}e^{2x} + 2x - \frac{1}{2}e^{-2x} + 3$	A1	
	[4]		
(ii)	$\mathrm{f}'(x) = e^{2x} + 2 + e^{-2x}$ $\mathrm{f}'(x) = \left(e^x + e^{-x}\right)^2$		
	$\mathrm{f}''(x) = 2e^{2x} - 2e^{-2x}$ $\mathrm{f}''(x) = 2\left(e^x + e^{-x}\right)\left(e^x - e^{-x}\right)$	B1	
	when $\mathrm{f}''(x) = 3$, $2e^{2x} - 2e^{-2x} = 3$		
	Let $e^{2x} = a$, $2a - \frac{2}{a} = 3$		
	$2a^2 - 2 = 3a$		
	$2a^2 - 3a - 2 = 0$		
	$(2a+1)(a-2) = 0$	M1	factorisation
	$a = -\frac{1}{2}$ $a = 2$		
	$e^{2x} = -\frac{1}{2}$ $e^{2x} = 2$	+DA1	Upon correct factorisation
	no solution $2x = \ln 2$		
	$x = \frac{1}{2}\ln 2 = \ln\sqrt{2} = 0.347$	+DA1	
	[4]		
	[8]		

10 A circle has the equation $x^2 + y^2 + 4x + 6y - 12 = 0$.

(i) Find the coordinates of the centre of the circle and the radius of the circle. [3]

The highest point of the circle is *A*.

(ii) State the equation of the tangent to the circle at *A*. [1]

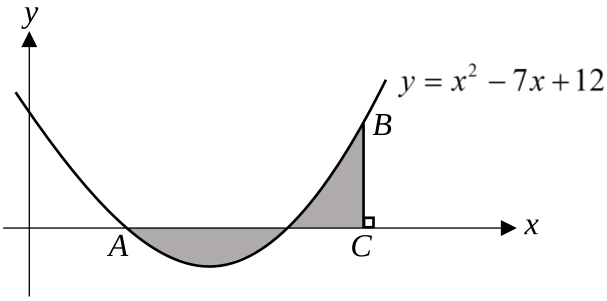
(iii) Determine whether the point $(0,\ -7)$ lies within the circle. [2]

The equation of a chord of the circle is $y = 7x - 14$.

(iv) Find the length of the chord. [5]

(i)	$x^2+y^2+4x+6y-12=0$ $x^2+y^2+2gx+2fy+c=0$ $2g=4 \qquad 2f=6$ $g=2 \qquad f=3$ Centre = $(-2,\ -3)$ Radius $=\sqrt{g^2+f^2-C}$ $=\sqrt{(-2)^2+(-3)^2-(-12)}$	$(x)^2+2(x)(2)+(2)^2+(y)^2+2(y)(3)+(3)^2$ $=12+(2)^2+(3)^2$ $(x+2)^2+(y+3)^2=25$	A1 M1
	Radius $=5$ units [3]		A1 ignore no unit
(ii)	$y=2$ (y= their y coord of centre +radius) [1]		B1 √
(iii)	The distance of the point from the centre of the cicle $=\sqrt{(0-(-2))^2+(-7-(-3))^2}$ $=\sqrt{20} \quad <\sqrt{25}$ Since it is lesser than the radius of the circle, it lies within the circle. [2]		M1√ their centre DA1
(iv)	$y=7x-14$ -------- (1) $x^2+y^2+4x+6y-12=0$ -------- (2) Sub (1) into (2), $x^2+(7x-14)^2+4x+6(7x-14)-12=0$ $x^2+49x^2-196x+196+4x+42x-84-12=0$ $50x^2-150x+100=0$ $x^2-3x+2=0$ $(x-1)(x-2)=0$ $x=1$ or $x=2$ Sub into (1), $y=-7$ or $y=0$ The length of the chord $=\sqrt{(1-2)^2+(-7-0)^2}$ $=\sqrt{50}$ $=5\sqrt{2}$ units [5] **[11]**		M1 Solving simultaneous equations M1 Factorizing B1 Either 1 pair correct or both x solutions are correct √M1 A1 accept 7.07

11



The diagram shows part of the curve of $y = x^2 - 7x + 12$ passing through the point *B* and meeting the *x*-axis at the point *A*.

(i) Find the gradient of the curve at *A*. [4]

The normal to the curve at *A* intersects the curve at *B*.

(ii) Find the coordinates of *B*. [4]

The line *BC* is perpendicular to the *x*-axis.

(iii) Find the area of the shaded region. [4]

(i)	$y = x^2 - 7x + 12$ $\quad = (x-3)(x-4)$ $\frac{dy}{dx} = 2x - 7$ *when* $x = 3,\ \frac{dy}{dx} = 2(3) - 7$ $\qquad\qquad = -1$ [4]	M1 B1 M1 A1	using smaller (positive) x value
(ii)	$\perp m = 1$ sub $m = 1$ and $(3,0)$ into $y = mx + c$ $0 = 1(3) + c$ $c = -3$ equation of normal: $y = x - 3$ $x^2 - 7x + 12 = x - 3$ or $(x-3)(x-4) = x - 3$ $x^2 - 8x + 15 = 0$ $\qquad x - 4 = 1$ $(x-3)(x-5) = 0$ $\qquad x = 5$ $x = 3 \quad x = 5$ $y = 2$ $B(5,2)$ [4]	M1 M1 M1 A1	sub ⊥m and their(3,0) curve and normal factorisation
(iii)	$\text{Area} = \left\lvert \int_3^4 x^2 - 7x + 12\,dx \right\rvert + \int_4^5 x^2 - 7x + 12\,dx$ $= \left\lvert \left[\frac{x^3}{3} - \frac{7x^2}{2} + 12x \right]_3^4 \right\rvert + \left[\frac{x^3}{3} - \frac{7x^2}{2} + 12x \right]_4^5$ $= \left\lvert \left(\frac{64}{3} - \frac{7(16)}{2} + 12(4) \right) - \left(\frac{27}{3} - \frac{7(9)}{2} + 12(3) \right) \right\rvert$ $+ \left(\frac{125}{3} - \frac{7(25)}{2} + 12(5) \right) - \left(\frac{64}{3} - \frac{7(16)}{2} + 12(4) \right)$ $= \left\lvert 13\frac{1}{3} - 13\frac{1}{2} \right\rvert + 14\frac{1}{6} - 13\frac{1}{3}$ $= \left\lvert -\frac{1}{6} \right\rvert + \frac{5}{6}$ $= 1$ sq unit [4] **[12]**	M1 B1 M1 A1	$\text{Area} = \left\lvert \int y\,dx \right\rvert + \int y\,dx$ √their limits from (i) and (ii) for integration substitution

12 A particle P moves in a straight line, so that, t seconds after passing through a fixed point O, its velocity, v m s^{-1}, is given by $v = \cos t - \sin 2t$, where $0 \leq t \leq \frac{\pi}{2}$. Find

(i) in terms of π, the values of t, when P is at instantaneous rest, [5]

(ii) the distance travelled by P from $t = 0$ to $t = \frac{\pi}{2}$, [6]

(iii) an expression for the acceleration of P in terms of t. [1]

(i)	$v = \cos t - \sin 2t$ when $v = 0$, $\cos t - \sin 2t = 0$ $\cos t - 2\sin t \cos t = 0$ $\cos t(1 - 2\sin t) = 0$ $\cos t = 0 \quad \sin t = \frac{1}{2}$ $t = \frac{\pi}{2} \quad t = \frac{\pi}{6}$ [5]	B1 B1 M1 A1+A1	For v=0 for double angle factorisation
(ii)	$s = \int \cos t - \sin 2t \, dt$ $= \sin t + \frac{1}{2}\cos 2t + c$ when $t = 0, s = 0 \quad 0 = \sin 0 + \frac{1}{2}\cos 0 + c$ $c = -\frac{1}{2}$ $s = \sin t + \frac{1}{2}\cos 2t - \frac{1}{2}$ when $t = \frac{\pi}{6}$, $s = \sin\frac{\pi}{6} + \frac{1}{2}\cos\frac{\pi}{3} - \frac{1}{2}$ $= \frac{1}{2} + \frac{1}{2}\left(\frac{1}{2}\right) - \frac{1}{2}$ $= \frac{1}{4}$ when $t = \frac{\pi}{2}$, $s = \sin\frac{\pi}{2} + \frac{1}{2}\cos\pi - \frac{1}{2}$ [6] $= 1 + \frac{1}{2}(-1) - \frac{1}{2}$ $= 0$ Distance travelled = $2\left(\frac{1}{4}\right)$ $= \frac{1}{2}$ m	B1 B1+B1 M1 M1 DA1	For $s = \int v \, dt$ Integration ignore no +c Sub either $t = \frac{\pi}{6}$ or $t = \frac{\pi}{2}$ For both s for $t = \frac{\pi}{6}$ and $t = \frac{\pi}{2}$ found
(iii)	$a = \frac{dv}{dt} = (-\sin t - 2\cos 2t) m / s^2$ [1] **[12]**	B1	

1 Express $\dfrac{8x^2 - 2x + 19}{(1-x)(4+x^2)}$ in partial fractions. [5]

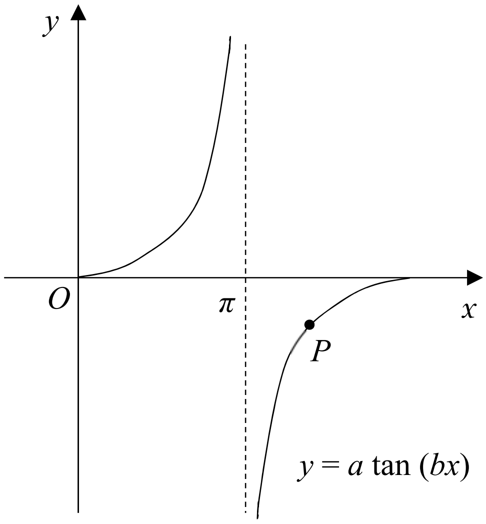
2 **(i)** On the same axes sketch the curves $y = -\sqrt{x}$ and $y = -\sqrt{32}\,x^3$. [2]

(ii) Find the x-coordinates of the points of intersection of the two curves. [2]

3 **(a)** Given that $\theta = \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$, express θ in terms of π.

Hence, find the exact value of $\sin 2\theta + \tan \theta$. [4]

(b)



The figure shows part of the graph of $y = a \tan(bx)$ and a point $P\left(\frac{3\pi}{2}, -2\right)$ marked. Find the value of each of the constants a and b. [2]

4 The equation of a curve is $y = e^x + 2e^{-x}$.

(i) Find the coordinates of the stationary point of the curve, leaving your answer in exact form. [4]

(ii) Determine the nature of this point. [2]

5 **(i)** Sketch the graph of $y = \left|4 - \frac{x}{2}\right| - 1$, indicating clearly the vertex and the intercepts on the coordinate axes. [3]

(ii) State the range of y. [1]

(iii) Find the values of x for $\left|4 - \frac{x}{2}\right| - 1 = 6$. [2]

(iv) The graph $y = \left|4 - \frac{x}{2}\right| - 1$ is reflected in the y-axis.
Write down the equation of the new graph. [1]

6 **(a)** Find the maximum and minimum values of $(1 - \cos A)^2 - 5$ and the corresponding value(s) of A where each occurs for $0° \leq A \leq 360°$. [4]

(b) A, B and C are angles of a triangle such that $\cos A = -\frac{1}{\sqrt{5}}$ and $\sin B = \frac{5}{13}$.

(i) State the range of values for A. [1]

(ii) Find the exact value of $\cos(A + B)$.

Hence find the exact value of $\cos C$. [4]

7 **(a)** **(i)** Show that $\frac{d}{dx}\left(\frac{\ln x}{4x}\right) = \frac{1 - \ln x}{4x^2}$. [3]

(ii) Integrate $\frac{\ln x}{x^2}$ with respect to x. [4]

(b) Given that $\int_1^5 f(x)\,dx = 8$, find $\int_1^2 f(x)\,dx - \int_5^2 [f(x) + 3x]\,dx$. [3]

8 **(a)** A curve C is such that $\frac{dy}{dx} = 8\cos 2x$ and $P\left(\frac{\pi}{3}, 2\sqrt{3} - 3\right)$ is a point on C.

(i) The normal to the curve at P crosses the x-axis at Q.

Find the coordinates of Q. [3]

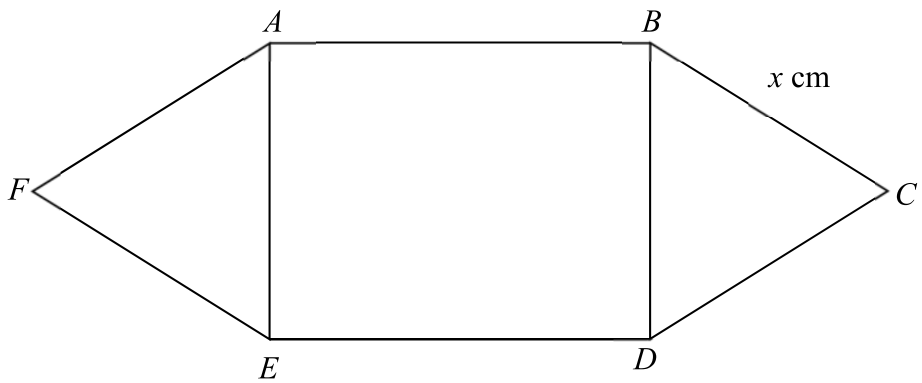
(ii) Find the equation of C. [3]

(b) Given that $y = \sin 4x$, show that $\frac{d^2y}{dx^2} \times \frac{dy}{dx} = -32 \sin 8x$. [4]

9 **(a)** Find the range of values of k for which $2x(2x + k) + 6 = 0$ has no real roots.[4]

(b) If p and q are roots of the equation $x^2 + 2x - 1 = 0$ and $p > q$,
express $\frac{q}{p^2}$ in the form $a + b\sqrt{2}$, where a and b are integers. [5]

10



A hexagon $ABCDEF$ has a fixed perimeter of 210 cm.
BCD and AFE are 2 equilateral triangles and $ABDE$ is a rectangle.
The length of BC is represented as x cm.

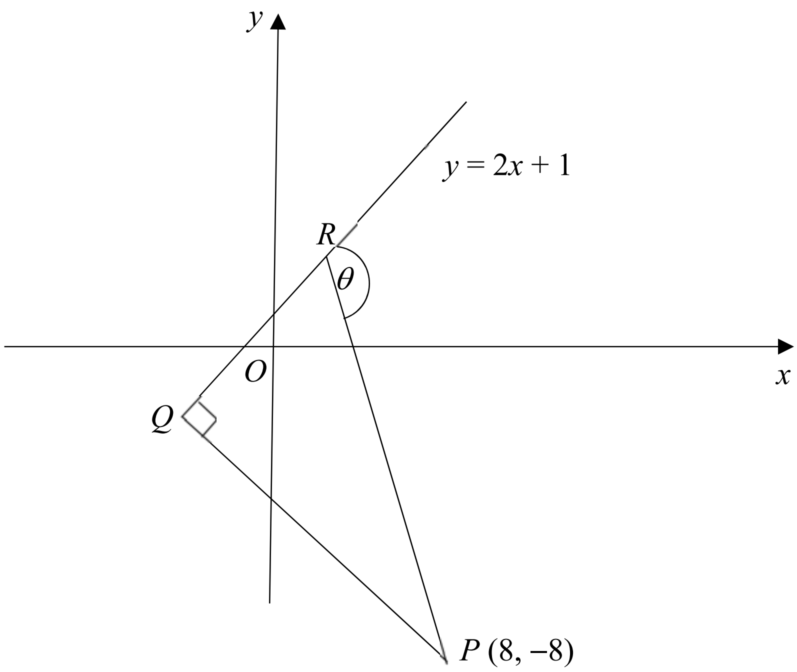
(i) Express AB in terms of x. [1]

(ii) Show that the area of the hexagon, H is given by

$$H = \left(\frac{\sqrt{3}}{2} - 2\right)x^2 + 105x.$$ [2]

(iii) Find the value of x for which H is a maximum. [4]

11



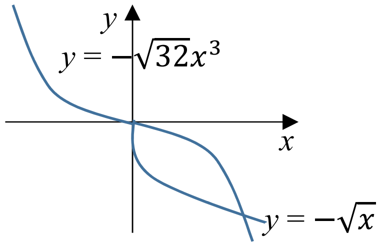
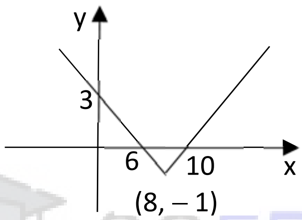
The diagram shows triangle PQR in which the point P is $(8, -8)$ and angle PQR is 90°. The gradient of PR is $-\frac{13}{8}$ and the equation of QR produced is $y = 2x + 1$.

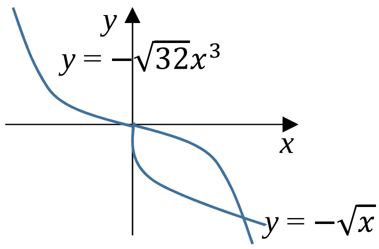
The line PR makes an angle θ with QR produced.

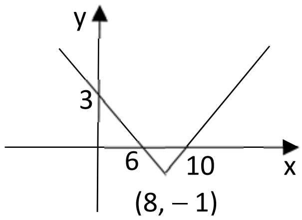
(i) Find the coordinates of Q. [4]

(ii) Find the value of θ. [3]

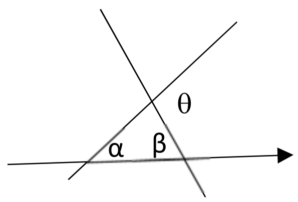
Answers

1	$\frac{5}{1-x} - \frac{3x+1}{4+x^2}$
2(i)	
2(ii)	$x = 0$ or $\frac{1}{2}$
3(a)	$\theta = -\frac{\pi}{3}$ $2\sin\theta\cos\theta + \tan\theta = -\frac{3}{2}\sqrt{3}$
3(b)	$a = 2$; $b = \frac{1}{2}$
4(i)	$(\ln\sqrt{2}, 2\sqrt{2})$ (ii) Minimum point
5(i)	
5(ii)	$y \geq -1$ (iii) $x = -6$ or 22
5(iv)	$y = \left\|4 + \frac{x}{2}\right\| - 1$
6(a)	Max value = –1 when A = 180° Min value = – 5 when A = 0°,360°
6(b)(i)	$90° < A < 180°$ or $\frac{\pi}{2} < A < \pi$
6(b)(ii)	$\cos(A + B) = -\frac{22}{13\sqrt{5}}$ $\cos C = \frac{22}{13\sqrt{5}}$
7(a)(ii)	$\int \frac{\ln x}{x^2}\,dx = = -\frac{1}{x} - \frac{\ln x}{x} + c$ (b) $39\frac{1}{2}$
8(a)(i)	$Q(12 - 8\sqrt{3} + \frac{\pi}{3}, 0)$ or (–0.809,0)
8(ii)	$y = 4\sin 2x - 3$
9(a)	$-\sqrt{24} < k < \sqrt{24}$
9(b)	$p = -1 + \sqrt{2}$, $q = -1 - \sqrt{2}$ $\frac{q}{p^2} = -7 - 5\sqrt{2}$
10(i)	$AB = 105 - 2x$
10(iii)	$x = 46.3$ Maximum H
11(i)	$Q(-2, -3)$ (ii) $\theta = 121.8°$

Qn	Working	Marks
1	$\dfrac{8x^2-2x+19}{(1-x)(4+x^2)}=\dfrac{A}{1-x}+\dfrac{Bx+C}{4+x^2}$ $8x^2-2x+19=A(4+x^2)+(Bx+C)(1-x)$ Sub $x=1$, $8-2+19=5A$ $\quad A=5$ Sub $x=0$, $19=4(5)+C$ $\quad C=-1$ Compare coeff of x^2, $8=A-B$ $\quad B=-3$ $\dfrac{8x^2-2x+19}{(1-x)(4+x^2)}=\dfrac{5}{1-x}-\dfrac{3x+1}{4+x^2}$	B1 correct PF M1 A2 For all 3correct A1 For 2 correct √ A1 Only if B1 awarded
	Total	**5 marks**
2(i)		G1 G1
2(ii)	$x^{\frac{1}{2}}=\sqrt{32}\,x^3$ $x=32x^6$ $x(1-32x^5)=0$ $x=0$ or $\frac{1}{2}$	M1 A1
	Total	**4 marks**
3(a)	$\theta=-\frac{\pi}{3}$ $2\sin\theta\cos\theta+\tan\theta=2\left(-\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right)+\left(-\sqrt{3}\right)$ $=-\frac{3}{2}\sqrt{3}$	B1 B1 value of cos θ B1 value of tan θ B1
3(b)	$a=2$ Period $=2\pi=\frac{\pi}{b}$ $\quad b=\frac{1}{2}$	B1 B1
	Total	**6 marks**
4(i)	$\frac{\mathrm{dy}}{\mathrm{dx}}=\mathrm{e}^x-2\mathrm{e}^{-x}=0$ $\mathrm{e}^{2x}=2$ $x=\ln\sqrt{2}$ $y=e^{ln\sqrt{2}}+2e^{-ln\sqrt{2}}$ $=\sqrt{2}+\frac{2}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}=2\sqrt{2}$ $\quad$ Point is $(\ln\sqrt{2},2\sqrt{2})$	M1 $\frac{\mathrm{dy}}{\mathrm{dx}}=0$ B1 Differentiate A1 value of x B1 *o.e.*
4(ii)	$\frac{d^2y}{dx^2}=\mathrm{e}^x+2\mathrm{e}^{-x}$ $x=\ln\sqrt{2}$, $\frac{d^2y}{dx^2}=2+\frac{2}{\sqrt{2}}>0$ Minimum point	M1 Knowing test Correct concl based on test √A1
	Total	**6 marks**

Qn	Working	Marks
5(i)		G1 vertex G1 x ints G1 y int
5(ii)	$y \geq -1$	B1
5(iii)	$\left\|4-\frac{x}{2}\right\|-1=6$ $\left\|4-\frac{x}{2}\right\|=7$ $4-\frac{x}{2}=7$ or $4-\frac{x}{2}=-7$ $x=-6$ or 22	M1 or by counting A1
5(iv)	$y=\left\|4+\frac{x}{2}\right\|-1$	B1
	Total	**7 marks**
6(a)	$(1-\cos A)^2-5$ Max value $=(1-(-1))^2-5=-1$ When $\cos A=-1$, $A=180°$ Min value $=(1-1)^2-5=-5$ When $\cos A=1$, $A=0°, 360°$	B1 B1 B1 B1
6(b)(i)	$90° < A < 180°$ or $\frac{\pi}{2} < A < \pi$	B1
6(b)(ii)	$\cos(A+B)=\cos A\cos B-\sin A\sin B$ $=-\frac{1}{\sqrt{5}}\left(\frac{12}{13}\right)-\frac{2}{\sqrt{5}}\left(\frac{5}{13}\right)$ $=-\frac{22}{13\sqrt{5}}$ $\cos C=\cos(180°-(A+B))$ $=-\cos(A+B)$ $=\frac{22}{13\sqrt{5}}$	B1 value of cos B B1 value of sin A B1 √B1 e
	Total	**9marks**

Qn	Working	Marks
7(a)(i)	$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\ln x}{4x}\right) = \frac{4x\left(\frac{1}{x}\right) - 4\ln x}{(4x)^2}$ $= \frac{4-4\ln x}{16x^2}$ $= \frac{1-\ln x}{4x^2}$ (shown)	M1 quotient rule M1 diff ln x B1 working seen
7(a)(ii)	$\int \frac{1-\ln x}{4x^2}\,dx = \frac{\ln x}{4x} + c_1$ $\frac{1}{4}\int \frac{\ln x}{x^2}\,dx = \int \frac{1}{4}x^{-2}\,dx - \frac{\ln x}{4x} + c_1$ $= \frac{x^{-1}}{-4} - \frac{\ln x}{4x} + c_1$ $\int \frac{\ln x}{x^2}\,dx == -\frac{1}{x} - \frac{\ln x}{x} + c$	B1 use integ^n as reverse of diff Ignore if +c is missing B1 rearrange terms B1 $\int x^{-2}\,dx$ B1 must have +c
7(b)	$\int_1^2 f(x)\,dx + \int_2^5 [f(x) + 3x]dx$ $= \int_1^2 f(x)\,dx + \int_2^5 f(x)dx + \int_2^5 3x\,dx$ $= 8 + \left[\frac{3x^2}{2}\right]_2^5$ $=8 + [\frac{3}{2}(25) - \frac{3}{2}(4)]$ $= \frac{79}{2} = 39\frac{1}{2}$	M1 switch limits and –ve becomes +ve B1 correct integral A1
	Total	**10 marks**
8(a)(i)	When $x = \frac{\pi}{3}$, $\frac{\mathrm{d}y}{\mathrm{d}x} = 8\cos\frac{2\pi}{3} = -4$ $\frac{0-(2\sqrt{3}-3)}{x-\frac{\pi}{3}} = \frac{1}{4}$ $Q(12 - 8\sqrt{3} + \frac{\pi}{3}, 0)$ or (–0.809,0)	B1 M1 A1
8(ii)	$y = 4\sin 2x + \mathrm{c}$ Sub $\left(\frac{\pi}{3}, 2\sqrt{3} - 3\right)$ $2\sqrt{3} - 3 = 4\sin\frac{2\pi}{3} + \mathrm{c}$ $2\sqrt{3} - 3 = 4\left(\frac{\sqrt{3}}{2}\right) + \mathrm{c}$ $y = 4\sin 2x - 3$	B1 ignore if +c missing M1 A1
8(iii)	$\frac{\mathrm{d}y}{\mathrm{d}x} = 4\cos 4x$ $\frac{d^2y}{dx^2} = -16\sin 4x$ $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} \times \frac{\mathrm{d}y}{\mathrm{d}x} = (-16\sin 4x)(4\cos 4x)$ $= -32(2\sin 4x\cos 4x)$ $= -32\sin 8x$	B1 $\frac{d}{dx}\sin x = \cos x$ B1 $\frac{d}{dx}\cos x = -\cos x$ B1 use of chain rule B1 2sin4xcos4x seen
	Total	**10 marks**

Qn	Working	Marks
9(a)	$2x(2x+k)+6=0$ $4x^2+2kx+6=0$ Discriminant < 0 $(2k)^2-4(4)(6)<0$ $k^2-24<0$ $\qquad (k-\sqrt{24})(k+\sqrt{24})<0$ $-\sqrt{24}<k<\sqrt{24}$	 B1 For D < 0 M1 correct sub M1 Solve ineq A1 (M0 if $k<\pm\sqrt{24}$)
9(b)	$x^2+2x-1=0$ $x=\frac{-2\pm\sqrt{2^2-4(1)(-1)}}{2}$ $p=-1+\sqrt{2}$, $q=-1-\sqrt{2}$ $\frac{q}{p^2}=\frac{-1-\sqrt{2}}{(-1+\sqrt{2})^2}$ $=\frac{-1-\sqrt{2}}{3-2\sqrt{2}}\times\frac{3+2\sqrt{2}}{3+2\sqrt{2}}$ $=\frac{-3-2(2)-3\sqrt{2}-2\sqrt{2}}{9-4(2)}$ $=-7-5\sqrt{2}$	 M1 A1 p > q M1 rationalise M1 simplify A1
	Total	**9 marks**
10(i)	$4x+2(AB)=210$ $AB=105-2x$	 B1
10(ii)	$H=2\left(\frac{1}{2}\right)x^2\sin 60+(105-2x)x$ $=\frac{\sqrt{3}}{2}x^2+105x-2x^2$ $=\left(\frac{\sqrt{3}}{2}-2\right)x^2+105x$ (shown)	B1 Area of Δ B1 sub & working
10(iii)	$\frac{dH}{dx}=2\left(\frac{\sqrt{3}}{2}-2\right)x+105$ $\frac{dH}{dx}=0$ $x=46.3$ $\frac{d^2H}{dx^2}=\sqrt{3}-4<0$ Maximum H	B1 M1 A1 B1 test & concl
	Total	**7marks**
11(i)	Eqn of PQ: $y-(-8)=-\frac{1}{2}(x-8)$ $y=-\frac{1}{2}x-4$ ----------(1) QR: $y=2x+1$ ----------(2) Solving simultaneously $Q(-2,-3)$	B1 correct m_{PQ} B1 form eqn M1 A1
11(ii)	$\tan\alpha=2$ $\alpha=63.43°$ $\tan\beta=\frac{13}{8}$ $\beta=58.39°$  $\theta=63.43°+58.39°$(ext ∠ of Δ) $=121.8°$	M1 use grads to Find angles M1 manipulate ∠s A1
	Total	**7 marks**

TANJONG KATONG SECONDARY SCHOOL
Preliminary Examination 2018
Secondary 4

CANDIDATE NAME			
CLASS		INDEX NUMBER	

ADDITIONAL MATHEMATICS **4047/02**

Paper 2 **Tuesday 28 August 2018**

2 hours 30 minutes

Additional Materials: Writing Paper
Graph Paper

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Write your answers on the writing paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal in the case of angles in degree, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 100.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots\ldots.(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

Answer **all** questions.

1 The amount of energy, E erg, generated in an earthquake is given by the equation $E = 10^{a+bM}$, where a and b are constants and M is the magnitude of the earthquake.

The table below shows some corresponding values of M and E.

M	1	2	3	4	5
E (erg)	2.0×10^{13}	6.3×10^{14}	2.0×10^{16}	6.3×10^{17}	2.0×10^{19}

(i) Plot lg E against M. [2]

(ii) Using your graph, find an estimate for the value of a and of b. [3]

(iii) Using your answers from **(ii)**, find the amount of energy generated, in erg, by an earthquake of magnitude 7. [2]

2 **(i)** Write down the expansion of $(3 - x)^3$ in ascending powers of x. [1]

(ii) Expand $(3 + 2x)^8$, in ascending powers of x, up to the term in x^3. [3]

(iii) Write down the expansion of $(3 - x)^3\,(3 + 2x)^8$ in ascending powers of x, up to x^2. [2]

(iv) By letting $x = 0.01$ and your expansion in **(iii)**, find the value of $2.99^3 \times 3.02^8$, giving your answer correct to 3 significant figures.
Show your workings clearly. [2]

(v) Explain clearly why the expansion in **(iii)** is not suitable for finding the value of $2^3 \times 5^8$. [2]

3 **(i)** By writing 3θ as $(2\theta + \theta)$, show that $\sin(3\theta) = 3\sin\theta - 4\sin^3\theta$. [3]

(ii) Solve $\sin(3\theta) = 3\sin\theta\cos\theta$ for $0° < \theta < 360°$. [5]

4 The equation $x^2 + bx + c = 0$ has roots α and β, where $b > 0$.

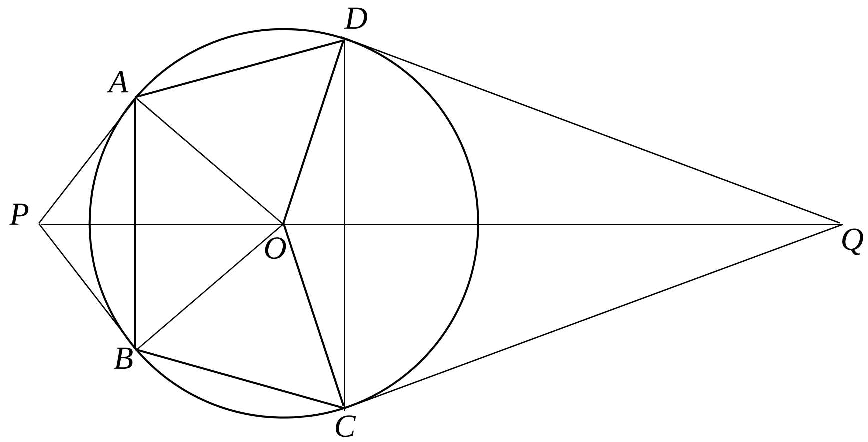
(i) Write down, in terms of b and/or c, the value of $\alpha + \beta$ and of $\alpha\beta$. [1]

(ii) Find a quadratic equation with roots α^2 and β^2, in terms of a and b. [3]

(iii) Find the relation between b and c for which the equation found in **(ii)** has two distinct roots. [2]

(iv) Give an example of values of b and c which satisfy the relation found in **(iii)**. [1]

5 In the diagram, A, B, C and D are points on the circle centre O.
AP and BP are tangents to the circle at A and B respectively.
DQ and CQ are tangents to the circle at D and C respectively.
POQ is a straight line.



(i) Prove that angle $COD = 2 \times$ angle CDQ. [3]

(ii) Make a similar deduction about angle AOB. [1]

(iii) Prove that $2 \times$ angle OAD = angle CDQ + angle BAP. [4]

6 **(i)** Differentiate $y = 2e^{3x}(1 - 2x)$ with respect to x. [3]

(ii) Find the range of values of x for which y is decreasing. [1]

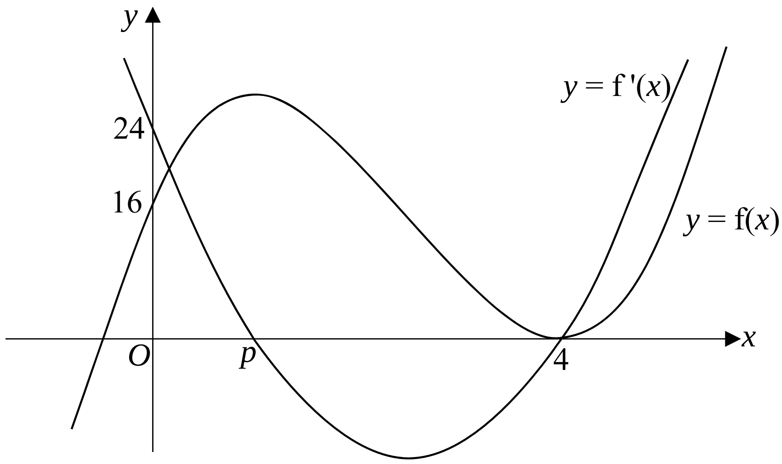
(iii) Given that x is decreasing at a rate of 5 units per second, find the rate of change of y at the instant when $x = -1.5$. [3]

7 **(i)** By using an appropriate substitution, express $2^{3a+1} - 2^{2a+2} + 2^a$ as a cubic function. [3]

(ii) Solve the equation $2^{3a+1} - 2^{2a+2} + 2^a = 0$. [5]

(iii) Find the range of values of k for which $2^{3a+1} - 2^{2a+2} + k(2^a) = 0$ has at least one real solution. [3]

8 The diagram shows the graphs of $y = f(x)$ and $y = f'(x)$.



The function $f(x) = ax^3 + bx^2 + 24x + 16$ has stationary points at $x = p$ and $x = 4$.

(i) Find an expression for $f'(x)$, in terms of a and b. [1]

(ii) Find the value of a and of b. [3]

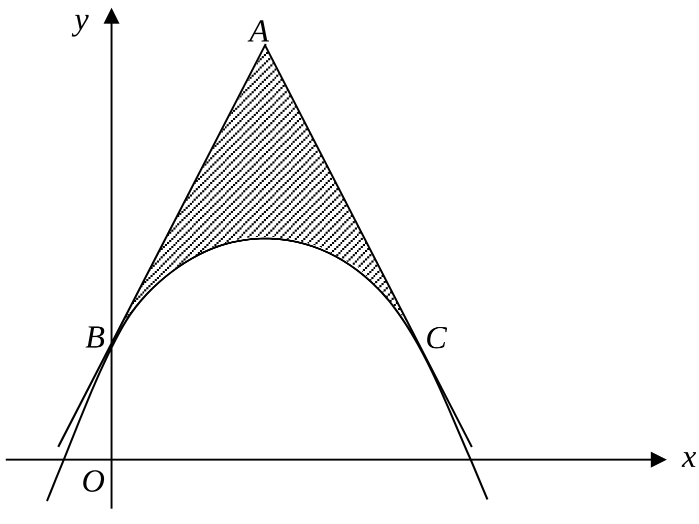
(iii) Find the value of p.
State the range of values of k, where $k > 0$ and $y = f(x) - k$ has only one real root. [3]

(iv) Find the minimum value of the gradient of $f(x)$. [2]

9 The diagram shows the graph of $y = -\frac{1}{2}(x-2)^4 + 16$.

AB and AC are tangents to the curve at B and C respectively.

B lies on the y-axis and $AB = AC$.



(i) Find the gradient function of the curve. [1]

(ii) Find the equation of the tangent at B.
Hence, state the coordinates of A. [3]

(iii) Find the area of the shaded region. [6]

[Turn over

10 A particle, P, travels along a straight line so that, t seconds after passing a fixed point O, its velocity, v m/s is given by

$$v = (12e^{kt} + 18), \text{ where } k \text{ is a constant.}$$

(i) Find the initial velocity of the particle. [1]

Two seconds later, its velocity is 40 m/s.

(ii) Show that $k = 0.3031$, correct to 4 significant figures. [3]

(iii) Sketch the graph of $v = 12e^{kt} + 18$, for $0 \le t \le 4$. [3]

(iv) Explain why the distance travelled by P during the 4 seconds does not exceed 180 metres. [2]

(v) Find the maximum acceleration of P during the interval $0 \le t \le 4$. [2]

11 A circle, C_1, with centre A, has equation $x^2 + y^2 - 8x - 4y - 5 = 0$.

(i) Find the coordinates of A and the radius of C_1. [3]

(ii) Show that (1, 6) lies on the circle. [1]

(iii) Find the equation of the tangent to the circle at (1, 6). [3]

The equation of the tangent to the circle at (1, 6) cuts the x-axis at B.

(iv) Find the coordinates of B. [2]

Another circle, C_2, has centre at B and radius r.

(v) Find the exact value of r given that circle C_2 touches circle C_1. [3]

End of Paper

Answers:

1 (i) $a = 11.7$ to 11.9, $b = 1.49$ to 1.51 (iii) $E = 2.0 \times 10^{22}$ Erg

2 (i) $27 - 9x + 3x^2 - x^3$ (ii) $6\,561 + 34\,992x + 81\,648x^2 + 108\,864x^3 + \ldots$

(iii) $177\,147 + 885\,735x + 1\,909\,251x^2 + \ldots$

(iv) 186 000

(v) For $2^3 \times 5^8$, need to use $x = 1$

Since 1 is large in comparison to 0.01, the value is inaccurate because a significantly large value is removed after the 3rd term.

3 (ii) 104.5°, 255.5°, 180°

4 (i) $\alpha + \beta = -b$, $\alpha\beta = c$ (ii) $x^2 - (b^2 - 2c)x + c^2 = 0$ o.e

(iii) $b^2 - 4c > 0$ (iv) $b = 5$, $c = 2$ o.e.

6 (i) $\frac{dy}{dx} = 2e^{3x}(1 - 6x)$ (ii) $x > \frac{1}{6}$ (iii) -1.11 units/sec

7 (i) $2x^3 - 4x^2 + x$ (o.e.) (ii) $a = 0.7771$ or -1.77 (iii) $k \le 2$

8 (i) $f'(x) = 3ax^2 + 2bx + 24$ (ii) $a = 2$, $b = -15$

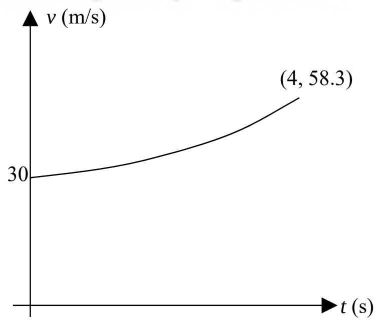
(iii) $p = 1$, $k > 27$ (iv) -13.5

9 (i) $\frac{dy}{dx} = -2(x - 2)^3$ (ii) Eq AB: $y = 16x + 8$, A is (2, 40)

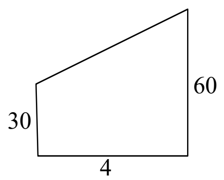
(iii) 38.4 units^2

10 (i) 30 m/s

(iii)



(iv) area of trapezium < 0.5(30 + 60) × 4 = 180

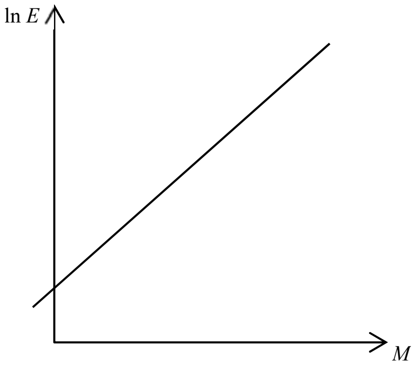


∴ distance travelled < 180 m

(v) max $a = 12.23$ m/s^2

11 (i) A is (4, 2), Radius = 5 units (iii) $4y - 3x = 21$ (o.e.)

(iv) (–7, 0) (v) $r = 5\sqrt{5} - 5$

<table>
<tr><th>Qn</th><th>Key Steps</th><th>Marks / Remarks</th><th></th></tr>
<tr><td>1(i)</td><td><table><tr><td>M</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr><tr><td>$\ln E$</td><td>13.3</td><td>14.8</td><td>16.3</td><td>17.8</td><td>19.3</td></tr></table></td><td>B1 TOV

B1 Line passes through pts</td><td></td></tr>
<tr><td>(ii)</td><td>lg $E = a + bM$
a = vertical intercept = 11.8
b = gradient (their rise/run)
= 1.5</td><td>B1 11.7 to 11.9
M1 working for gradient
A1 1.49 to 1.51</td><td></td></tr>
<tr><td>(iii)</td><td>lg $E = 11.8 + 1.5(7) = 22.3$
$E = 2.0 \times 10^{22}$ Erg</td><td>M1
A1 1.34×10^{22} to 2.95×10^{22}</td><td>7</td></tr>
<tr><td></td><td></td><td></td><td></td></tr>
<tr><td>2(i)</td><td>$(3 - x)^3 = 27 - 27x + 9x^2 - x^3$</td><td>B1</td><td></td></tr>
<tr><td>(ii)</td><td>$(3 + 2x)^8$
$= 3^8 + \binom{8}{1}(3)^7(2x) + \binom{8}{2}(3)^6(2x)^2 + \binom{8}{3}(3)^5(2x)^3$
$= 6\,561 + 34\,992x + 81\,648x^2 + 108\,864x^3 + \ldots$</td><td>B3 1m for each term (2nd to 4th)
−1m if 1st term missing
B0 is all not evaluated</td><td></td></tr>
<tr><td>(iii)</td><td>$(3 - x)^3\,(3 + 2x)^8$
= their (i) × their (ii)
$= 177\,147 + 767\,637x + 2\,854\,035x^2 + \ldots$</td><td>M1 choosing correct pairs
A1</td><td></td></tr>
<tr><td>(iv)</td><td>$2.99^3 \times 3.02^8$
$= 177\,147 + 767\,637(0.01) + 2\,854\,035(0.01)^2$
= 185108.7735
= 185 000</td><td>B1 Subn must be seen

B1 reject 184 956</td><td></td></tr>
<tr><td>(v)</td><td>For $2^3 \times 5^8$, need to use $x = 1$

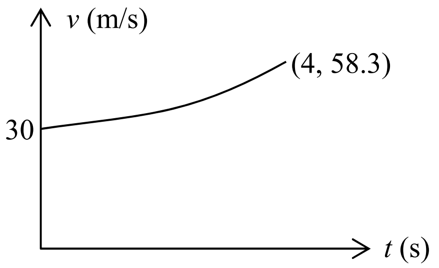
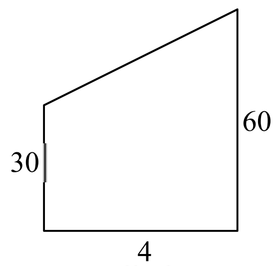
Since 1 is large in comparison to 0.01, the value is inaccurate because a significantly large value is removed after the 3rd term</td><td>B1 $x = 1$ seen

B1 o.e. "big" or "large" seen</td><td>10</td></tr>
<tr><td></td><td></td><td></td><td></td></tr>
</table>

Qn	Key Steps	Marks / Remarks	
3(i)	$\sin(\theta + 2\theta)$ $= \sin\theta\cos 2\theta + \cos\theta\sin 2\theta$ $= \sin\theta(1 - 2\sin^2\theta) + \cos\theta(2\sin\theta\cos\theta)$ $= \sin\theta(1 - 2\sin^2\theta) + 2\sin\theta\cos^2\theta$ $= \sin\theta(1 - 2\sin^2\theta) + 2\sin\theta(1 - \sin^2\theta)$ $= 3\sin\theta - 4\sin^3\theta$	B1 Use compound angle B1 Any double angle seen B1 Use identity AG	
(ii)	$\sin(3\theta) = 3\sin\theta\cos\theta$ $3\sin\theta - 4\sin^3\theta = 3\sin\theta\cos\theta$ $\sin\theta(3 - 4\sin^2\theta - 3\cos\theta) = 0$ $\sin\theta = 0 \quad \therefore \theta = 180^\circ$ or $3 - 4\sin^2\theta - 3\cos\theta = 0$ $3 - 4(1 - \cos^2\theta) - 3\cos\theta = 0$ $4\cos^2\theta - 3\cos\theta - 1 = 0$ $(4\cos\theta + 1)(\cos\theta - 1) = 0$ $\cos\theta = -\frac{1}{4}$ or $\cos\theta = 1$ (NA) Hence, $\theta = 104.5^\circ, 255.5^\circ$	B1 $\theta = 180^\circ$ seen M1 Solve a quadratic B1 Use identity B2 −1m for extra answer	**8**
4(i)	$\alpha + \beta = -b \qquad \alpha\beta = c$	B1 Both correct	
(ii)	$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$ $= b^2 - 2c$ $\alpha^2\beta^2 = c^2$ Eqn: $x^2 - (b^2 - 2c)x + c^2 = 0$	B1 Correct sum B1 Correct product B1 Equation seen	
(iii)	For 2 distinct roots, $(b^2 - 2c)^2 - 4c^2 > 0$ $b^2(b^2 - 4c) > 0$ Since $b^2 > 0$, hence $b^2 - 4c > 0$	B1 Correct D ok if $[-(b^2 - 2c)]^2$ or $(b^2 - 2c)^2$ B1 o.e.	
(iv)	$b = 5, c = 2$	B1 o.e.	**7**

Qn	Key Steps	Marks / Remarks	
5(i)	Let $\angle CDQ = a$ $\angle ODQ = 90°$ (tan $\perp$ rad) $\therefore\ \angle ODC = 90° - a$ $\therefore\ \angle COD = 180° - 2(90° - a)$ ($\angle$sum, ΔCOD)	B1 with reason B1 B1 with reason	
(ii)	$\angle AOB = 2 \times \angle BAP$	B1	
(iii)	From (i) and (ii), $2(\angle CDQ + \angle BAP) = \angle COD + \angle AOB$ $\angle CDQ + \angle BAP = \frac{1}{2}(\angle COD + \angle AOB)$ $= \angle AOP + \angle DOQ$ ($\perp$ prop of chord) $= 180° - \angle AOD$ $= 2\angle OAD$	B1 attempt to use (i) and (ii) B1B1 1m for reason B1	**8**
6(i)	$y = 2e^{3x}(1 - 2x)$ $\frac{dy}{dx} = 2e^{3x}(-2) + 6e^{3x}(1 - 2x)$ $= 2e^{3x}(1 - 6x)$	B1 Product Rule B1 Diff exponential fn B1 Simplify, ok if not factorised	
(ii)	For decreasing function, $\frac{dy}{dx} < 0$ $\therefore\ 1 - 6x < 0$ $x > \frac{1}{6}$ ✓	B1	
(iii)	Given that $\frac{dy}{dx} = -5$ units/s $\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$ $= 2e^{3x}(1 - 6x)(-5)$ $= 2e^{3(-1.5)}(1 + 6 \times 1.5)(-5)$ $= -1.11$ units/sec	B1 with negative seen B1 with subs seen B1	**7**

Qn	Key Steps	Marks / Remarks	
7(i)	$2^{3a+1} - 2^{2a+2} + 2^a$ Let $2^a = x$ $= 2 \times 2^{3a} - 4 \times 2^{2a} + 2^a$ $= 2x^3 - 4x^2 + x$	B1 Use of: $2^{p+q} = 2^p \times 2^q$ B1 Use of: $(2^p)^q = 2^{pq}$ B1	
(ii)	$2x^3 - 4x^2 + x = 0$ $x(2x^2 - 4x + 1) = 0$ $x = 0$, $\therefore 2^a = 0$ (rej) or $2x^2 - 4x + 1 = 0$ $x = \dfrac{-4 \pm \sqrt{16 - 4\times2\times1}}{4}$ $= 1.707$ or 0.2929 $2^a = 1.707$ or 0.2929 $a = \dfrac{\lg 1.707}{\lg 2}$ or $\dfrac{\lg 0.2929}{\lg 2}$ $= 0.7771$ or -1.77	B1 $x = 0$ seen M1 Solving quad with working seen A1 Both x M1 Using log (any base) A1 Both a	
(iii)	$2^{3a+1} - 2^{2a+2} + (k)2^a = 0$ has at least one root $\therefore$ $2x^2 - 4x + k = 0$ has at least one root $\therefore$ $16 - 4 \times 2 \times k \geq 0$ $k \leq 2$	M1 Using quad part of eqn B1 Correct D with subs A1	**11**
8(i)	$\text{f}(x) = ax^3 + bx^2 + 24x + 16$ $\text{f}'(x) = 3ax^2 + 2bx + 24$	B1	
(ii)	Sub (4, 0) into $\text{f}'(x) = 0$ $3a(16) + 2b(4) + 24 = 0$ $\therefore$ $48a + 8b + 24 = 0$ ……………………(1) Sub (4, 0) into $\text{f}(x)$ $a(64) + 16b + 24(4) + 16 = 0$ $\therefore$ $64a + 16b + 96 + 16 = 0$ …………….(2) $a = 2$, $b = -15$	B1 Sub into their $\text{f}'(x)$ and $\text{f}(x)$ M1 Solve simul eqn A1 Both	
(iii)	$\text{f}'(x) = 6x^2 - 30x + 24$ $= 6(x^2 - 5x + 4)$ $= 6(x-1)(x-4)$ $\therefore p = 1$ At $x = 1$, $\text{f}(x) = 2(1) - 15(1) + 24(1) + 16 = 27$ Hence, $k > 27$	B1 M1 Using their p A1	
(iv)	Min value of $\text{f}'(x) = 6(2.5)^2 - 30(2.5) + 24$ $= -13.5$	M1 Use $x = 2.5$ A1	**9**

Qn	Key Steps	Marks / Remarks	
9(i)	$y = -\frac{1}{2}(x-2)^4 + 16, \quad \therefore \frac{dy}{dx} = -2(x-2)^3$	B1 o.e.	
(ii)	Grad of $AB = -2(-8) = 16$ At B, $x = 0$, $\therefore y = 8$ Eqn AB: $y = 16x + 8$ $\therefore A$ is (2, 40)	B1 Grad AB seen B1 Eqn AB seen B1	
(iii)	Area $OBACD$ $= (8 + 40) \times 2$ $= 96$ units^2 Area bounded by curve and axes $= \int_0^4 \left(-\frac{1}{2}(x-2)^4 + 16\right) dx$ $= \left(-\frac{1}{10}(x-2)^5 + 16x\right)_0^4$ $= (-\frac{1}{10} \times 32 + 64) - (\frac{1}{10} \times 32)$ $= 57.6$ $\therefore$ shaded area – 96 – 57.6 = 38.4 units^2	M1 Using composite figures A1 B1 Knowing to use integral for area B1 Correct integration B1 Subs seen B1	**10**
10(i)	$v_0 = 12e^{k(0)} + 18 = 30$ m/s	B1 Sub need not be seen	
(ii)	$v_2 = 40 \quad \therefore 40 = 12e^{k(2)} + 18$ $e^{2k} = \frac{11}{6}$ $2k = \ln(\frac{11}{6})$ $k = 0.303\ 1$	B1 Sub into eqn B1 Using logarithm B1	
(iii)		B1 Shape B1 Label y-intercept B1 Label (4, 58.3)	
(iv)	Area under curve < Area of trapezium Area of trapezium $= 0.5(30 + 60) \times 4 = 180$  $\therefore$ distance travelled < 180 m	B1 Find relevant distance travelled using any suitable method B1 Making conclusion	
(v)	Max accn occurs at $t = 4$ where the gradient is most steep Max accn $= 0.3031 \times 12\ e^{0.3031(4)}$ $= 12.23$ m/s^2	M1 Knowing to differentiate A1	**11**

Qn	Key Steps	Marks / Remarks	
11(i)	$x^2 + y^2 - 8x - 4y - 5 = 0$ A is (4, 2) Radius = $\sqrt{4^2 + 2^2 + 5}$ = 5 (units)	B1 M1A1	
(ii)	$1^2 + 6^2 - 8(1) - 4(6) - 5 = 0$ Hence, (1, 6) lies on the circle.	B1 Subs seen and statement	
(iii)	Gradient of line joining (4, 2) and (1, 6) $= -\frac{4}{3}$ Eqn of tangent at (1, 6) is $y - 6 = -\frac{4}{3}(x - 1)$ $4y - 3x = 21$	B1 ⊥ grad seen B1 Find eqn B1 o.e.	
(iv)	At B, $y = 0$ $\therefore x = -7$ $\therefore B$ is (−7, 0)	M1 Finding x A1 Ordered pair seen	
(v)	Distance between centres $= \sqrt{11^2 + 2^2}$ $= \sqrt{125}$ $\therefore$ radius of C_2 $= \sqrt{125} - 5$ $= 5\sqrt{5} - 5$	M1 Find dist between centres M1 Using sum radii = distance A1	**12**

NAME:		( )		CLASS:	

YISHUN TOWN SECONDARY SCHOOL

PRELIMINARY EXAMINATION 2018
SECONDARY 4 EXPRESS / 5 NORMAL ACADEMIC
ADDITIONAL MATHEMATICS PAPER 1 (4047/01)

DATE : **7 AUGUST 2018** **DAY** : **TUESDAY**

DURATION: **2 h** **MARKS:** **80**

ADDITIONAL MATERIALS

Writing Paper x 6
Mathematics Cover Sheet x 1

READ THESE INSTRUCTIONS FIRST

Do not turn over the cover page until you are told to do so.
Write your name, class and class index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid/ tape.
Write your answers on the writing papers provided.

Answer **all** the questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
Calculators should be used where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

This question paper consists of **5** printed pages and 1 blank page

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the quadratic equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} .$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \cdots + \binom{n}{r} a^{n-r} b^r + \cdots + b^n.$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\cdots\cdots(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

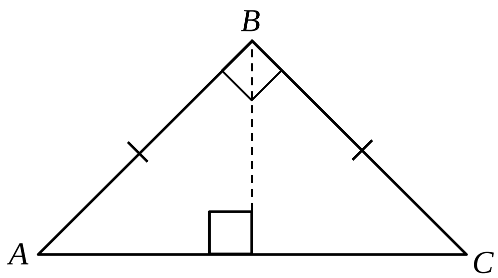
$$\text{Area of } \Delta = \frac{1}{2} bc \sin A$$

1 Find the range of values of a for which $x^2 + ax + 2(a - 1)$ is always greater than 1. [4]

2 Find the distance between the points of intersection of the line $2x + 3y = 8$ and the curve $y = 2x^2$, leaving your answer in 2 significant figures. [5]

3 Express $\dfrac{x^2 - 2x - 6}{x(x^2 - x - 6)}$ as a sum of 3 partial fractions. [5]

4 Triangle ABC is an right angled isosceles triangle with angle ABC as the right angle. The height from point B to the base AC is $\dfrac{3\sqrt{2}}{\sqrt{3}+\sqrt{6}}$. **Without using a calculator**, express the area of the triangle ABC in the form $a + b\sqrt{2}$, where a and b are integers. [5]



5 **(i)** Given $\sin(A + B) + \sin(A - B) = k \sin A \cos B$, find k. [2]

(ii) Hence, find the exact value of $\int_0^{\frac{\pi}{4}} \sin 2x \cos x \, dx$. [4]

6 **(a)** State the values between which the principal value of $\tan^{-1}x$ must lie. [1]

(b) The function f is defined by $f(x) = 3\cos ax + 1$, where a is a positive integer and $-\pi \le x \le \pi$.

(i) State the amplitude and the minimum value of f. [2]

(ii) Given that $f(x) = 1$ when $x = \dfrac{\pi}{4}$, find the smallest possible value of a [1]

(iii) Using the value of a found in part **(ii)**, state the period of f and sketch the graph of $y = f(x)$. [4]

7 The function f is defined by $f(x) = 6x^3 - kx^2 + 3x + 10$, where k is a constant.

(i) Given that $2x + 1$ is a factor of $f(x)$, find the value of k. [2]

(ii) Using the value of k found in part **(i)**, solve the equation $f(x) = 0$. [4]

8 Solve the equation

(i) $3\log_3 x - \log_x 3 = 2$, [5]

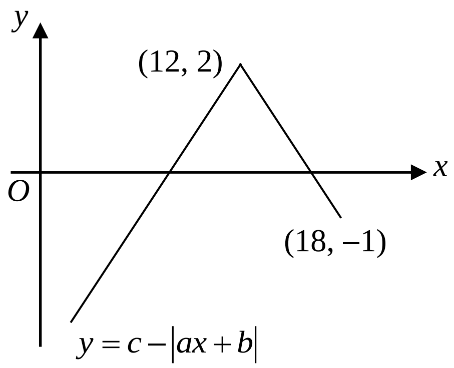
(ii) $2\log_2(1-2x) - \log_2(6-5x) = 0$. [4]

9 The equation of a curve is $y = \dfrac{2x^2}{x-1}$, $x > 1$.

(i) Find the coordinates of the stationary point of the curve. [4]

(ii) Use the second derivative test to determine the nature of the point. [3]

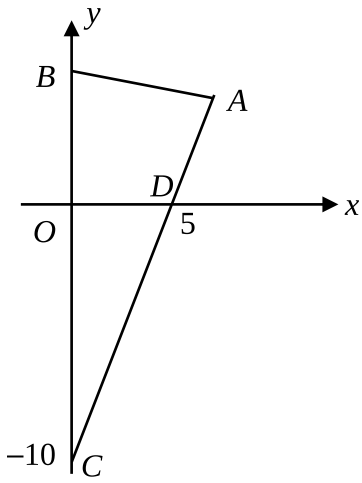
10 The diagram shows part of the graph $y = c - |ax + b|$ where $a > 0$. The graph has a maximum point (12, 2) and passes through the point (18, −1).



(i) Determine the value of each of a, b and c. [4]

(ii) State the set of value(s) of m for which the line $y = mx + 4$ cuts the graph $y = c - |ax + b|$ at exactly one point. [3]

11



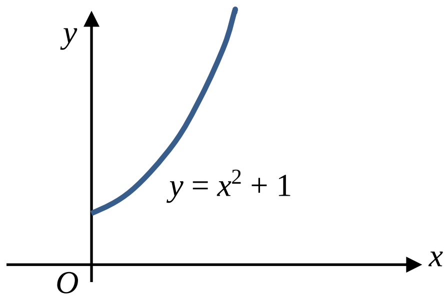
The diagram shows a triangle *ABC* in where points *B* and *C* are on the *y*-axis. The line *AC* cuts the *x*-axis at point *D* and the coordinates of point *C* and *D* are (0, −10) and (5, 0) respectively. $AD = \frac{2}{7}AC$ and points *A*, *B* and *D* are vertices of a rhombus *ABDE*.

(i) Show that the coordinates of *A* is (7, 4). [1]

(ii) Find the coordinates of *B* and *E*. [5]

(iii) Calculate the area of the quadrilateral *ABOD*. [2]

12



The diagram above shows part of the curve $y = x^2 + 1$. *P* is the point on the curve where $x = p$, $p > 0$. The tangent at *P* cuts the *x*-axis at point *Q* and the foot of the perpendicular from *P* to *x*-axis is *R*.

(i) Show that the area *A* of the triangle *PQR* is given by $A = \frac{p^3}{4} + \frac{p}{2} + \frac{1}{4p}$. [5]

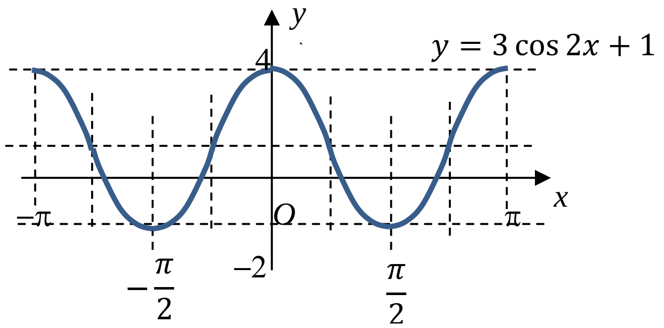
(ii) Obtain an expression for $\frac{dA}{dp}$. [1]

(iii) Find the least area of the triangle *PQR*, leaving your answer in 2 decimal places. [4]

End of Paper

YISHUN TOWN SECONDARY SCHOOL
2018 Preliminary Examination
Secondary Four Express / Five Normal
ADDITIONAL MATHEMATICS 4047/01

Answer Scheme

Qn	Answer
1	$2 < a < 6$
2	2.8 units
3	$\frac{x^2-2x-6}{x(x^2-x-6)} = \frac{1}{x} - \frac{1}{5(x-3)} + \frac{1}{5(x+2)}$
4	$18 - 12\sqrt{2}$
5(i)	$k = 2$ **5(ii)** $\frac{4-\sqrt{2}}{6}$
6(a)	$-\frac{\pi}{2} < \tan^{-1} x < \frac{\pi}{2}$ or $-90^0 < \tan^{-1} x < 90^0$
6(b)(i)	Amplitude = 3, minimum value = – 2 **6(ii)** $a = 2$
(iii)	Period = π 
7(i)	$k = 31$ **7(ii)** $x = 5$ or $\frac{2}{3}$ or $-\frac{1}{2}$
8(i)	$x = 0.693$ or 3
(ii)	$x = -\frac{5}{4}$
9(i)	(2, 8)
(ii)	$\frac{d^2y}{dx^2} = \frac{4}{(x-1)^3}$ Min point
10(i)	$a = \frac{1}{2}$, $b = -6$, $c = 2$
(ii)	$m = -\frac{1}{6}$ or $m > \frac{1}{2}$ or $m \le \frac{1}{2}$
11(i)	(7, 4)
(ii)	B (0, 5), E(12, – 1)
(iii)	27.5 units^2
12(ii)	$\frac{dA}{dp} = \frac{3}{4}p^2 + \frac{1}{2} - \frac{1}{4p^2}$ **12(ii)** 0.77 units^2

NAME:		( )		CLASS:	

YISHUN TOWN SECONDARY SCHOOL

PRELIMINARY EXAMINATION 2018
SECONDARY 4 EXPRESS / 5 NORMAL ACADEMIC
ADDITIONAL MATHEMATICS PAPER 2 (4047/02)

DATE :	**16 AUGUST 2018**	**DAY :**	**THURSDAY**
DURATION:	**2 h 30 min**	**MARKS:**	**100**

ADDITIONAL MATERIALS

Writing Paper x 8
Mathematics Cover Sheet x 1

READ THESE INSTRUCTIONS FIRST

Do not turn over the cover page until you are told to do so.
Write your name, class and class index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid/ tape.
Write your answers on the writing papers provided.

Answer **all** the questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
Calculators should be used where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

This question paper consists of **6** printed pages and 1 blank page.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the quadratic equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots\ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots\ldots(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$\text{Area of } \Delta = \frac{1}{2}bc \sin A$$

1. **(a)** Given that the roots of the equation $x^2 - 6x + k = 0$ differ by 2, find the value of *k*. [3]

(b) If α and β are the roots of the equation $x^2 + bx + 1 = 0$, where *b* is a non-zero constant, show that the equation with roots $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ is $x^2 - \left(b^2 - 2\right)x + 1 = 0$. [4]

2. If the first three terms in the expansion of $\left(1 - \frac{x}{2}\right)^n$ is $1 - 6x + ax^2$, find the value of *n* and of *a*. [4]

3. **(a)** Solve the equation $\sqrt{4 + \frac{3}{x}} = \frac{1}{\sqrt{x}} + 2$. [5]

(b) Given that $\frac{4}{n}(3x)^2\left(\frac{2}{9x^2}\right)^{n-2} \equiv \frac{m}{x^2}$, where $x \neq 0$, find the values of the constants *m* and *n*. [4]

4. A precious stone was purchased by a jeweler in the beginning of January 2003. The expected value, \$*V*, of the stone may be modelled by the equation $V = 6000\left(4^t\right) - 1000\left(16^t\right)$, where *t* is the number of years since the time of purchase. Find

(i) the expected value of the stone when $t = \frac{3}{4}$. [1]

(ii) the value(s) of *t* for which the expected value of the stone is \$8000. [3]

(iii) the range of values of *t* for which the expected value of the stone exceeds \$8000. [1]

5. The equation of a circle, *C*, is $x^2 + y^2 - 4ux + 2uy + 5\left(u^2 - 20\right) = 0$ where *u* is a positive constant.

(a) Given that $u = 6$, find the coordinates of the centre and the radius of the circle *C*. [3]

(b) Determine the value of *u* for which

(i) the circle, *C*, passes through the point $(-4, 4)$, [2]

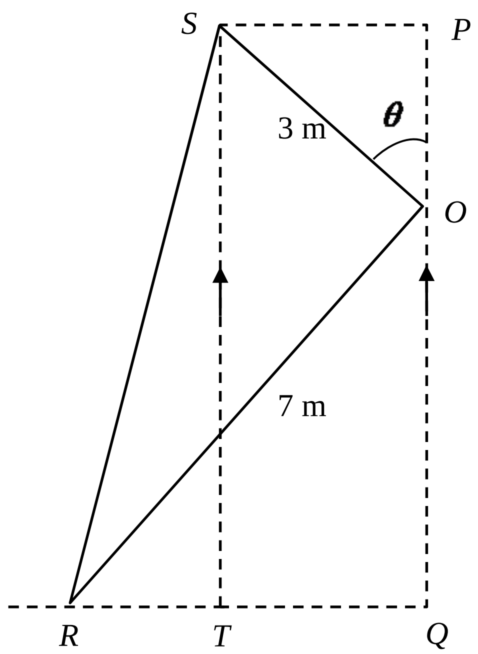
(ii) the line $x = 2$ is a tangent to the circle, *C*. [4]

6. The variables x and y are related by the equation $mx + ny - 3xy = 0$, where m and n are non-zero constants. When $\frac{1}{y}$ is plotted against $\frac{1}{x}$, a straight line is obtained. Given that the line passes through the points $(1,0)$ and $(-5,9)$, find the values of m and of n. [6]

7. **(i)** Prove that $\sin^4\theta - \cos^4\theta \equiv 1 - 2\cos^2\theta$. [3]

(ii) Hence solve $\sin^4\theta - \cos^4\theta - 3\cos\theta = 2$ for $0 < \theta < 2\pi$. [4]

8. In the diagram, $OS = 3$ m, $OR = 7$ m and angle SOR = angle SPO = angle $RQO = 90^\circ$. It is given that angle SOP is a variable angle θ where $0^\circ < \theta < 90^\circ$. The point T is on the line RQ such that ST is parallel to PQ.



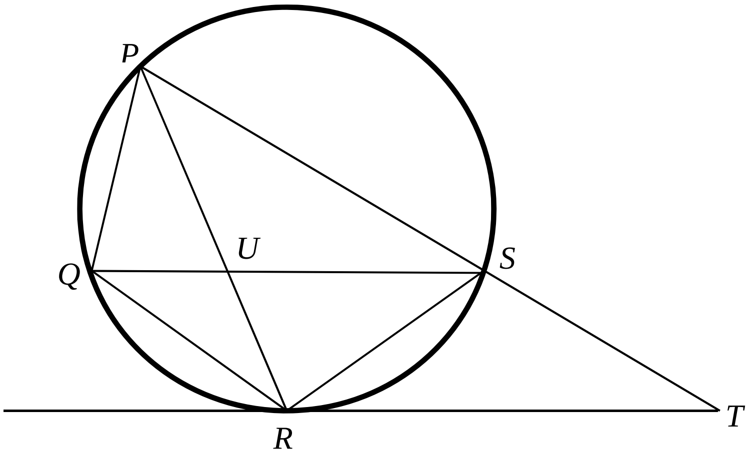
(i) Show that $PQ = 7\sin\theta + 3\cos\theta$. [1]

(ii) Show that the area of triangle RST is $\frac{21}{2}\cos 2\theta + 10\sin 2\theta$. [3]

(iii) Express the area of the triangle RST as $k\cos(2\theta - \alpha)$, where $k > 0$ and $0^\circ < \alpha < 90^\circ$. [4]

(iv) Hence find the maximum area of triangle RST and the corresponding value of θ. [3]

9. The diagonals of a cyclic quadrilateral *PQRS* intersect at a point *U*. The circle's tangent at *R* meets the line *PS* produced at *T*.



If $QR = RS$, prove the following.

(i) *QS* is parallel to *RT*. [3]

(ii) Triangles *PUS* and *QUR* are similar. [3]

(iii) $PU^2 - QU^2 = (PU \times PR) - (QU \times QS)$. [3]

10. It is given that $y = xe^{-x} - 2e^{-2x}$.

(i) Find $\frac{dy}{dx}$. [2]

(ii) If *x* and *y* can vary with time and *x* increases at the rate of 1.5 units per second at the instant when $x = \ln 2$, find the exact value of the rate of increase of *y* at this instant. [3]

11. A curve has the equation $y = \frac{\ln x}{x^2} - 2$.

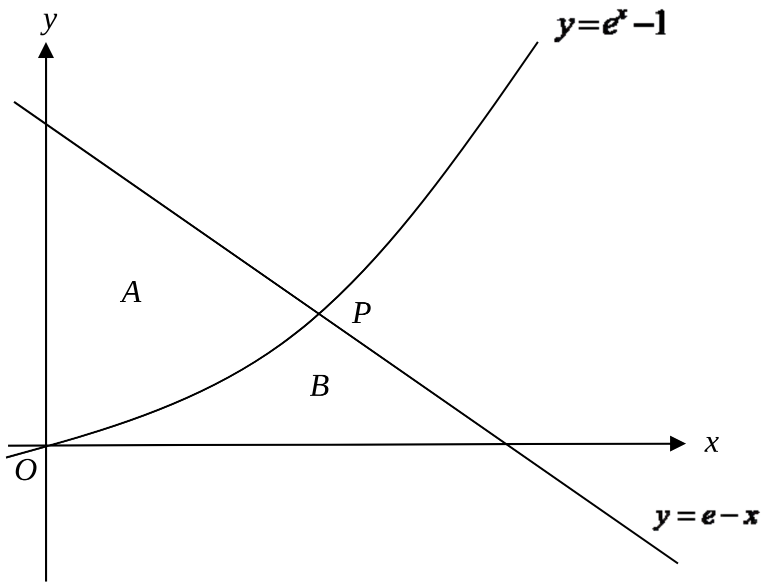
(i) Show that $\frac{dy}{dx} = \frac{1 - 2\ln x}{x^3}$. [2]

(ii) **(a)** The *x*-coordinate of a point *P* on the curve is 1. Find the equation of the tangent to the curve at *P*. [2]

(b) The tangent to the curve at the point *P* intersects the *x*-axis at *Q* and the *y*-axis at *R*. Calculate the shortest distance from the origin *O* to the line *QR*. [4]

(iii) Given that another curve $y = f(x)$ passes through the point $(1, -0.25)$ and is such that $f'(x) = \frac{\ln x}{x^3}$, find the function $f(x)$. [3]

12. The diagram shows the graphs of $y = e^x - 1$ and $y = e - x$. *P* is the point of intersection of the two graphs.



(i) Show that $\alpha = 1$ is a root to the equation $e\left(1 - e^{\alpha - 1}\right) - \alpha + 1 = 0$. [1]

(ii) Hence, find the coordinates of *P*. [2]

(iii) Find the area of region *A*, which is enclosed by the two graphs and the *y*-axis. [4]

(iv) Find the exact value of $\dfrac{\text{area of region } A}{\text{area of region } B}$, given that the area of region *B* is enclosed by the two graphs and the *x*-axis. [2]

13. A particle moves pass a point *A* in a straight line with a displacement of -4 m from a fixed point *O*. Its acceleration, *a* m/s^2 , is given by $a = \frac{t}{2}$, where *t* seconds is the time elapsed after passing through point *A*.

Given that the initial velocity is -1 m/s, find,

(i) the velocity when $t = 2$, [3]

(ii) the distance travelled by the particle in the first 5 seconds. [5]

END OF PAPER

YISHUN TOWN SECONDARY SCHOOL
2018 Preliminary Examination
Secondary Four Express / Five Normal
ADDITIONAL MATHEMATICS 4047/02

1(a)	$k = 8$
1(b)	Show $x^2 - (b^2 - 2)x + 1 = 0$
2	$n = 12$, $a = \frac{33}{2}$
3(a)	$x = \frac{1}{4}$
3(b)	$n = 4$, $m = \frac{4}{9}$
4(i)	$8970
4(ii)	$t = \frac{1}{2}, 1$
4(iii)	$\frac{1}{2} < t < 1$
5(i)	Centre is $(12, -6)$ Radius = 10 units
5(ii)(a)	$u = 2$
5(ii)(b)	$u = 6$
6	$m = 2$, $n = 3$
7(i)	Show $\sin^4\theta - \cos^4\theta \equiv 1 - 2\cos^2\theta$
7(ii)	$\theta = \frac{2\pi}{3}, \pi, \frac{4\pi}{3}$
8(i)	Show $PQ = 7\sin\theta + 3\cos\theta$
8(ii)	Show Area = $\frac{21}{2}\cos 2\theta + 10\sin 2\theta$
8(iii)	Area = $\frac{29}{2}\cos(2\theta - 43.6^\circ)$
8(iv)	Max area of triangle $RST = \frac{29}{2}$ m^2, $\theta = 21.8^\circ$
9(i)	Show QS is parallel to RT
9(ii)	Show Triangles PUS and QUR are similar
9(iii)	Show $PU^2 - QU^2 = (PU \times PR) - (QU \times QS$
10(i)	$\frac{dy}{dx} = (1 - x)e^{-x} + 4e^{-2x}$
10(ii)	$\left.\frac{dy}{dt}\right\|_{x=\ln 2} = \frac{9}{4} - \frac{3}{4}\ln 2$
11(i)	$\frac{dy}{dx} = \frac{1 - 2\ln x}{x^3}$
11(ii)(a)	$y = x - 3$
11(ii)(b)	$h = \frac{3\sqrt{2}}{2}$ units
11(iii)	$\text{f}(x) = -\frac{1 + 2\ln x}{4x^2}$
12(ii)	$P(1, e - 1)$
12(iii)	Area of Region $A = \frac{3}{2}$ units^2
12(iv)	$\frac{\text{Area of Region } A}{\text{Area of Region } B} = \frac{3}{e^2 - 3}$
13(i)	Velocity = 0 m/s
13(ii)	Total distance travelled $= 8\frac{1}{12}$ m

ZHONGHUA SECONDARY SCHOOL

PRELIMINARY EXAMINATION 2018

SECONDARY 4E/5N

Candidate's Name	Class	Register Number

ADDITIONAL MATHEMATICS 4047/01

PAPER 1

11 September 2018

2 hours

Additional Materials: Writing paper, Graph paper (2 sheets)

READ THESE INSTRUCTIONS FIRST

Write your index number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the presentation, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is **80**.

For Examiner's Use:

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \cdots + \binom{n}{r}a^{n-r}b^r + \cdots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!\,r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1.$$
$$\sec^2 A = 1 + \tan^2 A.$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A.$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

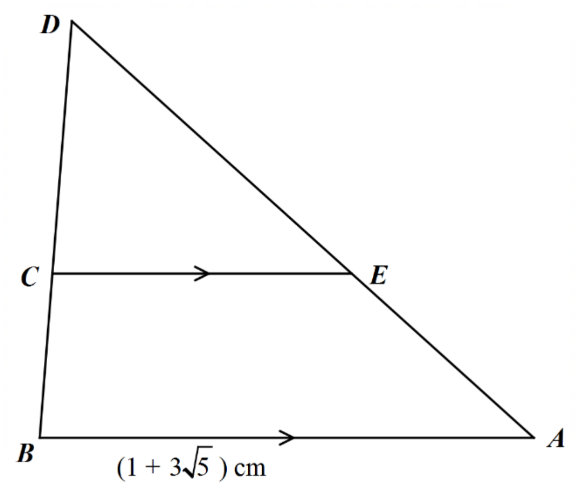
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

Answer all the questions

1.



AB is parallel to EC and $AB = (1+3\sqrt{5})$ cm. E is a point on AD such that $AE : ED = \sqrt{5} : 3$.
Find

(i) $\dfrac{EC}{AB}$ in the form of $a + b\sqrt{5}$, where a and b are rational numbers. [3]

(ii) the length of EC in the form of $c + d\sqrt{5}$, where c and d are integers. [3]

2. The equation of a curve is $y = (k+2)x^2 - 10x + 2k + 1$, where k is a constant.

(i) In the case where $k = 1$, sketch the graph of $y = (k+2)x^2 - 10x + 2k + 1$, showing the x- and y- intercepts and its turning point clearly. [3]

(ii) Find the range of values of k for which the curve meets the line $y = 2x + 3$. [5]

3. (a) Express $\dfrac{3x^3 - 5}{x^2 - 1}$ in partial fractions. [5]

(b) Solve the equation $|21 - 18x| - |7 - 6x| = 4x - 1$. [4]

4. The equation of a curve is $y = 2x(x-1)^3$.

(i) Find the coordinates of the stationary points of the curve. [5]

(ii) Determine the nature of each of these points using the first derivative test. [3]

5. (i) On the same diagrams, sketch the graphs $y = \dfrac{4}{x^2}$, $x > 0$ and $y = 3x^{\frac{1}{2}}$, $x \geq 0$. [2]

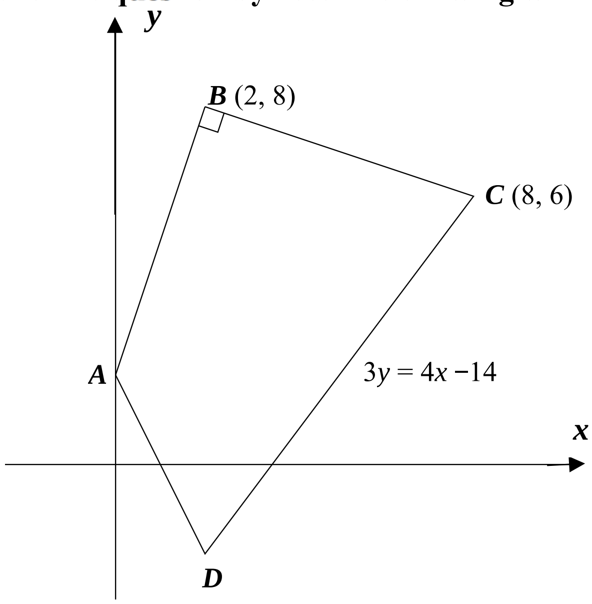
(ii) Find the value of the constant k for which the $x-$coordinate of the point of intersection of your graphs is the solution to the equation $x^5 = k$. [2]

[Turn over

6. (i) Prove that $\dfrac{1}{3\tan^2\theta+3}=\dfrac{\cos^2\theta}{3}$. [2]

(ii) Show that $\displaystyle\int_0^{\frac{\pi}{3}}\frac{\sec^2\theta\cos 2\theta}{3\tan^2\theta+3}\,\mathrm{d}\theta=\frac{\sqrt{3}}{12}$. [4]

7. **Solutions to this question by accurate drawing will not be accepted.**



The diagram above shows a quadrilateral *ABCD*. Point *B* is (2, 8) and point *C* is (8, 6).

The point *D* lies on the perpendicular bisector of *BC* and the point *A* lies on the *y*-axis.

The equation of *CD* is $3y=4x-14$ and angle $ABC = 90^\circ$. Find

(i) the equation of *AB*, [2]

(ii) the coordinates of *A*, [1]

(iii) the equation of the perpendicular bisector of *BC*, [3]

(iv) the coordinates of *D*, [3]

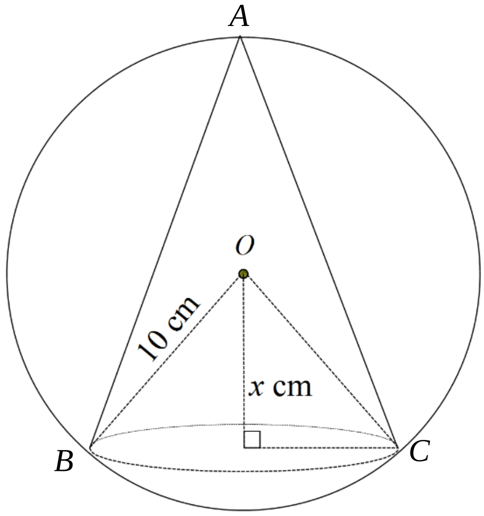
8. (i) Show that $\dfrac{\mathrm{d}}{\mathrm{d}x}(x^2\ln x-3x)=x+2x\ln x-3$. [2]

(ii) Evaluate $\displaystyle\int_1^4 x\ln x\,\mathrm{d}x$. [4]

9. A curve is such that the gradient function is $1+\dfrac{1}{2x^2}$. The equation of the tangent at point *P* on the curve is $y=3x+1$. Given that the *x*-coordinate of *P* is positive, find the equation of the curve. [7]

[Turn over

10.



A right circular cone, *ABC*, is inscribed in a sphere of radius 10 cm and centre *O*.

The perpendicular distance from *O* to the base of the cone is *x* cm.

$\left[\text{Volume of cone} = \frac{1}{3}\pi r^2 h\right]$

(i) Show that volume, *V*, of the cone is $V = \frac{1}{3}\pi(100 - x^2)(10 + x)$. [2]

(ii) If *x* can vary, find the value of *x* for which *V* has a stationary value. [3]

(iii) Find this stationary volume. [1]

(iv) Determine whether the volume is a maximum or minimum. [2]

11. (a) Find, in radians, the two principal values of *y* for which $2\tan^2 y + \tan y - 6 = 0$. [4]

(b) The height, *h* m, above the ground of a carriage on a carnival ferris wheel is modelled by the equation $h = 7 - 5\cos(8t)$, where *t* in the time in minutes after the wheel starts moving.

(i) State the initial height of the carriage above ground. [1]

(ii) Find the greatest height reached by the carriage. [1]

(iii) Calculate the duration of time when the carriage is 9 m above the ground. [3]

END OF PAPER

4E5N 2018 Prelim AMath paper 1 Marking Scheme

<table>
<tr><td>1i</td><td>

ΔABD is similar to ΔECD.

$\therefore \frac{CE}{BA} = \frac{DE}{DA}$

$\frac{CE}{BA} = \frac{3}{3+\sqrt{5}}$ [M1] ratio seen

$= \frac{3}{3+\sqrt{5}} \times \frac{3-\sqrt{5}}{3-\sqrt{5}}$ [B1] correct conjugate surd

$= \frac{9-3\sqrt{5}}{3^2-5}$

$= \frac{9-3\sqrt{5}}{4}$ [A1]

</td><td></td></tr>
<tr><td>ii</td><td>

$EC = \frac{9-3\sqrt{5}}{4} \times \left(1+3\sqrt{5}\right)$ [M1]

$= \frac{1}{4}\left[9\left(1+3\sqrt{5}\right) - 3\sqrt{5}\left(1+3\sqrt{5}\right)\right]$

$= \frac{1}{4}\left(9+27\sqrt{5}-3\sqrt{5}-9\times 5\right)$ [M1] expansion seen

$= \frac{1}{4}\left(-36+24\sqrt{5}\right)$

$= -9+6\sqrt{5}$ [A1]

</td><td></td></tr>
<tr><td>2i</td><td colspan="2">

When $k = 1$,

$y = 3x^2 - 10x + 3$

$= 3\left(x^2 - \frac{10}{3}\right) + 3$

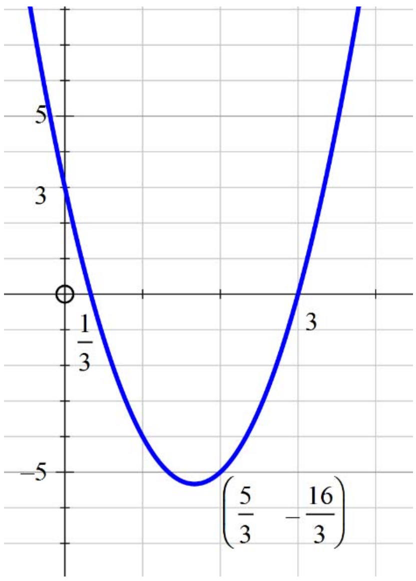
$= 3\left[\left(x - \frac{10}{6}\right)^2 - \left(\frac{10}{6}\right)^2\right] + 3$

$= 3\left(x - \frac{5}{3}\right)^2 - \frac{25}{3} + 3$

$= 3\left(x - \frac{5}{3}\right)^2 - \frac{16}{3}$

Turning point $\left(\frac{5}{3}, -\frac{16}{3}\right)$

When $y = 0$, $x = 3$ or $\frac{1}{3}$



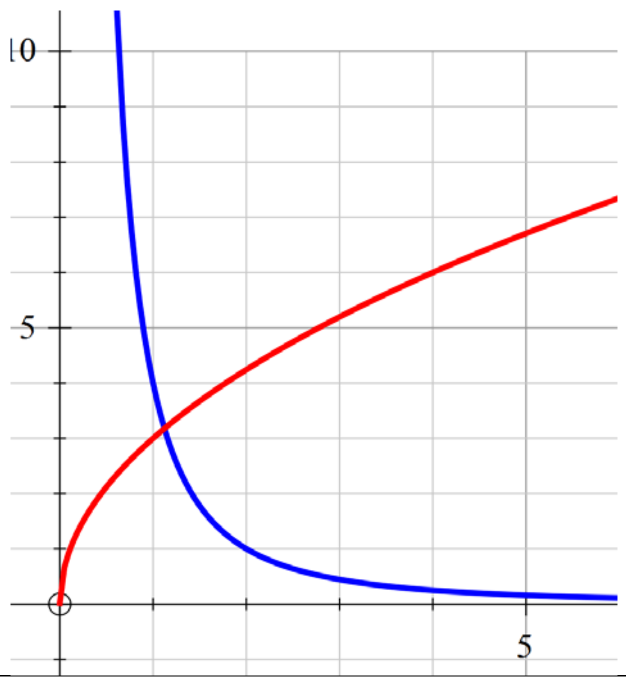
[B1] y-intercept

[B1] x-intercepts

[B1] turning point

</td></tr>
</table>

ii	$(k+2)x^2-10x+2k+1=2x+3$ [M1] substitution $(k+2)x^2-12x+2k-2=0$ $b^2-4ac\ge 0$ [B1] $(-12)^2-4(k+2)(2k-2)\ge 0$ $144-8(k^2+k-2)\ge 0$ $-8k^2-8k+160\ge 0$ $k^2+k-20\le 0$ $(k+5)(k-4)\le 0$ [M1] factorisation $-5\le k\le 4$ and $k\ne -2$ [A1] [A1]	
3i	By long division [M1] $\frac{3x^3-5}{x^2-1}=3x+\frac{3x-5}{x^2-1}$ [A1] $\frac{3x-5}{(x+1)(x-1)}=\frac{A}{x+1}+\frac{B}{x-1}$ $3x-5=A(x-1)+B(x+1)$ [M1] any acceptable method to find A and B $x=1$: $3(1)-5=2B$ $B=-1$ $x=-1$: $-3-5=-2A$ $A=4$ [A1] correct A and B $\therefore \frac{3x^3-5}{x^2-1}=3x+\frac{4}{x+1}-\frac{1}{x-1}$ [A1]	
ii	$\lvert 21-18x\rvert-\lvert 7-6x\rvert=4x-1$ $\lvert 3(7-6x)\rvert-\lvert 7-6x\rvert=4x-1$ [B1] factorise 3 $3\lvert 7-6x\rvert-\lvert 7-6x\rvert=4x-1$ $2\lvert 7-6x\rvert=4x-1$ $\lvert 7-6x\rvert=\frac{4x-1}{2}$ $7-6x=\frac{4x-1}{2}$ or $7-6x=\frac{-4x+1}{2}$ [B1] either one seen $x=\frac{15}{16}$ or $x=\frac{13}{8}$ [A1] [A1]	

4i	$y = 2x(x-1)^3$ $\frac{dy}{dx} = 2x\left[3(x-1)^2\right] + 2(x-1)^3$ [M1] product rule $= 6x(x-1)^2 + 2(x-1)^3$ [A1] $= 2(x-1)^2(3x+x-1)$ $= 2(x-1)^2(4x-1)$ For $\frac{dy}{dx} = 0$ $2(x-1)^2(4x-1) = 0$ [M1] $x = 1$ or $x = \frac{1}{4}$ $y = 0$ or $y = -\frac{27}{128}$ $(1,0)$ and $\left(\frac{1}{4}, -\frac{27}{128}\right)$ [A1] [A1]	
ii	By first derivative test, [M1] $(1,0)$ is a point of inflexion and $\left(\frac{1}{4}, -\frac{27}{128}\right)$ is a min. point [A1], [A1]	
5i	 $y = 3x^{\frac{1}{2}}$ [B1] $y = \frac{4}{x^2}$ [B1]	

ii	$3x^{\frac{1}{2}} = \frac{4}{x^2}$ [M1] substitution $x^{\frac{1}{2}}.x^2 = \frac{4}{3}$ $x^{\frac{5}{2}} = \frac{4}{3}$ $x^5 = \left(\frac{4}{3}\right)^2$ [M1] squaring $= \frac{16}{9}$ $\therefore k = \frac{16}{9}$ [A1]	
6i	$\text{LHS} = \frac{1}{3\tan^2\theta + 3}$ [B1] apply correct identity $= \frac{1}{3\left(\sec^2\theta - 1\right) + 3}$ $= \frac{1}{3\sec^2\theta}$ [B1] able to simplify $= \frac{\cos^2\theta}{3}$ $= \text{RHS}$	
ii	$\int_0^{\frac{\pi}{3}} \frac{\sec^2\theta\cos 2\theta}{3\tan^2\theta + 3}\,d\theta = \int_0^{\frac{\pi}{3}} \frac{\cos^2\theta}{3}\left(\frac{1}{\cos^2\theta}\right)\cos 2\theta\,d\theta$ [M1] substitution of $\frac{1}{3\tan^2\theta+3}$ $= \frac{1}{3}\int_0^{\frac{\pi}{3}} \cos 2\theta\,d\theta$ [B1] $\sec^2\theta = \frac{1}{\cos^2\theta}$ $= \frac{1}{3}\left[\frac{\sin 2\theta}{2}\right]_0^{\frac{\pi}{3}}$ [B1] correct integration of $\cos 2\theta$ $= \frac{1}{6}\left(\sin\frac{2\pi}{3} - \sin 0\right)$ $= \frac{1}{6}\left(\sin\frac{\pi}{3} - 0\right)$ $= \frac{1}{6}\left(\frac{\sqrt{3}}{2}\right)$ [B1] $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$ $= \frac{\sqrt{3}}{12}$ (shown)	

7i	Grad. BC $=\frac{8-6}{2-8}$ $=-\frac{1}{3}$ Grad. $AB = 3$ [B1] Eqn AB is $\frac{y-8}{x-2}=3$ $\therefore y=3x+2$ [B1]	
ii	When $x = 0$, $y = 2$ A (0, 2) [B1]	
iii	Grad. of perpendicular bisector = 3 Midpt. $BC=\left(\frac{2+8}{2},\ \frac{8+6}{2}\right)$ [M1] midpoint formula $=(5,\ 7)$ Eqn is $\frac{y-7}{x-5}=3$ [M1] $y=3x-8$ [A1]	
iv	$3y=4x-14$ $3(3x-8)=4x-14$ [M1] substitution $9x-24=4x-14$ $5x=10$ $x=2$ [A1] $y=3(2)-8$ $=-2$ $D\ (2,-2)$ [A1]	
8i	$\frac{\mathrm{d}}{\mathrm{d}x}(x^2\ln x-3x)=x^2\left(\frac{1}{x}\right)+2x\ln x-3$ [B1] $\frac{1}{x}$ seen $=x+2x\ln x-3$ [B1] product seen	
ii	$\int_1^4 x+2x\ln x-3\ \mathrm{d}x=\left[x^2\ln x-3x\right]_1^4$ [M1] reverse differentiation $\int_1^4 x-3\ \mathrm{d}x+\int_1^4 2x\ln x\ \mathrm{d}x=4^2\ln 4-3(4)-(0-3)$ $\left[\frac{x^2}{2}-3x\right]_1^4+2\int_1^4 x\ln x\ \mathrm{d}x=16\ln 4-12+3$ [A1] $\left[\frac{x^2}{2}-3x\right]$ seen $2\int_1^4 x\ln x\ \mathrm{d}x=16\ln 4-9-\left[\frac{4^2}{2}-3(4)-\frac{1}{2}+3\right]$ [A1] simplification $=16\ln 4-\frac{15}{2}$ or -14.7 (3s.f.) [A1]	

9	$\frac{dy}{dx}=1+\frac{1}{2x^2}$ $=1+\frac{1}{2}x^{-2}$ $y=\int\left(1+\frac{1}{2}x^{-2}\right)dx$ [M1] $=x+\frac{1}{2}\left(\frac{x^{-1}}{-1}\right)+c$ $=x-\frac{1}{2x}+c$ [A1] Since $\frac{dy}{dx}=3$ $1+\frac{1}{2x^2}=3$ [M1] $\frac{1}{2x^2}=2$ $x^2=\frac{1}{4}$ $x=\pm\frac{1}{2}$ (reject $-\frac{1}{2}$) [A1] When $x=\frac{1}{2}$, $y=3\left(\frac{1}{2}\right)+1$ $=\frac{5}{2}$ [A1] At $\left(\frac{1}{2},\frac{5}{2}\right)$, $\frac{5}{2}=\frac{1}{2}-\frac{1}{2(0.5)}+c$ [M1] attempt to find c $c=3$ $y=x-\frac{1}{2x}+3$ [A1]	
10i	Radius of cone $=\sqrt{10^2-x^2}$ $=\sqrt{100-x^2}$ [B1] Volume of cone $=\frac{1}{3}\pi r^2h$ $=\frac{1}{3}\pi\left(\sqrt{100-x^2}\right)^2(x+10)$ $=\frac{1}{3}\pi\left(100-x^2\right)(x+10)$ [B1] application of formula and substitution	

ii	$\frac{dV}{dx}=\frac{1}{3}\pi\left[-2x\left(x+10\right)+100-x^2\right]$ [M1] product rule $=\frac{1}{3}\pi\left[-20x-2x^2+100-x^2\right]$ $=\frac{1}{3}\pi\left(-3x^2-20x+100\right)$ For stationary V, $\frac{dV}{dx}=0$ [M1] $\frac{1}{3}\pi\left(-3x^2-20x+100\right)=0$ $3x^2+20x-100=0$ $(x+10)(3x-10)=0$ $x=-10$ (rejected), $x=\frac{10}{3}$ [A1]	
iii	$V=\frac{1}{3}\pi\left(100-\frac{100}{9}\right)\left(\frac{10}{3}+10\right)$ $=1241.123$ $=1240\text{ cm}^3$ (3s.f.) [B1]	
iv	$\frac{d^2V}{dx^2}=\frac{1}{3}\pi(-6x-20)$ [M1] Since $\frac{d^2V}{dx^2}<0$, V is a maximum. [A1]	
11a	$2\tan^2 y+\tan y-6=0$ $\left(2\tan y-3\right)\left(\tan y+2\right)=0$ [M1] factorisation $\tan y=\frac{3}{2}$ or $\tan y=-2$ [A1] either one $y=\tan^{-1}\left(\frac{3}{2}\right)$ $y=\tan^{-1}\left(-2\right)$ $=0.9827$ $=-1.1071$ ≈ 0.983 (3s.f.) ≈ -1.11 (3s.f.) [A1] [A1]	
bi	Initial height = 2 m [B1]	
ii	Greatest height = 7 – 5(–1) = 12 m [A1]	

iii	$7-5\cos 8t=9$ [M1] $\cos 8t=-\frac{2}{5}$ $\alpha=1.1592$ $8t=1.9823,\ 4.300$ $t=0.2477,\ 0.5375$ [A1] $\text{Duration}=0.5375-0.2477$ $=0.2898$ ≈ 0.290 minutes (3s.f.) [A1]	

ZHONGHUA SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2018
SECONDARY 4E/5N

Candidate's Name	Class	Register Number

ADDITIONAL MATHEMATICS 4047/02

PAPER 2

14 September 2018
2 hours 30 minutes

Additional Materials: Writing paper, Graph paper (1 sheet)

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the presentation, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is **100**.

For Examiner's Use
100

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \cdots + \binom{n}{r}a^{n-r}b^r + \cdots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \dfrac{n!}{(n-r)!\,r!} = \dfrac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1.$$
$$\sec^2 A = 1 + \tan^2 A.$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A.$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

1. (i) Given that $u = 4^x$, express $4^x = 9 - 5 \times 4^{1-x}$ as a quadratic equation in u. [2]

   (ii) Hence find the values of x for which $4^x = 9 - 5 \times 4^{1-x}$, giving your answer, where appropriate, to 1 decimal place. [4]

   (iii) Determine the values of k for which $4^x = k - 5 \times 4^{1-x}$ has no solution. [3]

2. (i) By using long division, divide $2x^4 + 5x^3 - 8x^2 - 8x + 3$ by $x^2 + 3x - 1$. [2]

   (ii) Factorise $2x^4 + 5x^3 - 8x^2 - 8x + 3$ completely. [2]

   (iii) Hence find the exact solutions to the equation [4]

$$32p^4 + 40p^3 - 32p^2 - 16p + 3 = 0.$$

3. The roots of the quadratic equation $8x^2 - 4x + 1 = 0$ are $\frac{1}{\alpha^2\beta}$ and $\frac{1}{\alpha\beta^2}$. Find a quadratic equation with roots α^3 and β^3. [7]

4. (i) Write down the general term in the binomial expansion of $\left(2x^2 - \frac{p}{x}\right)^{10}$, where p is a constant. [1]

.

   (ii) Given that the coefficient of x^8 in the expansion of $\left(2x^2 - \frac{p}{x}\right)^{10}$ is negative $\frac{10}{3}$ times the coefficient of x^5. Show that the value of p is $\frac{1}{2}$. [5]

   (iii) Showing all your working, use the value of p in part (ii), to find the constant term in the expansion of $(2x-1)\left(2x^2 - \frac{p}{x}\right)^{10}$. [5]

5. (a) (i) Show that $\sin 3x = \sin x(4\cos^2 x - 1)$ [3]

   (ii) Solve the equation $3\sin 3x = 16\cos x \sin x$ for $0 \le x \le 2\pi$ [5]

   (b) Differentiate $\cos 2x\,(\tan^2 x - 1)$ with respect to x. No simplification is required. [3]

6 The equation of a curve is $y = x^3 - 4x^2 + px + q$ where p and q are constants. The equation of the tangent to the curve at the point $A(-1, 5)$ is $15x - y + 20 = 0$.

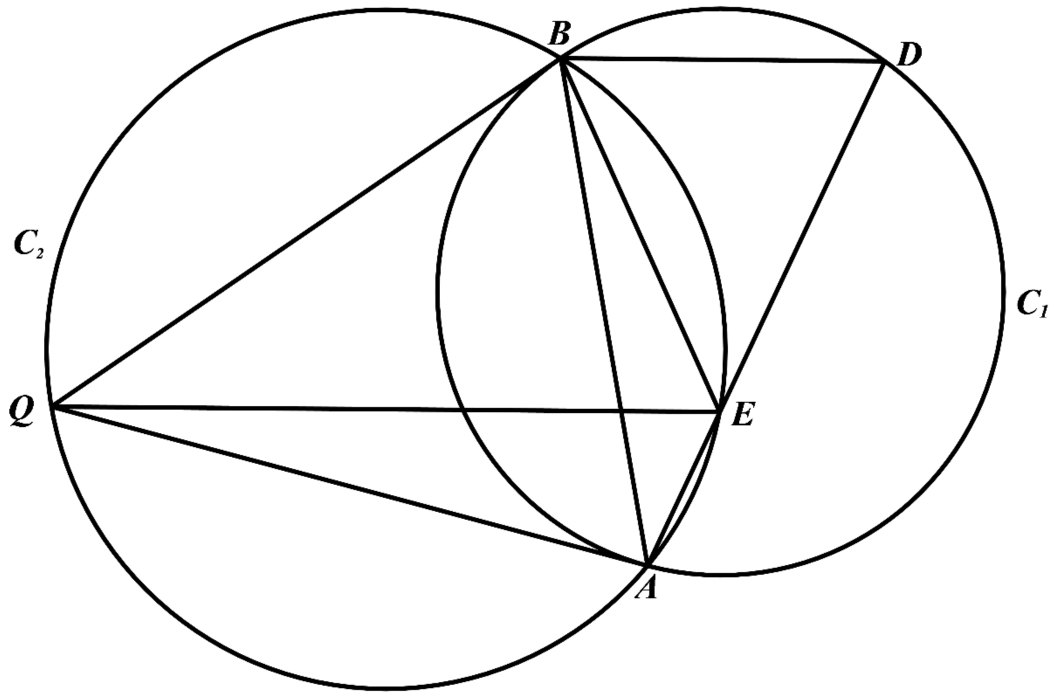
(i) Find the values of p and of q. [4]

(ii) Determine the values of x for which y is an increasing function. [3]

(iii) Find the range of values of x for which the gradient is decreasing. [2]

(iv) A point P moves along the curve in such a way that the x-coordinate of P increases at a constant rate of 0.02 units per second. Find the possible x-coordinates of P at the instant that the y-coordinate of P is increasing at 1.9 units per second. [4]

7.



The diagram shows two intersecting circles, C_1 and C_2. C_1 passes through the vertices of the triangle ABD. The tangents to C_1 at A and B intersect at the point Q on C_2. A line is drawn from Q to intersect the line AD at E on C_2.

Prove that

(i) QE bisects angle AEB, [4]

(ii) $EB = ED$, [2]

(iii) BD is parallel to QE. [2]

8. The number, N, of E. Coli bacteria increases with time, t minutes. Measured values of N and t are given in the following table.

t	2	4	6	8	10
N	3215	3446	3693	3959	4243

It is known that N and t are related by the equation $N = N_o\,(2)^{kt}$, where N_o and k are constants.

(i) Plot lg N against t and draw a straight line graph. The vertical lg N axis should start at 3.40 and have a scale of 2 cm to 0.02. [3]

(ii) Use your graph to estimate the values of N_o and k. [4]

(iii) Estimate the time taken for the number of bacteria to increase by 25%. [2]

9. A man was driving along a straight road, towards a traffic light junction. When he saw that the traffic light had turned amber, he applied the brakes to his car and it came to a stop just before the traffic light junction. The velocity, v m/s, of the car after he applied the brakes is given by $v = 40e^{-\frac{1}{3}t} - 15$, where t, the time after he applied the brakes, is measured in seconds.

(i) Calculate the initial acceleration of the car. [2]

(ii) Calculate the time taken to stop the car. [2]

(iii) Obtain an expression, in terms of t, for the displacement of the car, t seconds after the brakes has been applied. [3]

(iv) Calculate the braking distance. [1]

10. The points $P(4, 6)$, $Q(-3, 5)$ and $R(4, -2)$ lie on a circle.

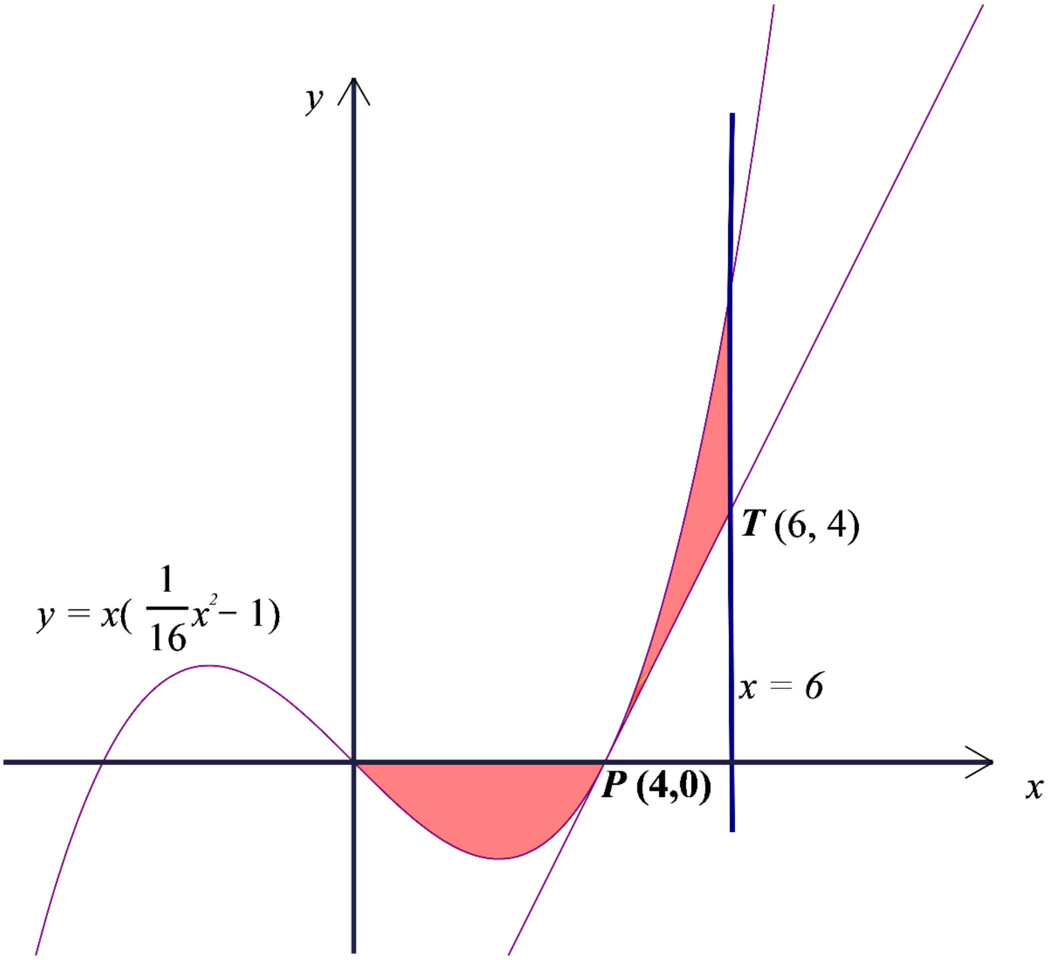
(i) Find the equation of the perpendicular bisector of PQ. [3]

(ii) Show that the centre of the circle is $(1, 2)$ and find the radius of the circle. [3]

(iii) State the equation of the circle. [1]

(iv) Find the equation of the tangent to the circle at R. [3]

11. The diagram shows part of the curve $y = x\left(\frac{1}{16}x^2 - 1\right)$. The curve cuts the x-axis at P(4, 0). The tangent to the curve at P meets the vertical line $x = 6$ at $T(6, 4)$.
Showing all your workings, find the total area of the shaded regions. [6]



End of paper

1	(i)	$u^2 - 9u + 20 = 0$
	(ii)	$x = 1$
		$x = 1.2$`
	(iii)	$-\sqrt{80} < k < \sqrt{80}$
2	(i)	$\mathbf{2x^2 - x - 3}$
	(ii)	$(x^2+3x-1)(2x-3)(x+1)$
	(iii)	$p = \frac{-3 \pm \sqrt{13}}{4}$ M1 $p = \frac{3}{4}$ or $p = -\frac{1}{2}$
3		$x^2 + 4x + 8 = 0$
4	(i)	$\binom{10}{r}(2x^2)^{10-r}\left(-\frac{p}{x}\right)^r$
	(ii)	$\frac{\binom{10}{4}2^6}{\binom{10}{5}2^5} \times \frac{3}{10} = p$ $p = \frac{1}{2}$ AG
	(iii)	-15
5a	(ii)	$x = 0,\ \pi,\ 2\pi$ or $x = 1.74$ or 4.54
5b		$2\cos 2x \tan x \sec^2 x - 2\sin 2x(\tan^2 x - 1)$
6	(i)	$p = 4$ $q = 14$
	(ii)	$x < \frac{2}{3}$ or $x > 2$
	(iii)	$x < \frac{4}{3}$
	(iv)	$x = -\frac{13}{3}$ or $x = 7$

8	(ii)	$N_o = 2992$ accept also 2990 $k = 0.05$
	(iii)	time taken= 6.4 mins
9	(i)	$-\frac{40}{3}\text{ m/s}^2$
	(ii)	2.94s
	(iii)	$s = -120e^{-\frac{1}{3}t} - 15t + 120$
	(iv)	30.9m
10	(i)	$y = -7x + 9$
	(ii)	$r = 5$ units
	(iii)	$(x-1)^2 + (y-2)^2 = 25$
	(iv)	$y = \frac{3}{4}x - 5$
11		$\frac{25}{4}\text{ units}^2$

ZHONGHUA SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2018
SECONDARY 4E/5N

Candidate's Name	Class	Register Number
Marking Scheme		

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PAPER 2

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For Examiner's Use
100

Setter: Mrs Koh SH
Vetted by: Mrs See YN, Mr Poh WB

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where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!\,r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1.$$

$$\sec^2 A = 1 + \tan^2 A.$$

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$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

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$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

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Answer all the questions

Given that

1.	(i)	Given that $u = 4^x$, express $4^x = 9 - 5 \times 4^{1-x}$ as a quadratic equation in u.	[2]
	(ii)	Hence find the values of x for which $4^x = 9 - 5 \times 4^{1-x}$, giving your answer, where appropriate, to 1 decimal place.	[4]
	(iii)	Determine the values of k for which $4^x = k - 5 \times 4^{1-x}$ has no solution.	[3]

1	Solutions	Remarks
(i)	(i) $u = 9 - 5 \times \frac{4}{u}$	M1
[2]	$u^2 - 9u + 20 = 0$	A1
(ii)	(ii) $(u-4)(u-5) = 0$	M1
[4]	$u = 4$ or $u = 5$	
	$4^x = 4$ or $4^x = 5$	
	$x = 1$ A1 or $x \lg 4 = \lg 5$	M1 taking lg
	$x = \frac{\lg 5}{\lg 4} = 1.16$	A1
(iii)	(iii) $u = k - \frac{5 \times 4}{u}$	
[3]	$u^2 - ku + 20 = 0$	
	For no real roots, $(-k)^2 - 4(1)(20) < 0$	B1
	$(k - \sqrt{80})(k + \sqrt{80}) < 0$	M1
	$-\sqrt{80} < k < \sqrt{80}$	A1

2.	(i)	By using long division, divide $2x^4+5x^3-8x^2-8x+3$ by x^2+3x-1.	[2]

2	(i)	$2x^2 - x - 3$	M1 A1
	[2]	$x^2+3x-1 \overline{)\ 2x^4+5x^3-8x^2-8x+3}$	
		$-\ (2x^4+6x^3-2x^2)$	
		$-x^3-6x^2-8x$	
		$-(-x^3-3x^2+x)$	
		$-3x^2-9x+3$	
		$-(-3x^2-9x+3)$	
		0	

	(ii)	Factorise $2x^4+5x^3-8x^2-8x+3$ completely.	[2]

2	(ii)	$2x^4+5x^3-8x^2-8x+3 = (x^2+3x-1)(2x^2-x-3)$	B1
[2]		$= (x^2+3x-1)(2x-3)(x+1)$	A1

	(iii)	Hence find the exact solutions to the equation $32p^4+40p^3-32p^2-16p+3=0.$	[4]

2	(iii)	Let $x=2p$	
[4]		$2(2p)^4+5(2p)^3-8(2p)^2-8(2p)+3=0$	
		$((2p)^2+3(2p)-1)(2(2p)-3)(2p+1)=0$ either B1	
		$(4p^2+6p-1)(4p-3)(2p+1)=0$ or	
		$(4p^2+6p-1)=0$ or $(4p-3)=0$ or $(2p+1)=0$	
		$p=\dfrac{-6\pm\sqrt{36-4(4)(-1)}}{2(4)}$ M1 $p=\dfrac{3}{4}$ or $p=-\dfrac{1}{2}$ [A1 for both ans]	
		$=\dfrac{-3\pm\sqrt{13}}{4}$ A1	

3. The roots of the quadratic equation $8x^2 - 4x + 1 = 0$ are $\frac{1}{\alpha^2\beta}$ and $\frac{1}{\alpha\beta^2}$. Find a quadratic equation with roots α^3 and β^3 .	[7]

3. [7]

$$\frac{1}{\alpha^2\beta} + \frac{1}{\alpha\beta^2} = \frac{1}{2} \text{ ---- (1)}$$

$$\frac{1}{\alpha^3\beta^3} = \frac{1}{8} \text{ --------(2)}$$

B1

From (2), $\alpha\beta = \sqrt[3]{8} = 2$ B1

From (1), $\frac{\beta+\alpha}{\alpha^2\beta^2} = \frac{1}{2}$

$\alpha + \beta = \frac{1}{2} \times 4$

$= 2$ B1

$\alpha^3 + \beta^3 = (\alpha+\beta)(\alpha^2 - \alpha\beta + \beta^2)$ B1

$= (\alpha+\beta)\left[(\alpha+\beta)^2 - 3\alpha\beta)\right]$

B1

$= 2\left[2^2 - 3\times 2)\right]$

$= -4$ B1

$\alpha^3\beta^3 = 8$

Equation is $x^2 + 4x + 8 = 0$ A1

4. (i) Write down the general term in the in the binomial expansion of $\left(2x^2 - \frac{p}{x}\right)^{10}$.	[1]

4 [1] (i) General term $= \binom{10}{r}(2x^2)^{10-r}\left(-\frac{p}{x}\right)^r$ A1

	(ii)	Given that the coefficient of x^8 in the expansion of $\left(2x^2 - \frac{p}{x}\right)^{10}$ is negative $\frac{10}{3}$ times the coefficient of x^5. Show that the value of p is $\frac{1}{2}$.	[5]

4 (ii) [5]

For x^8, $x^{20-2r-r} = x^8$,

$20 - 3r = 8$

$r = 4$ $\qquad$ x^{20-3r} seen or any method (M1)

For x^5, $x^{20-2r-r} = x^5$,

$20 - 3r = 5$

$r = 5$ $\qquad$ A1 for any correct value of r

$$\binom{10}{4}(2)^{10-4}\left(-\frac{1}{2}\right)^4 = -\frac{10}{3}\binom{10}{5}(2)^{10-5}\left(-\frac{1}{2}\right)^5$$

B1 $\qquad$ B1

$$\frac{\binom{10}{4}2^6}{\binom{10}{5}2^5} \times \frac{3}{10} = p \quad \text{M1}$$

$p = \frac{1}{2}$ AG

4	(iii)	Showing all your working, use the value of p found in part (i), find the constant term in the expansion of $(2x-1)\left(2x^2-\frac{p}{x}\right)^{10}$.	[5]

4 (iii) [5] $\left(2x^2-\frac{1}{2x}\right)^{10}$

For x^0, $20-3r=0$

$r=\frac{20}{3}$ (not an integer)

No constant term in $\left(2x^2-\frac{1}{2x}\right)^{10}$ — B1

4(ii) For x^{-1}, $20-3r=-1$

$r=7$ — M1

$$(2x+1)\left(\binom{10}{7}(2x^2)^3\left(-\frac{1}{2x}\right)^7+\ldots\ldots\right)$$

B1

constant term $= 2x\binom{10}{7}(2x^2)^3\left(-\frac{1}{2x}\right)^7$ M1

$= -15$ A1

5.(a)	(i)	Show that $\sin 3x = \sin x(4\cos^2 x-1)$	[3]

5 (a) (i) [3] LHS $=\sin(x+2x)$ Addition formula M1

$=\sin x\cos 2x+\cos x\sin 2x$

$=\sin x(2\cos^2 x-1)+\cos x\times 2\sin x\cos x$

using $\cos 2x = 2\cos^2 x-1$ or $\sin 2x = 2\sin x\cos x$ B1

$=\sin x\left(2\cos^2 x-1+2\cos^2 x\right)$

Factorisation B1

$=\sin x\left(4\cos^2 x-1\right)$

	(ii)	Solve the equation $3\sin 3x = 16\cos x\sin x$ for $0 \le x \le 2\pi$	[5]

5(a) (ii) [5] $3\sin 3x = 16\cos x\sin x$

$3\sin x\left(4\cos^2 x - 1\right) = 16\cos x\sin x$

$\sin x\left(12\cos^2 x - 16\cos x - 3\right) = 0$ factorisation with $\sin x$ seen M1

$\sin x\left(6\cos x + 1\right)\left(2\cos x - 3\right) = 0$ correct factorisation of quad exp B1

$\sin x = 0$ or $\cos x = -\frac{1}{6}$ or $\cos x = \frac{3}{2}$(rejected) A1

$x = 0,\ \pi,\ 2\pi$ or $x = \pi - 1.40335,\ \pi + 1.40335$

$= 1.74$ or 4.54

A1 A1

5(b)	Differentiate $\cos 2x\,(\tan^2 x - 1)$ with respect to x. No simplification is required	[3]

5(b) [3] $\frac{\mathrm{d}}{\mathrm{d}x}\left[\cos 2x\left(\tan^2 x - 1\right)\right]$

$= \cos 2x\left(2\tan x\sec^2 x\right) + \left(\tan^2 x - 1\right)\left(-2\sin 2x\right)$ M1 product rule

B1 B1

$= 2\cos 2x\tan x\sec^2 x - 2\sin 2x\left(\tan^2 x - 1\right)$

6	The equation of a curve is $y = x^3 - 4x^2 + px + q$ where p and q are constants. The		
	equation of the tangent to the curve at the point $A(-1, 5)$ is $15x - y + 20 = 0$.		
	(i)	Find the values of p and of q.	[4]

6 (i) [4] $\frac{dy}{dx} = 3x^2 - 8x + p$ B1

At $A(-1, 5)$, equation of the tangent is $y = 15x + 20$

gradient = 15

$3(-1)^2 - 8(-1) + p = 15$ M1

$11 + p = 15$

$p = 4$ A1

substitute $p = 4$, $x = -1$, $y = 5$ into equation of curve

$5 = -1 - 4 - 4 + q$

$q = 14$ A1

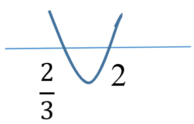
	(ii)	Determine the values of x for which y is an increasing function.	[3]

6(ii) [3] For y to be an increasing function,

$\frac{dy}{dx} > 0$

$3x^2 - 8x + 4 > 0$ B1(with value of p substituted)

$(3x - 2)(x - 2) > 0$ M1



$x < \frac{2}{3}$ or $x > 2$ A1

6	(iii)	Find range of values of x for which the gradient is decreasing.	[2]

6(iii) [2] For decreasing gradient,

$\frac{d^2y}{dx^2} < 0$ either

or M1

$6x - 8 < 0$

$x < \frac{4}{3}$ A1

6	(iv)	A point P moves along the curve in such a way that the x-coordinate of P increases at a constant rate of 0.02 units per second. Find the possible x-coordinates of P at the instant that the y-coordinate of P is increasing at 1.9 units per second.	[4]

6(*iv*) [4]

$$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$$

$$1.9 = \frac{dy}{dx} \times (0.02) \quad \text{M1}$$

$$\frac{dy}{dx} = \frac{1.9}{0.02}$$

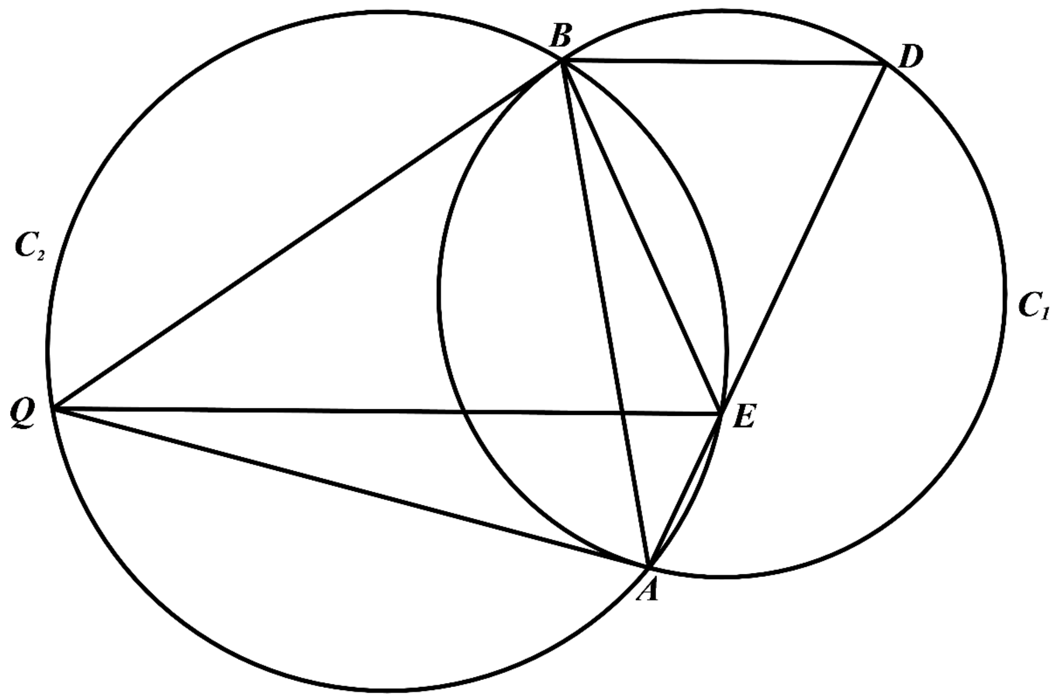
$$= 95$$

$$3x^2 - 8x + 4 = 95 \quad \text{M1 (quadratic equation in } x\text{)}$$

$$3x^2 - 8x - 91 = 0$$

$$(3x+13)(x-7) = 0$$

$$x = -\frac{13}{3} \text{ or } x = 7 \quad \text{A2}$$

7.			
	 The diagram shows two intersecting circles, C_1 and C_2. C_1 passes through the vertices of the triangle ABD. The tangents to C_1 at A and B intersect at the point Q on C_2. A line is drawn from Q to intersect the line AD at E on C_2. Prove that		
	(i)	*QE* bisects angle *AEB*	[4]
	(i)	$EB = ED$.	[2]
	(ii)	*BD* is parallel to *QE*.	[2]

7.(*i*)[4] *Let* $\angle QEA = x^\circ$

$\angle QBA = \angle QEA$ (angles in same segment in C_2) B1

$= x^\circ$

$QB = QA$ (tangents to C_1 from external point Q) B1

$\angle QAB = \angle QBA$ (base angles of isosceles triangle) B1

$= x^\circ$

$\angle QEB = \angle QAB$ (angles in the same segment in C_2)

$= x^\circ$

$\therefore \angle QEB = \angle QEA$

Hence QE bisects angle *AEB*. B1

7(*ii*) $\angle QBA = x^\circ$ (from (i))

$\angle ADB = \angle QBA$ (angles in alternate segment in C_1) either

$= x^\circ$

$\angle AEB = 2x^\circ$ (from (i))

$\angle DBE = \angle AEB - \angle ADB$ (exterior angle of triangle *BDE*) or B1

$= 2x^\circ - x^\circ$

$= x^\circ$

$\therefore \angle ADB = \angle EDB = \angle DBE = x^\circ$ (base angles of isosceles triangle BDE) B1

Hence $EB = ED$

(iii) [2] From (i) $\angle EBD = \angle QEB = x^\circ$ B1

$\therefore \angle EBD$ and $\angle QEB$ are alternate angles of parallel lines. (alternate angles are equal) B1

BD is parallel to QE

8.	The number, N, of E. Coli bacteria increases with time, t minutes. Measured values of N and t are given in the following table.	

t	2	4	6	8	10
N	3215	3446	3693	3959	4243

	It is known that N and t are related by the equation $N = N_o\,(2)^{kt}$, where N_o and k are constants.		
	(i)	Plot lg N against t and draw a straight line graph. The vertical lg N axis should start at 3.40 and have a scale of 2 cm to 0.02.	[3]
	(ii)	Use your graph to estimate the values of N_o and k.	[4]
	(iii)	Estimate the time taken for the number of bacteria to increase by 25%.	[2]

8. (i) [3] On graph paper

8(ii) [4] $N = N_o\left(2\right)^{kt}$

$\lg N = \lg N_o + kt\lg 2$

$\lg N$-intercept = 3.476 M1

$\lg N_o = 3.476$

$N_o = 2992$ accept also 2990 A1

gradient = $\dfrac{3552-3476}{5-0}$ M1(with points used to find gradient labelled on graph)

= 0.0152

$k\lg 2 = 0.0152$

$k = \dfrac{0.0152}{\lg 2}$

= 0.05 A1

(iii) [2] when $N = 125\%$ of 2992

= 3740 (to 4 sf)

$\lg N = \lg 3740$

= 3.573(M1)

From graph, time taken= 6.4 mins A1

9.	A man was driving along a straight road, towards a traffic light junction. When he saw that the traffic light had turned amber, he applied the brakes to his car and it came to a stop just before the traffic light junction. The velocity, v m/s, of the car after he applied the brakes is given by $v = 40e^{-\frac{1}{3}t} - 15$, where t is the time after he applied the brakes, is measured in seconds.		
	(i)	Calculate the initial acceleration of the car.	[2]
	(ii)	Calculate the time taken to stop the car.	[3]
	(iii)	Obtain an expression, in term of t , for the displacement of the car, t seconds after the brakes has been applied.	[3]
	(iv)	Calculate the braking distance.	[1]

9 [9]

(i) $v = 40e^{-\frac{1}{3}t} - 15$

$a = \frac{dv}{dt} = -\frac{40}{3}e^{-\frac{1}{3}t}$ B1

Initial acceleration = $-\frac{40}{3}$ m/s^2 A1

(ii) when $v = 0$

$40e^{-\frac{1}{3}t} - 15 = 0$ M1

$e^{-\frac{1}{3}t} = \frac{3}{8}$

$-\frac{t}{3} = \ln\frac{3}{8}$ (M1 taking logarithm)

$t = -3\ln\frac{3}{8}$

= 2.94s (A1)

(iii) $s = \int\left(40e^{-\frac{1}{3}t} - 15\right)dt$ M1

$= -120e^{-\frac{1}{3}t} - 15t + c$ B1

when t = 0, s = 0, where s is the displacement from the point where the brakes was applied.

$c = 120$

$s = -120e^{-\frac{1}{3}t} - 15t + 120$ A1

(iv) Substitute $t = -3\ln\frac{3}{8}$, Braking distance = $-120\left(\frac{3}{8}\right) - 15\left(-3\ln\frac{3}{8}\right) + 120$

= 30.9m (to 3 sf) A1

10.	The points $P(4,6)$, $Q(-3,5)$ and $R(4,-2)$ lie on a circle.		
	(i)	Find the equation of the perpendicular bisector of *PQ*.	[3]
	(ii)	Show that the centre of the circle is $(1,2)$ and find the radius of the circle.	[3]
	(iii)	State the equation of the circle.	[1]
	(iv)	Find the equation of the tangent to the circle at *R*.	[3]

10. [10] (i) midpoint of $PQ = \left(\frac{1}{2}, \frac{11}{2}\right)$ B1

gradient of $PQ = \frac{1}{7}$

gradient of perpendicular bisector of $PQ = -7$ B1

Equation of perpendicular bisector of PQ is

$$y - \frac{11}{2} = -7\left(x - \frac{1}{2}\right)$$

$y = -7x + 9$ A1

(ii) Equation of perpendicular bisector of PR is $y = 2$

B1

Alternatively use :Equation of perpendicular bisector of QR is $y = x + 1$

Since perpendicular bisector of chords passes through centre of circle,

for centre of circle, substitute $y = 2$ into $y = -7x + 9$

$2 = -7x + 9$ M1 solving simultaneous equations

$7x = 7$

$x = 1$

centre = (1, 2) AG

Alternative method : centre = $(a,\ -7a + 9)$ B1

$RC = PC$ M1 forming an equation in a

r = distance between centre and P

$= \sqrt{(4-1)^2 + (6-2)^2}$

= 5 units A1

(iii) Equation of circle is $(x-1)^2 + (y-2)^2 = 25$ A1

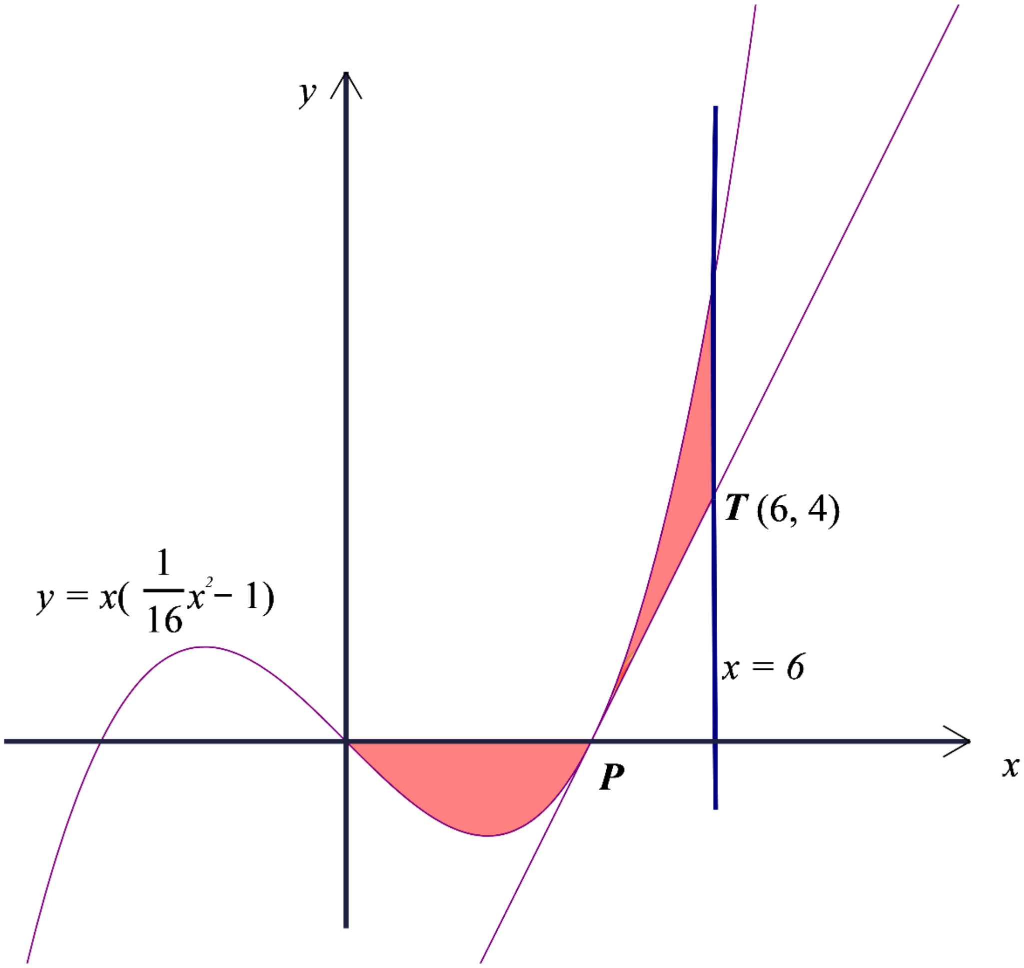
(iv) gradient of normal at $R = \frac{2-(-2)}{1-4} = -\frac{4}{3}$ M1

gradient of tangent at $R = \frac{3}{4}$ M1

Equation of tangent at R is $y + 2 = \frac{3}{4}(x-4)$

$y = \frac{3}{4}x - 5$ A1

11.	The diagram shows part of the curve $y = x\left(\frac{1}{16}x^2 - 1\right)$. The curve cuts the x-axis at P(4, 0). The tangent to the curve at P meets the vertical line $x = 6$ at $T(6, 4)$. Showing all your workings, find the total area of the shaded regions.	[6]



Area of total shaded regions = $-\int_0^4 \left(\frac{x^3}{16} - x\right) dx + \int_4^6 \left(\frac{x^3}{16} - x\right) dx - \frac{1}{2} \times 2 \times 4$

B1 B1 B1

$= \left[-\frac{1}{16} \times \frac{x^4}{4} + \frac{x^2}{2}\right]_0^4 + \left[\frac{1}{16} \times \frac{x^4}{4} - \frac{x^2}{2}\right]_4^6 - 4$ M1 correct integration

$= -\frac{1}{64} \times 4^4 + \frac{1}{2} \times 4^2 + \left(\frac{6^4}{64} - \frac{6^2}{2}\right) - \left(\frac{4^4}{64} - \frac{4^2}{2}\right) - 4$

M1 correct substitution of upper and lower limits

$= \frac{25}{4}$ units^2 A1

End of paper